Higher Grade Prelim 2001/2002 **Marking Scheme - Paper I**

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	**ans: p = -3 , q = 8 : x = 4** **5 marks** • 1 Setting up synth. division • 2 Obtaining first equation • 3 Obtaining second equation • 4 Solving system for p and q • 5 Sub. (say p in quotient) for 3rd root	• 1 1 | 1 p -6 q • 2 $p+q=5$ • 3 $4p+q=-4$ • 4 $p=-3$, $q=8$ • 5 $x^2-2x-8=0 \Rightarrow (x+2)(x-4)=0$ $x=4$ is missing root
2.	**(a) ans: statement** **1 mark** • 1 Explanation **(b) ans: 20** **2 marks** • 1 Method used • 2 For calculating limit **(c) ans: $U_0 = 15$** **3 marks** • 1 For initial equating and finding U_1 • 2 For recurrence with U_1 in place • 3 For answer	(a) • 1 Because $-1 < a < 1$ (or equiv.) (b) • 1 $L = \frac{b}{1-a}$ • 2 $L = \frac{8}{1-0\cdot6} = \frac{8}{0\cdot4} = \frac{80}{4} = 20$ (c) • 1 $20 - U_1 = 3 \ \therefore \ U_1 = 17$ • 2 $17 = 0\cdot6U_0 + 8$ • 3 $9 = 0\cdot6U_0 \Rightarrow U_0 = \frac{9}{0\cdot6} = \frac{90}{6}$ $U_0 = 15$
3.	**(a) ans: $f'(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2x^{\frac{1}{2}}}$** **4 marks** • 1 For dealing with fraction & expanding • 2 For first term differentiated • 3 For 2nd term diff. • 4 For writing with +ve indices **(b) ans: $2\frac{3}{4}$** **2 marks** • 1 For substituting • 2 For calculating answer	(a) • 1 $x^{-\frac{1}{2}}(x^2 - x) \Rightarrow x^{\frac{3}{2}} - x^{\frac{1}{2}}$ • 2 $\frac{3}{2}x^{\frac{1}{2}}$ ..... • 3 ..... $- \frac{1}{2}x^{-\frac{1}{2}}$ • 4 $f'(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2x^{\frac{1}{2}}}$ (b) • 1 $f'(4) = \frac{3}{2}\sqrt{4} - \frac{1}{2\sqrt{4}}$ (or equiv.) • 2 $f'(4) = 2\frac{3}{4}$
4.	**ans: $y = 3x - 16$** **5 marks** • 1 For points A & B • 2 Gradient of AB • 3 Gradient of perpendicular • 4 For mid-point of AB • 5 For equation of perpen. bisector	• 1 A(12,0) , B(0,4) • 2 $m_{AB} = -\frac{1}{3}$ • 3 $m_{per.} = 3$ • 4 M(6,2) • 5 $y - 2 = 3(x - 6)$
5.	**(a) ans: $f(g(2)) = -2$** **1 mark** • 1 For answer **(b) ans: $g(f(x)) = 8 - x$** **2 marks** • 1 For substitution • 2 For simplifying	(a) • 1 $f(g(2)) = -2$ (b) • 1 $g(8-2x) = \frac{1}{2}\big((8-2x)+8\big)$ (or equiv.) • 2 $g(f(x)) = \frac{1}{2}(16-2x)$ $= 8 - x$

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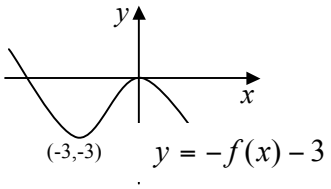
5.	**(c) ans: proof** **3 marks** • 1 For knowing how to find inverse • 2 For finding inverse • 3 For final statement (equating)	• 1 *To find* $f^{-1}(x) \Rightarrow y = 8 - 2x$ • 2 $2x = 8 - y \Rightarrow x = \frac{1}{2}(8 - y)$ • 3 $f^{-1}(x) = \frac{1}{2}(8 - x) = \frac{1}{2}[g(f(x))]$
6.	**(a) ans: C(2,5)** **1 mark** • 1 For answer **(b) ans: proof** **3 marks** • 1 For gradient of CM • 2 For perp. gradient of chord AB • 3 For equ. of chord and rearranging **(c) ans: A(-1,4) , B(3,2)** **4 marks** • 1 For attempting to solve a system • 2 For expanding and simplifying • 3 For factorising and finding *y* coords. • 4 For completing points	(a) • 1 C(2,5) (b) • 1 $M_{cm} = \frac{5-3}{2-1} = 2$ • 2 $M_{AB} = -\frac{1}{2}$ • 3 $y - 3 = -\frac{1}{2}(x - 1)$ $2y - 6 = -x + 1$ $x = 7 - 2y$ (c) • 1 $(7 - 2y)^2 + y^2 - 4(7 - 2y) - 10y + 19 = 0$ • 2 $5y^2 - 30y + 40 = 0$ • 3 $(5(y - 4)(y - 2) - 0 \ \therefore \ y = 4 , y = 2$ • 4 $y = 4$ *then* $x = -1$, $y = 2$ *when* $x = 3$
7.	**ans:**  **3 marks**	• 1 For reflection in *x*-axis • 2 For translation (3-down) • 3 For (-3,-3) marked [also (3,-3) ?]
8.	**ans: a = 2** **5 marks** • 1 For integrating • 2 For sub. limits • 3 Simplifying to quadratic • 4 discarding other root • 5 answer	• 1 $\left[10x - x^2\right]_a^{2a} = 8$ • 2 $(20a - 4a^2) - (10a - a^2) = 8$ • 3 $3a^2 - 10a + 8 = 0$ • 4 $a = 2$ *or* ~~$a = \frac{4}{3}$~~ discard • 5 $a = 2$
9.	**(a) ans:** $y = -x + 3$ **4 marks** • 1 Completing point of tangency • 2 Differentiating to find *m* • 3 Finding gradient of tangent • 4 Point + *m* in equation **(b) ans: proof** **2 marks** • 1 Finding where tan. cuts *x*-axis • 2 Showing that this point satisfies equation of curve	(a) • 1 $y = 1(1 - 5 + 6) = 2 \ \therefore \ T(1,2)$ • 2 $\frac{dy}{dx} = m = 3x^2 - 10x + 6$ • 3 $m = 3(1^2) - 10(1) + 6 = -1$ • 4 $y - 2 = -1(x - 1)$ various methods ..... (b) • 1 When $y = 0$ then $x = 3$ • 2 Sub $x = 3$ in equ. of curve $y = 3(3^2 - 5(3) + 6) = 3(0) = 0$

Total 50 marks

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	**(a) ans:** $y = x - 2$, $y = -x + 4$ **4 marks** • 1 For gradient of AC • 2 For equation of AC • 3 For finding gradient of altitude • 4 For equation of altitude **(b) ans:** P(3,1) **3 marks** • 1 Knowing to solve a system • 2 Finding first coordinate • 3 Finding second coord. **(c) ans:** D(6,-2) **1 mark** • 1 Answer **(d) ans:** 18° **3 marks** • 1 For knowing and using $\tan\theta = m$ • 2 For angle between CD and horoz. • 3 For 45^o and subtraction to ans.	(a) • 1 $m_{AC} = \frac{3+8}{5+6} = 1$ • 2 $y - 3 = 1(x - 5)$ • 3 $m_{alt} = -1$ • 4 $y - 7 = -1(x + 3)$ (b) • 1 $x - 2 = -x + 4$ • 2 $2x = 6 \Rightarrow x = 3$ • 3 $y = 3 - 2 = 1$ (c) • 1 $\rightarrow 6 \downarrow 6 \therefore \rightarrow 3 \downarrow 3$ from P, D(6,-2) (d) • 1 $m_{AC} = 1 \therefore \tan^{-1} 1 = \theta = 45^\circ$ • 2 $m_{CD} = \frac{-2+8}{6+6} = 0{\cdot}5 \therefore \tan^{-1} 0{\cdot}5 = 26{\cdot}6^\circ$ • 3 $45 - 26{\cdot}6 = 18^\circ$
2.	**ans:** $\{19{\cdot}5\,,160{\cdot}5\,,210\,,330\}$ **5 marks** • 1 For correct substitution • 2 For re-arranging to quadratic • 3 Factorising to two roots • 4 Two ans. from one root • 5 Two ans. from second root	• 1 $\sin x - 3(1 - 2\sin^2 x) + 2 = 0$ • 2 $6\sin^2 x + \sin x - 1 = 0$ • 3 $\sin x = \frac{1}{3}$ *or* $\sin x = -\frac{1}{2}$. • 4 $19{\cdot}5^o$, $160{\cdot}5^o$ • 5 210^o , 330^o
3.	**(a) ans:** proof **3 marks** • 1 For expanding original functions • 2 For correct substitution • 3 For expanding to answer **(b) ans:** x = 1 , proof **4 marks** • 1 Knowing to use synthetic division • 2 Finding the root , $x = 1$ • 3 Using x = 1 again ! • 4 Showing remaining quotient has no roots	(a) • 1 $f(x) = x^2 + 3x + 2$, $g(x) = x^2 - 2x$ • 2 $h(x) = (x^2 - 2x)^2 + 3(x^2 - 2x) + 2$ • 3 $h(x) = x^4 - 4x^3 + 4x^2 + 3x^2 - 6x + 2$ (b) • 1 $\begin{array}{c\|ccccc} & 1 & -4 & 7 & -6 & 2 \\ \hline \end{array}$ • 2 $\begin{array}{c\|ccccc} & 1 & -4 & 7 & -6 & 2 \\ 1 & & 1 & -3 & 4 & -2 \\ \hline & 1 & -3 & 4 & -2 & 0 \end{array}$ • 3 1 again leaves quot. $x^2 - 2x + 2$ • 4 for $b^2 - 4ac = -4 \therefore$ no more roots
4.	**(a) ans:** $C_1(-8,-6)$, $C_2(16,4)$ **2 marks** • 1 For first centre • 2 For second centre **(b) ans:** $r_1 = 10$, $r_2 = 10$, $d = 26$ **4 marks** • 1 Finding r of C_1 • 2 Finding r of C_2 • 3 For method (dist. form , pyth, etc.) • 4 For correct distance	(a) • 1 $C_1(-8,-6)$ • 2 $C_2(16,4)$ (b) • 1 $r = \sqrt{(-8)^2 + (-6)^2 - 0} = \sqrt{100} = 10$ • 2 $r = \sqrt{100} = 10$ • 3 $d = \sqrt{(x_2 - x_1)^2 \ .....}$ etc. • 4 $d = \sqrt{676} = 26$

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4.	**(c) ans:** $C_3(4,-1)$, $r = 3$ **3 marks** $(x-4)^2 + (y+1)^2 = 9$ • 1 For centre • 2 For radius • 3 For sub. into equ. to answer	(c) • 1 Centre must be mid-pt $C_3(4,-1)$ • 2 $r = (26-20) \div 2 = 3$ • 3 $(x-4)^2 + (y+1)^2 = 9$ or $x^2 + y^2 - 8x + 2y + 8 = 0$
5.	**(a) ans:** $U_5 \approx 14 \cdot 402$ Ah **3 marks** • 1 knowing to set-up recurrence • 2 correct a and b • 3 calculations and answer **(b) ans:** **3 marks** • 1 Knowing how to find limit • 2 Calculating limit • 3 Any sensible statement	(a) • 1 $U_1 = aU_0 + b$ • 2 $U_1 = 0 \cdot 76(U_0) + 3$ • 3 $U_5 \approx 14 \cdot 402$ (b) • 1 $L = \dfrac{b}{1-a}$ (or equivalent) • 2 $L = \dfrac{3}{1-0 \cdot 76} = 12 \cdot 5$ • 3 Lifespan good since $12 \cdot 51$ very close to limit. (more than 2 years) *Some students will calculate lifespan using an advanced calculator*
6.	**(a) ans:** P(1,-2) , R(3,0) **6 marks** • 1 Preparing to differentiate • 2 Knowing to solve deriv. to zero • 3 Differentiating • 4 Solving to answer for x coord. of P • 5 Finding y coord. of P • 6 Finding root (coords. of R) **(b) ans:** Area = $2 \cdot 55$ units^2 **5 marks** • 1 For setting up correct integral • 2 For integrating first term • 3 Integrating 2nd term • 4 Substituting limits • 5 Calculations to answer	(a) • 1 $f(x) = x^{\frac{3}{2}} - 3x^{\frac{1}{2}}$ • 2 S.P. when $f'(x) = 0$ (stated or impl.) • 3 $f'(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{1}{2}}$ (or equivalent) • 4 $\dfrac{3\sqrt{x}}{2} - \dfrac{3}{2\sqrt{x}} = 0$ $(\times\ 2\sqrt{x})$ $3x - 3 = 0 \ \ \therefore\ x = 1$ • 5 $y = \sqrt{1}(1-3) = -2$ • 6 $x - 3 = 0$, $x = 3$ (b) • 1 $A = -\int_1^3 x^{\frac{3}{2}} - 3x^{\frac{1}{2}}\ dx$ • 2 ..... $\frac{2}{5}x^{\frac{5}{2}}$ (or equivalent) • 3 ..... $2x^{\frac{3}{2}}$ (or equivalent) • 4 $A = -\left[\frac{2}{5}(3^{\frac{5}{2}}) - 2(3^{\frac{3}{2}})\right] - \left[\frac{2}{5} - 2\right]$ • 5 $A = -[6 \cdot 24 - 10 \cdot 39] - [-1 \cdot 6] = 2 \cdot 56$
7.	**(a) ans:** proof **4 marks** • 1 For discriminant statement • 2 Correct a, b and c • 3 Substituting and simplifying • 4 Statement regarding a perfect square **(b) ans:** $k = -\frac{3}{2}$, $x = 1$ **4 marks** • 1 For solving to zero and value of k • 2 Substituting k in quadratic • 3 Simplifying quadratic • 4 Solving for x	(a) • 1 For real roots $b^2 - 4ac \geq 0$ • 2 $a = \frac{3}{4}$, $b = -(k+3)$, $c = -(k^2 + 2k)$ • 3 $(k+3)^2 + 3(k^2 + 2k) \geq 0$ $4k^2 + 12k + 9 \geq 0$ • 4 $b^2 - 4ac = (2k+3)^2$ *never* $-ve$, etc. (b) • 1 $2k + 3 = 0 \ \ \therefore\ k = -\frac{3}{2}$ • 2 $\frac{3}{4}x^2 - (-\frac{3}{2} + 3)x - (\frac{9}{4} - 3) = 0$ • 3 $\frac{3}{4}x^2 - \frac{3}{2}x + \frac{3}{4} = 0 \Rightarrow 3x^2 - 6x + 3 = 0$ • 4 $3(x^2 - 2x + 1) = 3(x-1)(x-1) = 0$ $\therefore\ x = 1$

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8.	**(a) ans:** proof **4 marks** • 1 For attempting to use pythagoras • 2 For length x • 3 For length $(4-x)$ • 4 For expansion to answer **(b) ans:** $x = 2$, $OP_{min} = \sqrt{8}$ **4 marks** • 1 For removing common factor • 2 Completing the square with $x^2 - 4x$ • 3 Tidying to final form • 4 Answer for replacement and minimum *(discretion for minimum* 8 *instead of* $\sqrt{8}$*)*	(a) • 1 $OP^2 = a^2 + b^2$ (stated or implied) • 2 $OP^2 = x^2 + ...$ • 3 $OP^2 = ... + (4-x)^2$ • 4 $OP^2 = x^2 + 16 - 8x + x^2$ $= 2x^2 - 8x + 16$ (b) • 1 $2(x^2 - 4x) + 16$ • 2 $[(x-2)^2 - 4]$ • 3 $OP^2 = 2(x-2)^2 + 8$ • 4 minimum when x = 2 minimum value of $OP^2 = 8$ $\therefore OP_{min} = \sqrt{8}$ *NB... Pupils may use differentiation to answer this part of the question .... assign marks accordingly.*
		Total 65 marks