SCOTTISH
QUALIFICATIONS
AUTHORITY

2003 Mathematics

Advanced Higher

Finalised Marking Instructions

2003 Mathematics

Advanced Higher – Section A

Finalised Marking Instructions

Advanced Higher 2003: Section A Solutions and marks

A1. (a) Given $f(x) = x(1+x)^{10}$, then

$$f'(x) = (1+x)^{10} + x.10(1+x)^9 \qquad \mathbf{1,1}$$

$$= (1+11x)(1+x)^{9\,*} \qquad \mathbf{1}$$

(b) Given $y = 3^x$, then

$$\ln y = x \ln 3 \qquad \mathbf{1}$$

$$\frac{1}{y}\frac{dy}{dx} = \ln 3 \qquad \mathbf{1}$$

$$\frac{dy}{dx} = \ln 3\, y = \ln 3 \,.\, 3^x. \qquad \mathbf{1}$$

A2.

$$S_n = \sum_{k=1}^{n} u_k = \sum_{k=1}^{n} (11 - 2k)$$

$$= \sum_{k=1}^{n} 11 - 2\sum_{k=1}^{n} k \qquad \mathbf{1}$$

$$= 11n - 2 \times \tfrac{1}{2}n(n+1) \qquad \mathbf{1,1}$$

$$= -n^2 + 10n.$$

$$-n^2 + 10n = 21 \qquad \mathbf{1}$$

$$(n-3)(n-7) = 0$$

The sum is 21 when there are 3 terms and when there are 7 terms.† **1**

Alternative for first 2/3 marks

Using results for Arithmetic Series.

$$a = 9,\ d = -2 \qquad \mathbf{1}$$

$$S_n = \frac{n}{2}[18 + (n-1)(-2)] \qquad \mathbf{1}$$

A3.

$$y^3 + 3xy = 3x^2 - 5$$

$$3y^2\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 6x \qquad \mathbf{1,1}$$

$$\frac{dy}{dx} = \frac{6x - 3y}{3y^2 + 3x} = \frac{2x - y}{y^2 + x}$$

Thus at (2, 1), the gradient is 1 **1**

and an equation is $(y - 1) = 1(x - 2)$. **1**

i.e. $x = y + 1$ or $y = x - 1$.

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* The $(1+x)^9$ must be pulled out.

† If trial and error is used, both values are needed for the last mark.

A4. Let $z = x + iy$.

$$z + i = (x + iy) + i = x + (1 + y)i \qquad \mathbf{1}$$

$$\therefore \quad |z + i| = \sqrt{x^2 + (1 + y)^2} \qquad \mathbf{1}$$

$$\therefore \quad x^2 + (1 + y)^2 = 4$$

which is a circle, centre $(0, -1)$ radius 2. **1**

(The centre could be given as $-i$.)

A5.

$$x = 1 + \sin\theta$$

$$dx = \cos\theta \, d\theta \qquad \mathbf{1}$$

$$\theta = 0 \Rightarrow x = 1; \; \theta = \pi/2 \Rightarrow x = 2 \qquad \mathbf{1}$$

$$\int_0^{\pi/2} \frac{\cos\theta}{(1 + \sin\theta)^3} \, d\theta = \int_1^2 \frac{1}{x^3} \, dx \qquad \mathbf{1}$$

$$= \int_1^2 x^{-3} \, dx$$

$$= \left[\frac{x^{-2}}{-2} \right]_1^2 \qquad \mathbf{1}$$

$$= \left[\frac{-1}{8} - \frac{-1}{2} \right] \left(= \frac{3}{8} \right)^* \qquad \mathbf{1}$$

A6.

$$x + y + 3z = 1$$
$$3x + ay + z = 1$$
$$x + y + z = -1.$$

Hence

$$x + y + 3z = 1$$

$$(a - 3)y - 8z = -2$$

$$-2z = -2 \qquad \mathbf{2}^\dagger$$

When $a \neq 3$, we can solve to give a unique solution.

$$z = 1; \qquad y = \frac{6}{a - 3}; \qquad x = -2 + \frac{6}{3 - a}. \qquad \mathbf{2E1}$$

When $a = 3$, we get $z = \frac{1}{4}$ from the second equation but $z = 1$‡ from the third, i.e. inconsistent§ . **2**

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* optional

† 1 off for lower triangular form

‡ 1 for identifying the two values for z

§ 1 for conclusion

A7.

$$f(x) = \frac{x}{1 + x^2}$$

$$f'(x) = \frac{(1 + x^2) - 2x^2}{(1 + x^2)^2}$$ **1,1**

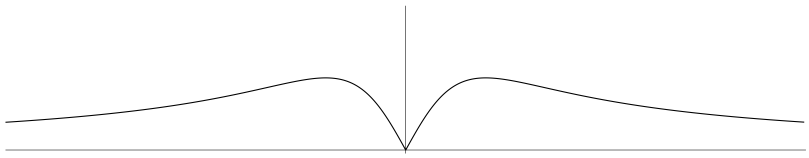
$$= \frac{1 - x^2}{(1 + x^2)^2}$$

$f'(x) = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1.$ **1**

The graph of $f(x)$ has two stationary values:

$(1, \tfrac{1}{2})$ and $(-1, -\tfrac{1}{2})$ **1**

and passes through $(0, 0)$.



1

Thus g has two turning points $(1, \tfrac{1}{2})$ and $(-1, \tfrac{1}{2})$ **1**

and its third critical value is $(0, 0)$ {as its gradient is discontinuous}. **1**

A8. Statement A is true: $p(n) = n(n+1)$ and one of n and $(n + 1)$ must be even. **3**

Or: n^2 and n are either both odd or both even. In either case $n^2 + n$ is even.*

Statement B is false: when $n = 1$, $n^2 + n = 2$. **1**

A9.

$$\frac{1}{w} = \frac{1}{\cos\theta + i\sin\theta} = \frac{1}{\cos\theta + i\sin\theta} \times \frac{\cos\theta - i\sin\theta}{\cos\theta - i\sin\theta} = \frac{\cos\theta - i\sin\theta}{\cos^2\theta - i^2\sin^2\theta}$$ **1**

$$= \frac{\cos\theta - i\sin\theta}{1} = \cos\theta - i\sin\theta^{\dagger}$$

$$w^k + w^{-k} = w^k + \left(w^k\right)^{-1}$$

$$= (\cos\theta + i\sin\theta)^k + \frac{1}{(\cos\theta + i\sin\theta)^k}$$ **1**

$$= \cos k\theta + i\sin k\theta + \frac{1}{\cos k\theta + i\sin k\theta}$$ **1**

$$= \cos k\theta + i\sin k\theta + \cos k\theta - i\sin k\theta$$ **1**

$$= 2\cos k\theta$$

$$\left(w + w^{-1}\right)^4 = w^4 + 4w^2 + 6 + 4w^{-2} + w^{-4}$$ **2E1**

$$(2\cos\theta)^4 = \left(w^4 + w^{-4}\right) + 4\left(w^2 + w^{-2}\right) + 6$$ **1,1**

$$16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$$ **1**

so $$\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}.$$

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* First mark needs attempt at justification; alternative proofs (e.g. induction) are acceptable

† needs justifying.

A10. (a)

$$I_1 = \int_0^1 xe^{-x}dx = \left[x\int e^{-x}dx - \int 1.\int e^{-x}dx.\,dx\right]_0^1 \qquad \textbf{2E1}$$

$$= \left[-xe^{-x} - e^{-x}\right]_0^1$$

$$= -e^{-1} - e^{-1} - (0 - 1) \qquad \textbf{1}$$

$$\left(= 1 - \frac{2}{e} = 0.264\right).^*$$

(b)

$$\int_0^1 x^n e^{-x}dx = \left[x^n\int e^{-x}dx - \int\left(nx^{n-1}\int e^{-x}dx\right)dx\right]_0^1 \qquad \textbf{3E1}$$

$$= \left[-x^n e^{-x}\right]_0^1 + \left[n\int x^{n-1}e^{-x}dx\right]_0^1$$

$$= -e^{-1} - (-0) + n\int_0^1 x^{n-1}e^{-x}dx \qquad \textbf{1}$$

$$= nI_{n-1} - e^{-1}$$

(c)

$$I_3 = 3I_2 - e^{-1} \qquad \textbf{1}$$

$$= 3(2I_1 - e^{-1}) - e^{-1} \qquad \textbf{1}$$

$$= 3(2 - 4e^{-1} - e^{-1}) - e^{-1} \qquad \textbf{1}$$

$$= 6 - 16e^{-1} \approx 0.1139.$$

A11.

$$\tfrac{dV}{dt} = V(10 - V)$$

$$\int \frac{dV}{V(10 - V)} = \int 1\,dt \qquad \textbf{1}$$

$$\frac{1}{10}\int \frac{1}{V} + \frac{1}{10 - V}\,dV = \int 1\,dt \qquad \textbf{2}$$

$$\frac{1}{10}\left(\ln V - \ln(10 - V)\right) = t + C \qquad \textbf{1}$$

$$\frac{1}{10}\ln V - \frac{1}{10}\ln(10 - V) = t + C$$

$V(0) = 5$, so

$$\tfrac{1}{10}\ln 5 - \tfrac{1}{10}\ln 5 = 0 + C$$

$$C = 0 \qquad \textbf{1}$$

$$\ln V - \ln(10 - V) = 10t$$

$$\ln\left(\frac{V}{10 - V}\right) = 10t$$

$$\frac{V}{10 - V} = e^{10t}$$

$$V = 10e^{10t} - Ve^{10t}$$

$$V\left(1 + e^{10t}\right) = 10e^{10t} \qquad \textbf{2E1}$$

$$V = \frac{10e^{10t}}{1 + e^{10t}}$$

$$V = \frac{10e^{10t}}{1 + e^{10t}} = \frac{10}{e^{-10t} + 1} \qquad \textbf{1}$$

$$\to 10 \text{ as } t \to \infty. \qquad \textbf{1}$$

[END OF MARKING INSTRUCTIONS]

* optional

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2003 Mathematics

Advanced Higher – Section B

Finalised Marking Instructions

Advanced Higher 2003: Section B Solutions and marks

B1. Let

$$\frac{x-3}{4} = \frac{y-2}{-1} = \frac{z+1}{2} = t$$

then

$$x = 3 + 4t$$

$$y = 2 - t \qquad \textbf{2E1}$$

$$z = -1 + 2t.$$

Thus

$$2(3+4t) + (2-t) - (-1+2t) = 4 \qquad \textbf{1}$$

$$9 + 5t = 4$$

$$t = -1$$

so the point is $(-1,\ 3,\ -3)$. **1**

B2.

$$A^2 = 4A - 3I$$

$$A^3 = 4A^2 - 3A \qquad \textbf{1}$$

$$= 16A - 12I - 3A$$

$$= 13A - 12I \qquad \textbf{1}$$

$$A^4 = 13A^2 - 12A \qquad \textbf{1}$$

$$= 52A - 39I - 12A$$

$$= 40A - 39I \qquad \textbf{1}$$

i.e. $p = 40, q = -39$.

Alternative

$$A^4 = \left(A^2\right)^2 \qquad \textbf{1}$$

$$= 16A^2 - 24A + 9I \qquad \textbf{1}$$

$$= 64A - 48I - 24A + 9I \qquad \textbf{1}$$

$$= 40A - 39I^{*} \qquad \textbf{1}$$

B3. Let λ represent a fixed point, then

$$\lambda = \frac{1}{2}\left\{\lambda + \frac{7}{\lambda}\right\} \qquad \textbf{1}$$

$$2\lambda = \lambda + \frac{7}{\lambda}$$

$$\lambda = \frac{7}{\lambda}$$

$$\lambda^2 = 7. \qquad \textbf{1}$$

The fixed points are $\sqrt{7}$ and $-\sqrt{7}$.† **1**

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* Using $I = 1$ at any stage costs a mark.

† For full marks, exact values are needed. Iterative method giving ±2.645 gets 2 marks.

B4.

	either	or	
$f(x)$	$= \sin^2 x$		$f(0) = 0$
$f'(x)$	$= 2\sin x \cos x$	$= \sin 2x$	$f'(0) = 0$
$f''(x)$	$= 2\cos^2 x - 2\sin^2 x$	$= 2\cos 2x$	$f''(0) = 2$
$f'''(x)$	$= -4\cos x \sin x - 4\sin x \cos x$	$= -4\sin 2x$	$f'''(0) = 0$
$f''''(x)$	$= -8\cos^2 x + 8\sin^2 x$	$= -8\cos 2x$	$f''''(0) = -8$

2E1

$$f(x) = 0 + 0.x + 2.\frac{x^2}{2} + 0.\frac{x^3}{6} - 8.\frac{x^4}{24}$$ **1**

$$= x^2 - \frac{1}{3}x^4$$ **1**

Since $\cos^2 x + \sin^2 x = 1$,

$$\cos^2 x = 1 - x^2 + \tfrac{1}{3}x^4$$ **1**

OR

$$\sin x = x - \frac{1}{3!}x^3 + \ \ldots$$ **1**

$$\therefore (\sin x)^2 = \left(x - \frac{1}{6}x^3 + \ \ldots\right)\left(x - \frac{1}{6}x^3 + \ \ldots\right)$$ **1**

$$= x^2 - 2 \times \frac{1}{6}x^4 + \ \ldots = \ \ldots$$ **1,1**

B5. (a) When $n = 1$, LHS = 0, RHS = $0 \times 1 \times 2 = 0$. Thus true when $n = 1$. **1**

Assume $\sum_{r=1}^{k} 3(r^2 - r) = (k-1)k(k+1)$ and consider the sum to $k + 1$. **1**

$$\sum_{r=1}^{k+1} 3(r^2 - r) = 3\left((k+1)^2 - (k+1)\right) + \sum_{r=1}^{k} 3(r^2 - r)$$

$$= 3(k+1)^2 - 3(k+1) + (k-1)k(k+1)$$ **1**

$$= (k+1)\left[3k + 3 - 3 + k^2 - k\right]$$

$$= (k+1)(k^2 + 2k) = k(k+1)(k+2)$$

$$\left((k+1) - 1\right)(k+1)\left((k+1) + 1\right).$$ **1**

Thus true for $k + 1$. Since true for 1, true for all $n \geqslant 1$.

(b)

$$\sum_{r=11}^{40} 3(r^2 - r) = \sum_{r=1}^{40} 3(r^2 - r) - \sum_{r=1}^{10} 3(r^2 - r)$$ **1**

$$= 39 \times 40 \times 41 - 9 \times 10 \times 11$$

$$= 63960 - 990$$

$$= 62970.$$ **1**

B6. Consider first

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0.$$

Auxiliary equations is

$$m^2 - 4m + 4 = 0 \qquad \mathbf{1}$$

$$(m - 2)^2 = 0 \qquad \mathbf{1}$$

Thus the complementary function is

$$y = (A + Bx)e^{2x}. \qquad \mathbf{1}$$

To find the particular integral, let $f(x) = ae^x$. Then **1**

$$f'(x) = ae^x \text{ and } f''(x) = ae^x. \qquad \mathbf{1}$$

$$ae^x - 4ae^x + 4ae^x = ae^x$$

$$\text{so} \qquad a = 1. \qquad \mathbf{1}$$

Therefore the general solution is

$$y = (A + Bx)e^{2x} + e^x \qquad \mathbf{1}$$

$$\frac{dy}{dx} = Be^{2x} + 2(A + Bx)e^{2x} + e^x \qquad \mathbf{1}$$

Initial conditions give

$$2 = A + 1$$

$$1 = B + 2A + 1 \qquad \mathbf{1}$$

i.e. $A = 1$ and $B = -2$. **1**

The required solution is

$$y = (1 - 2x)e^{2x} + e^x.$$

[END OF MARKING INSTRUCTIONS]

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2003 Mathematics

Advanced Higher – Section C

Finalised Marking Instructions

Advanced Higher 2003: Section C Solutions and marks

C1. P(Breast cancer | Mammogram positive)

$$= \frac{\text{P(Breast cancer and Mammogram positive)}}{\text{P(Mammogram positive)}} \qquad \mathbf{1}$$

$$= \frac{\text{P(Mammogram positive | Breast cancer)P(Breast cancer)}}{\text{P(M+ | BC).P(BC)} + \text{P(M+}|\,\overline{\text{BC}}\text{)} \,.\, \text{P(}\,\overline{\text{BC}}\text{)}} \qquad \mathbf{1}$$

$$= \frac{0{\cdot}9 \times 0{\cdot}01}{0{\cdot}9 \times 0{\cdot}01 + 0{\cdot}1 \times 0{\cdot}99} \qquad \mathbf{1}\ \ \mathbf{1}$$

$$= \frac{0{\cdot}009}{0{\cdot}108} = \frac{1}{12}\left(= 0.083\right)^{*} \qquad \mathbf{1}$$

C2. (a) $X \sim \text{Bin}(20,\ 0{\cdot}25)$ **1,1**

(b) $\text{P}(X \leqslant 3) = 0{\cdot}2252$ **1**

(c) The hypothesis p = 0.25 cannot be rejected at the 5% significance level **1**
since the probability calculated in (b) exceeds 0.05. **1**
Thus there is no evidence from the data to support the manager's belief. **1**†

C3. (a)

$$Y = \frac{5}{9}(X - 32)$$

$$\Rightarrow \text{E}(Y) = \frac{5}{9}(104 - 32) = 40 \qquad \mathbf{M1},\mathbf{1}$$

$$\text{V}(Y) = \left(\frac{5}{9}\right)^2 \times 1{\cdot}2^2 \qquad \mathbf{1}$$

$$\Rightarrow \sigma = \frac{2}{3} \qquad \mathbf{1}$$

(b) For central 95% probability, $z = 1{\cdot}96$. **1**
$\Rightarrow$ limits are $40 \pm 1{\cdot}96 \times 0{\cdot}667$‡
i.e. $(38{\cdot}7,\ 41{\cdot}3)$ **1**

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* optional
† for conclusion
‡ not sufficient, limits have to be evaluated

C4. (a)

$$\hat{p} \pm 1{\cdot}96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$ **1,1**

(b)(i) We require

$$2 \times 1{\cdot}96\sqrt{\frac{0{\cdot}3 \times 0{\cdot}7}{n}}\left(= \frac{1.8}{\sqrt{n}}\right)^{*}$$ **1,1**

(ii)

$$\frac{1.8}{\sqrt{n}} \leqslant 0.1$$ **1**

$$n \geqslant \left(\frac{1.8}{0.1}\right)^2 \Rightarrow n \geqslant 324$$ **1**

Thus a sample size of 324 or greater is required.

C5. (a)

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{530 - 502}{63/\sqrt{25}}$$

$$= 2{\cdot}22$$ **1**

$H_0 : \mu = 502$

$H_1 : \mu > 502$ **1**

The critical region is $z > 2{\cdot}33$ **1**

Since 2·22 lies outside in the critical region, the null hypothesis is accepted **1**

i.e. there is no evidence of an increase **1**

(b) p-value = $P(Z > 2{\cdot}22) = 0{\cdot}0132$ **1**

which is greater that 0·01 confirming the conclusion **1**

(c) The central limit theorem guarantees that sample means will be **1**

approximately normally distributed so the test will still be valid **1**

[END OF MARKING INSTRUCTIONS]

* optional

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2003 Mathematics

Advanced Higher – Section D

Finalised Marking Instructions

Advanced Higher 2003: Section D Solutions and marks

D1. $L(2{\cdot}5)$

$$= \frac{(2{\cdot}5-4)(2{\cdot}5-6)}{(-3)(-5)}3{\cdot}2182 + \frac{(2{\cdot}5-1)(2{\cdot}5-6)}{(3)(-2)}4{\cdot}0631 + \frac{(2{\cdot}5-1)(2{\cdot}5-4)}{(5)(2)}3{\cdot}1278$$

$= 1{\cdot}1264 + 3{\cdot}5552 - 0{\cdot}7038 = 3{\cdot}9778$ **3**

D2.

$$f(x) = \ln(3+2x) \qquad f'(x) = \frac{2}{(3+2x)} \qquad f''(x) = \frac{-4}{(3+2x)^2}$$

Taylor polynomial is

$$p(x) = p(1+h) = \ln 5 - \frac{2h}{5} - \frac{2h^2}{25}.$$ **3**

For ln 5·4, take $f(1{\cdot}2)$, $h = 0{\cdot}2$; $p(1{\cdot}2) = 1{\cdot}6094 + 0{\cdot}08 - 0{\cdot}0032 = 1{\cdot}6862$. **2**

Coefficient of h in Taylor polynomial is substantially smaller than 1.
Hence $f(x)$ is likely to be very insensitive to small changes in x near $x = 1$. **1**

D3.

$$\Delta^2 f_0 = \Delta f_1 - \Delta f_0 = (f_2 - f_1) - (f_1 - f_0) = f_2 - 2f_1 + f_0$$

$$\Delta^3 f_0 = (f_3 - 2f_2 + f_1) - (f_2 - 2f_1 + f_0) = f_3 - 3f_2 + 3f_1 - f_0$$ **2**

Maximum error is $\varepsilon + 3\varepsilon + 3\varepsilon + \varepsilon = 8\varepsilon$. **1**

This occurs when f_1 and f_3 have been rounded up and f_0 and f_2 rounded down by the maximum amount, or vice versa. **1**

D4. (a) Difference table is:

i	x	$f(x)$	diff1	diff2	diff3
0	0·3	1·298	−103	231	18
1	0·6	1·195	128	249	20
2	0·9	1·323	377	269	19
3	1·2	1·700	646	288	
4	1·5	2·346	934		
5	1·8	3·280			

3

(b) $\Delta^2 f_3 = 0{\cdot}288$ **1**

(c) Third degree polynomial would be suitable.
(Differences are approximately constant (well within rounding error).) **1**

(d) $p = 0{\cdot}1$

$$f(0{\cdot}63) = 1{\cdot}195 + 0{\cdot}1(0{\cdot}128) + \frac{(0{\cdot}1)(-0{\cdot}9)}{2}(0{\cdot}249) + \frac{(0{\cdot}1)(-0{\cdot}9)(-1{\cdot}9)}{6}(0{\cdot}020)$$

$$= 1{\cdot}195 + 0{\cdot}013 - 0{\cdot}011 - 0{\cdot}001 = 1{\cdot}196$$ **3**

(from $f(0{\cdot}3)$ with $p = 1{\cdot}1$,
$f(0{\cdot}63) = 1{\cdot}298 - 0{\cdot}113 + 0{\cdot}013 - 0{\cdot}000 = 1{\cdot}198$).

D5. (a)

$$\int_{x_0}^{x_1} f(x)\,dx = \int_0^1 f(x_0 + ph)h\,dp = h\int_0^1 \left[f(x_0) + f'(x_0)ph + \tfrac{1}{2}f''(x_0)p^2h^2\right]dp$$

$$= h\left[f(x_0)p + \frac{f'(x_0)hp^2}{2} + \frac{f''(x_0)h^2p^3}{6}\right]_0^1$$

$$= h\left[f(x_0) + \frac{f'(x_0)h}{2} + \frac{f''(x_0)h^2}{6}\right]$$

$$= h\left[f(x_0) + \frac{1}{2}f(x_1) - \frac{1}{2}f(x_0) - \frac{f''(x_0)h^2}{4} + \frac{f''(x_0)h^2}{6}\right]$$

$$= \frac{h(f_0 + f_1)}{2} - \frac{h^3 f''(x_0)}{12} \text{ (using } f(x_1) = f(x_0) + hf'(x_0) + \tfrac{1}{2}h^2 f''(x_0) + \ldots)$$

5

First term is trapezium rule; second term is principal truncation error.

(b) Trapezium rule calculation is:

x	$f(x)$	m	$m\,f(x)$
$\pi/4$	0·5554	1	0·5554
$5\pi/16$	0·8163	2	1·6326
$3\pi/8$	1·0884	2	2·1768
$7\pi/16$	1·3480	2	2·6960
$\pi/2$	1·5708	1	1·5708
			8·6316

Hence $I = 8{\cdot}6316 \times \pi/32 \approx 0{\cdot}8474$. **3**

(c) $f''(x) = 2\cos x - x\sin x$ whose magnitude has maximum on $[\pi/4, \pi/2]$ at $x = \pi/2$ since $f''(\pi/2) = -\pi/2 = -1.571$ and $f''(\pi/4) = 0{\cdot}859$ and $f'''(x) \neq 0$ on the interval.
$|\text{maximum truncation error}| = (\pi/16)^2 \times \pi/4 \times 1{\cdot}571/12 \approx 0{\cdot}0040$. **2**

Hence estimate for I is $I = 0{\cdot}85$. **1**

[END OF MARKING INSTRUCTIONS]

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2003 Mathematics

Advanced Higher – Section E

Finalised Marking Instructions

Advanced Higher 2003: Section E Solutions and marks

E1. (a) Given that $\frac{d^2s}{dt^2} = a$, $\frac{ds}{dt} = at + c$.

Since $\frac{ds}{dt} = U$ when $t = 0$, $c = U$. Thus $\frac{ds}{dt} = U + at$. **1**

$$\Rightarrow s = \int (U + at)\,dt = Ut + \tfrac{1}{2}at^2 + c'.$$

When $t = 0$, $s = 0$ so $c' = 0$, hence $s = Ut + \frac{1}{2}at^2$. **1**

(b) Use $s = \frac{1}{2}gt^2$. When $s = H$, $t = 6$ so

$$H = 18g.$$ **1**

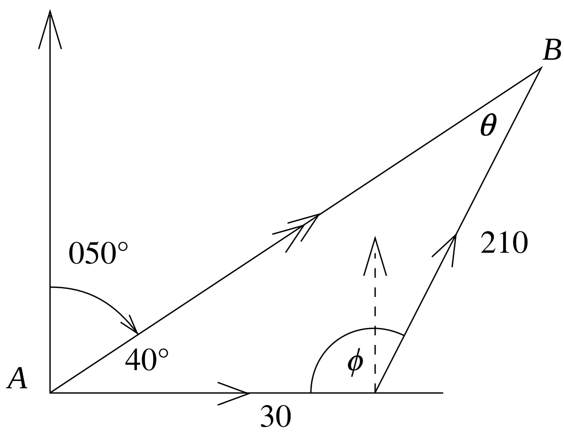
Hence, when $s = \frac{1}{2}H$

$$\tfrac{1}{2}gt^2 = \tfrac{1}{2}H$$

$$t^2 = 18$$

$$t = 3\sqrt{2} \approx 4{\cdot}2 \text{ seconds}$$ **1**

E2.



1

By the sine rule

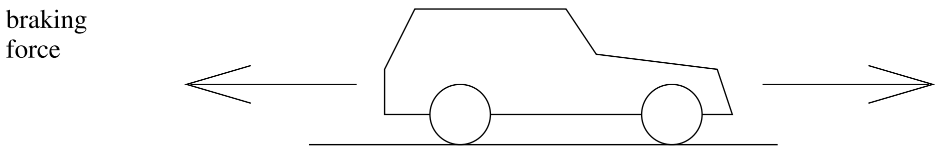
$$\frac{\sin\theta°}{30} = \frac{\sin 40°}{210}$$ **1**

$$\Rightarrow \sin\theta° = 0{\cdot}092$$

$$\Rightarrow \theta° \approx 5{\cdot}3°$$ **1**

Then $\phi = 180 - (40 + 5{\cdot}3) = 134{\cdot}7$
and the required bearing is $134{\cdot}7° - 90° = 044{\cdot}7°$. **1**

E3.



(a) By Newton II

$$m\frac{d^2s}{dt^2} = -2m\left(1 + \frac{t}{4}\right)$$ **1**

so $$\frac{d^2s}{dt^2} = -2\left(1 + \frac{t}{4}\right)$$

Integrating gives $$\frac{ds}{dt} = -2t - \frac{t^2}{4} + c.$$

Since $\frac{ds}{dt} = 12$ when $t = 0$, $c = 12$. So

$$\frac{ds}{dt} = 12 - 2t - \frac{t^2}{4}. \qquad (*)$$ **1**

The car is stationary when $\frac{ds}{dt} = 0$, i.e. when

$$\frac{t^2}{4} - 2t - 12 = 0$$ **1**

$$\Rightarrow \quad t^2 + 8t - 48 = 0$$

$$\Rightarrow \quad (t + 12)(t - 4) = 0$$ **1**

$$\Rightarrow t = 4$$ **1**

(As $t \geqslant 0$, the root -12 is ignored.)

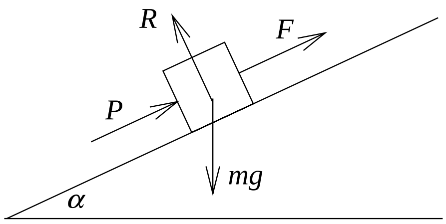
(b) Integrating (*) gives

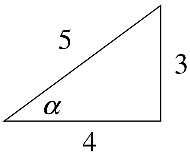
$$s = 12t - t^2 - \frac{t^3}{12} \qquad (\text{as } s(0) = 0)$$ **1**

The stopping distance is

$$s(4) = (48 - 16 - \tfrac{16}{3}) = 26\tfrac{2}{3} \text{ m}.$$ **1**

E4.





(a) Resolving perpendicular to the plane

$$R = mg\cos\alpha = \tfrac{4}{5}mg$$

and hence $F = \mu R = \frac{4}{5}\mu mg$. **1**

Resolving parallel to the plane

$$mg\sin\alpha = P + F = P + \tfrac{4}{5}\mu mg$$ **1**

$$\Rightarrow \quad P = mg\left(\tfrac{3}{5} - \tfrac{4}{5}\mu\right)$$ **1**

$$= \frac{1}{5}mg(3 - 4\mu).$$

(b) As in (a) $F = \frac{4}{5}\mu mg$

Resolving parallel to the plane

$$2P = \frac{3}{5}mg + \frac{4}{5}\mu mg$$ **1,1**

$$P = \frac{mg}{10}(3 + 4\mu).$$

To find μ, equate these expressions for P.

$$\tfrac{1}{10}(3 + 4\mu) = \tfrac{1}{5}(3 - 4\mu)$$ **1**

$$\Rightarrow \quad 3 + 4\mu = 6 - 8\mu$$

$$\Rightarrow \quad \mu = \frac{1}{4}.$$ **1**

E5. (a)

$$\mathbf{V} = V(\cos 45°, \sin 45°) = \frac{V}{\sqrt{2}}(1, 1).$$

From the equations of motion

$$\ddot{x} = 0 \quad \Rightarrow \quad x = \frac{V}{\sqrt{2}}t$$ **1**

$$\ddot{y} = -g \quad \Rightarrow \quad y = \frac{V}{\sqrt{2}}t - \frac{1}{2}gt^2.$$ **1**

Substituting $t = \dfrac{\sqrt{2}x}{V}$, gives **1**

$$y = \frac{V}{\sqrt{2}}\frac{\sqrt{2}x}{V} - \frac{1}{2}g\left(\frac{\sqrt{2}x}{V}\right)^2$$ **1**

$$= x - \frac{gx^2}{V^2}.$$

(b) To hit A, we require $y = h$ when $x = 10h$. **1**

$$\Rightarrow \quad 10h - \frac{g(10h)^2}{V^2} = h$$ **1**

$$\Rightarrow \quad 9h = \frac{g}{V^2}(10h)^2$$

$$\Rightarrow \quad \frac{V^2}{gh} = \frac{10^2}{9}$$

$$\Rightarrow \quad V = \frac{10}{3}\sqrt{gh}.$$ **1**

(b) At B: $y < h$ when $x = 11h$ **1**

$$\Rightarrow \quad 11h - \frac{g(11h)^2}{V^2} < h$$ **1**

$$\Rightarrow \quad 10 < \frac{gh\,11^2}{V^2}$$

$$\Rightarrow \quad \frac{V^2}{gh} < \frac{11^2}{10}$$

$$\Rightarrow \quad \frac{V}{\sqrt{gh}} < \frac{11}{\sqrt{10}}.$$ **1**

So

$$\frac{10}{3} < \frac{V}{\sqrt{gh}} < \frac{11}{\sqrt{10}}.$$

[END OF MARKING INSTRUCTIONS]