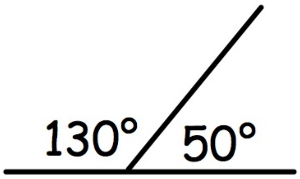
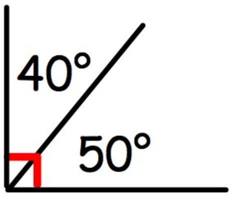
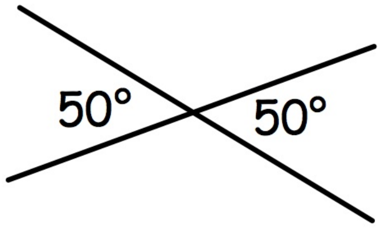
PROPERTIES OF SHAPES



supplementary

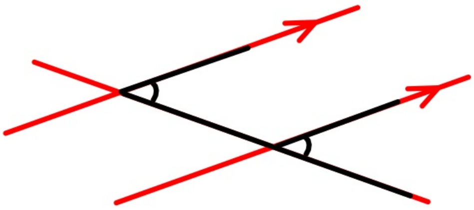


complementary

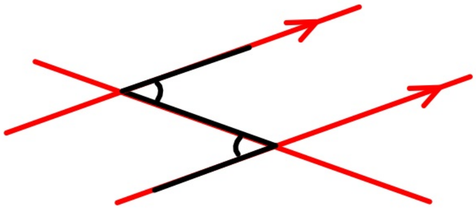


vertically opposite

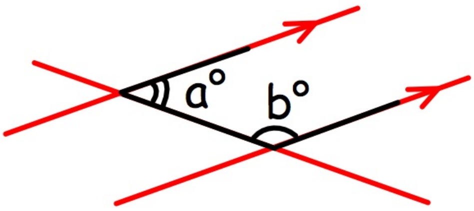
PARALLEL LINES:



corresponding angles



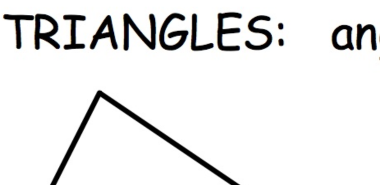
alternate angles



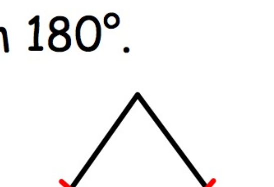
$a° + b° = 180°$

allied angles

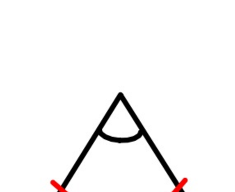
TRIANGLES: angle sum 180°.



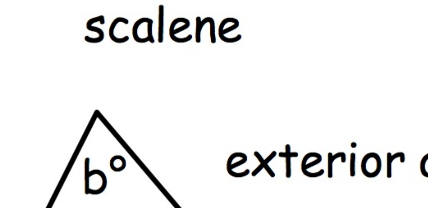
scalene



isosceles



equilateral

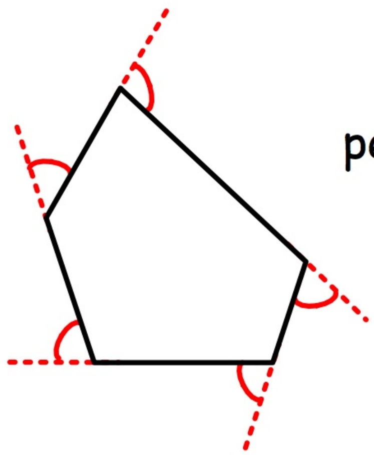


exterior angle = sum of interior opposite angles

POLYGONS: n-sided polygon:

interior angles sum to $(n - 2) \times 180°$

exterior angles sum to 360°



pentagon: interior sum $3 \times 180° = 540°$

exterior sum 360°

REGULAR POLYGON

All sides equal and all angles equal.

angle at the centre

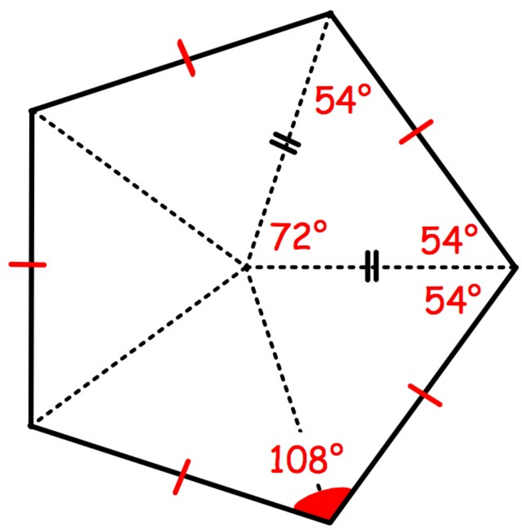
$360° \div 5 = 72°$

isosceles Δ

$180° - 72° = 108°$

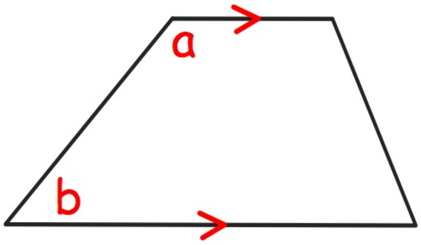
$108° \div 2 = 54°$

interior angle $54° \times 2 = 108°$

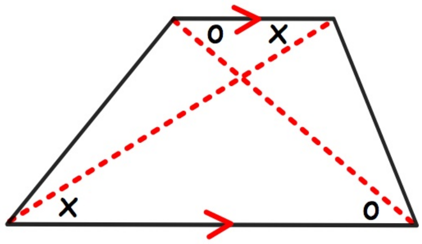


QUADRILATERALS: angle sum 360°.

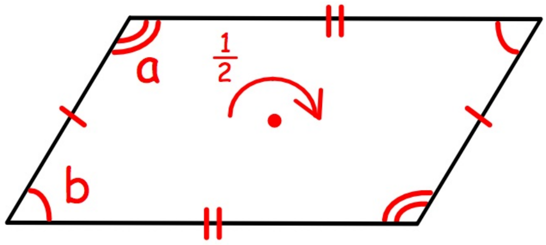
TRAPEZIUM:

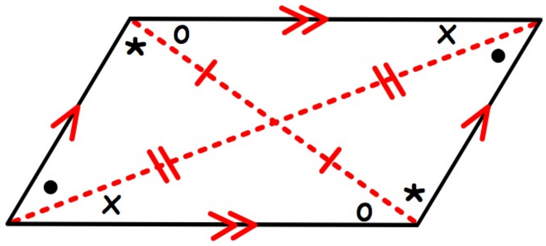


$a + b = 180°$

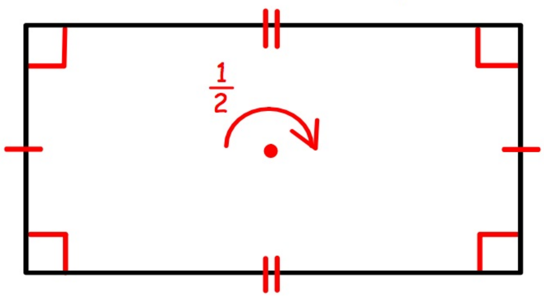


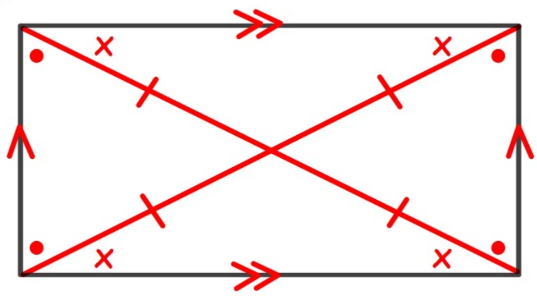
PARALLELOGRAM: a trapezium



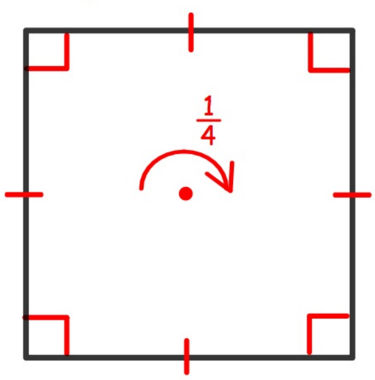


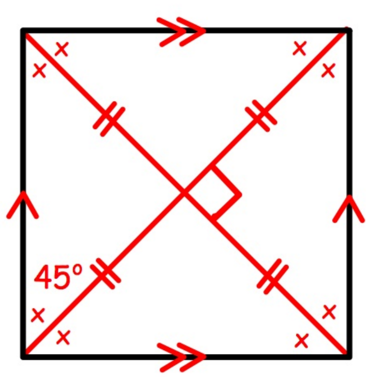
RECTANGLE: a parallelogram



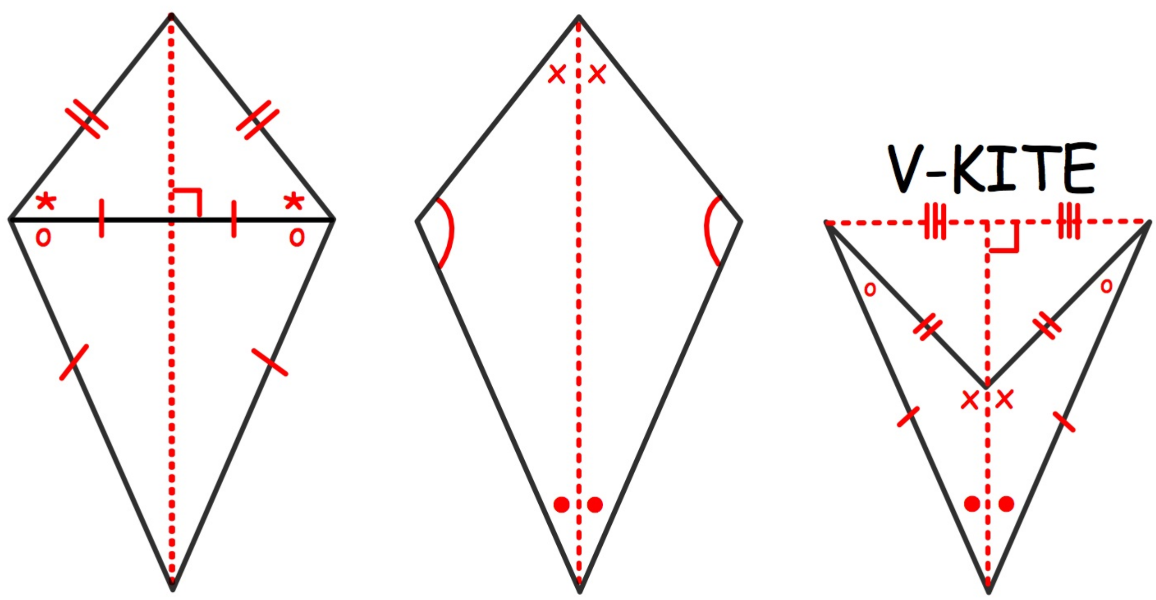


SQUARE: a rectangle

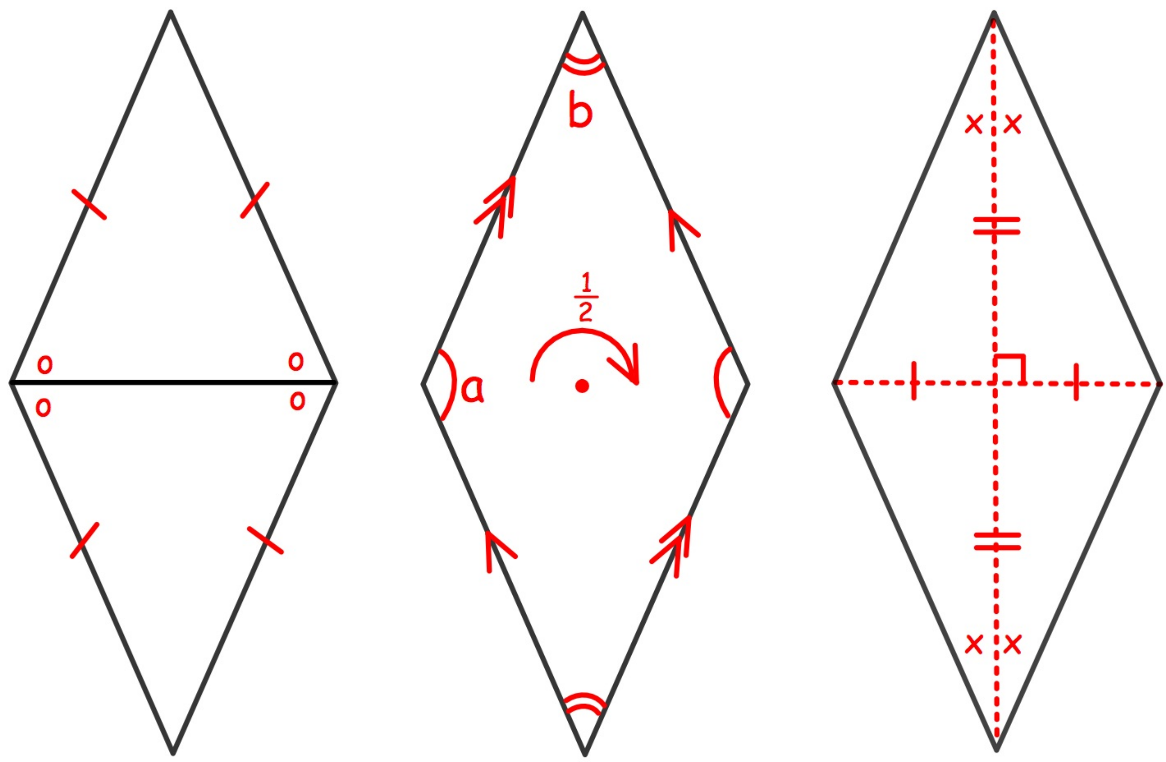




KITE



RHOMBUS a kite and a parallelogram

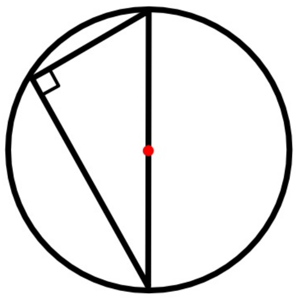


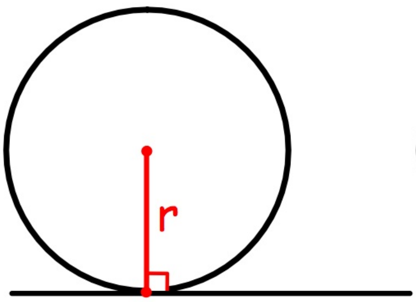
CIRCLE PROPERTIES

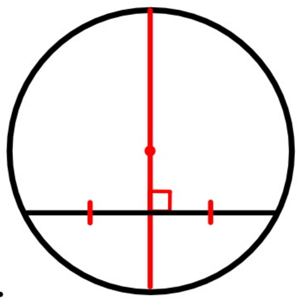
The angle in a semicircle is a right angle.

A tangent and the radius drawn to the point of contact form a right angle.

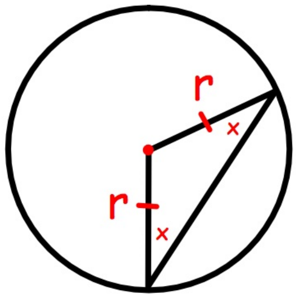
The perpendicular bisector of a chord is a diameter.



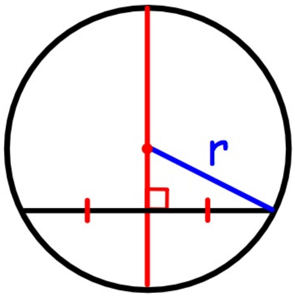




isosceles triangle

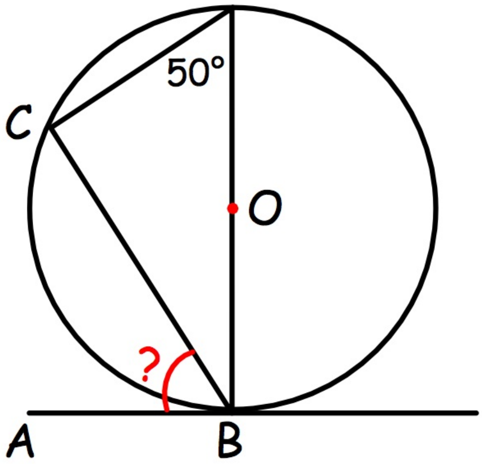


right-angled Δ

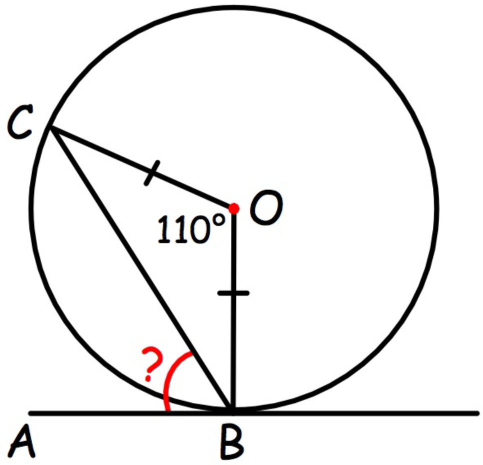


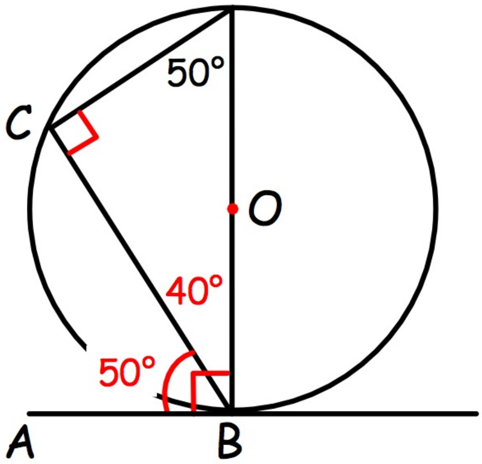
Calculate the size of angle ABC.

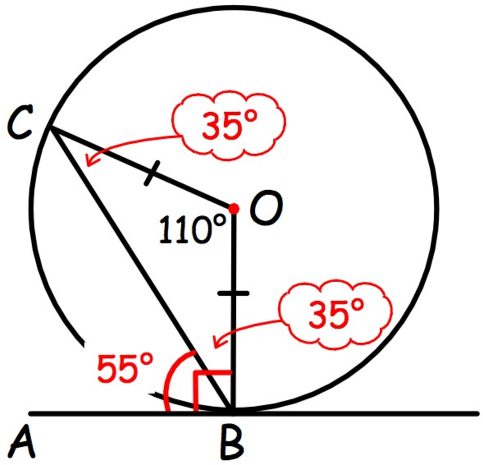
(1)



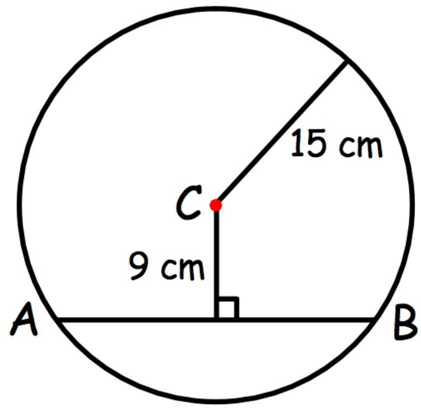
(2)



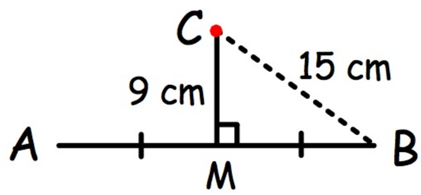




(3) Find the length of chord AB, which is 9 cm from the centre.



PYTH. THM.



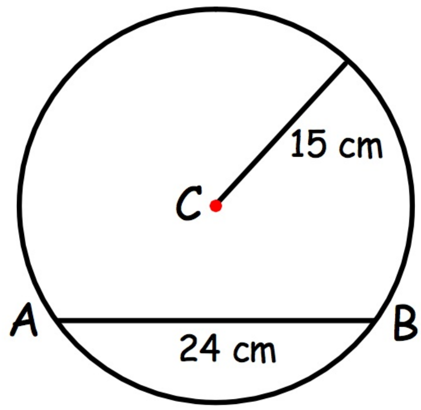
$MB^2 = 15^2 - 9^2$
$= 225 - 81$
$= 144$

$MB = \sqrt{144}$
$= 12$

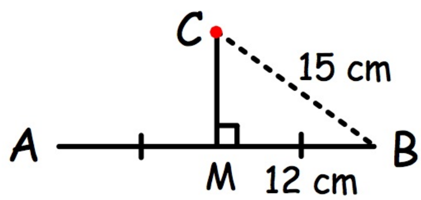
$AB = 2 \times 12$ cm
= **24 cm**

(4) Chord AB is 24 cm long.

Find the distance of chord AB from the centre.



PYTH. THM.



$MC^2 = 15^2 - 12^2$
$= 225 - 144$
$= 81$

$MC = \sqrt{81}$
$= 9$

distance **9 cm**

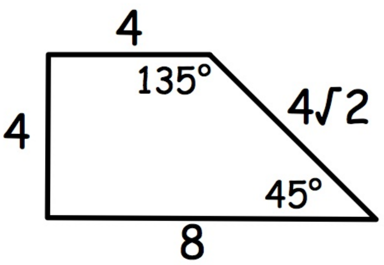
SIMILAR SHAPES

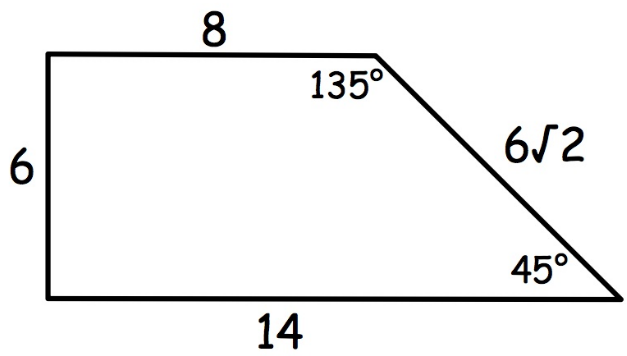
Are enlargement or reductions of each other:

(i) angles are unchanged - shapes are EQUIANGULAR

(ii) sides are enlarged/reduced by a SCALE FACTOR.

Shapes can be equiangular but NOT similar.

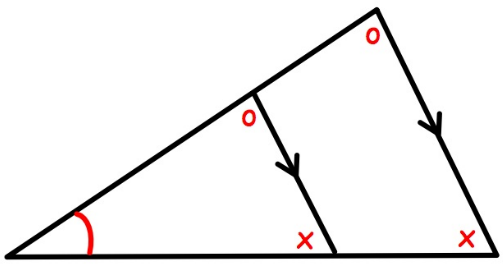


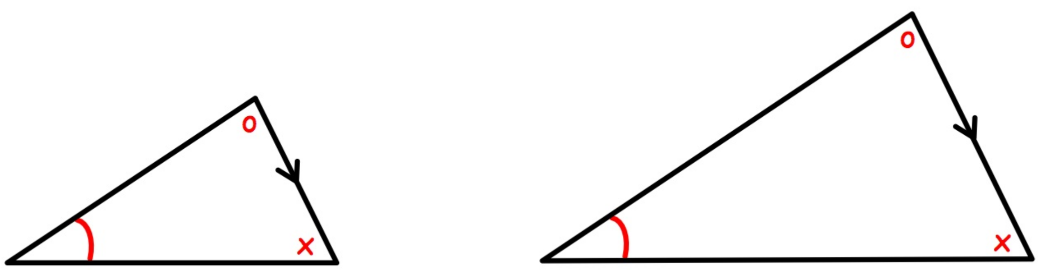


Sides are not scaled.

TRIANGLES ARE SPECIAL

EQUIANGULAR triangles are SIMILAR.





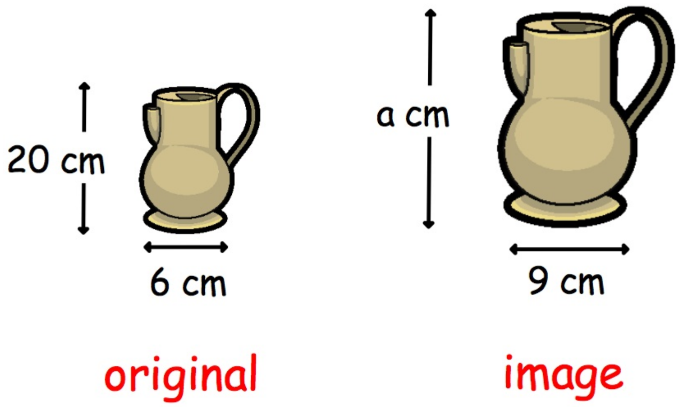
Δs EQUIANGULAR so SIMILAR.

ENLARGE/REDUCE $\quad$ Scale Factor = $\dfrac{\text{image size}}{\text{original size}}$

ENLARGEMENT: $\quad SF > 1$

REDUCTION: $\quad 0 < SF < 1$

(1) One jug is an enlargement of the other.



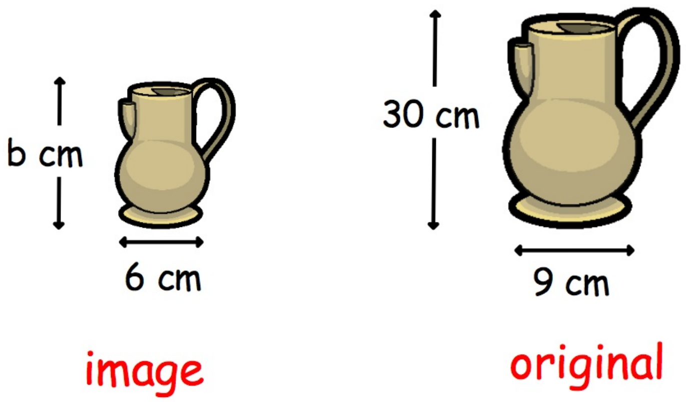
$SF = {}^9/_6 = {}^3/_2$

$a = 20 \times {}^3/_2$

$= 20 \div 2 \times 3$

$= 30$

(2) One jug is a reduction of the other.



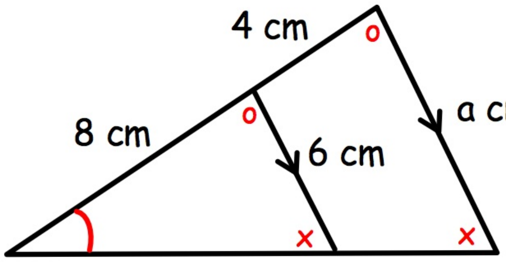
$SF = {}^6/_9 = {}^2/_3$

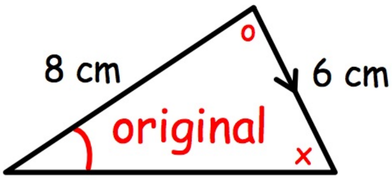
$b = 30 \times {}^2/_3$

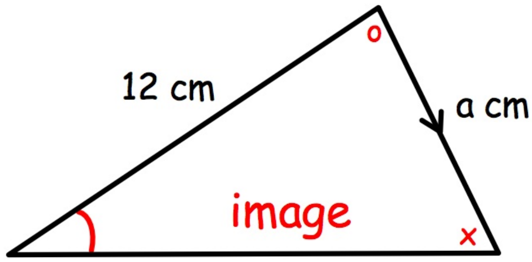
$= 30 \div 3 \times 2$

$= 20$

(3) Find a.







$SF = {}^{i}/_{o}$

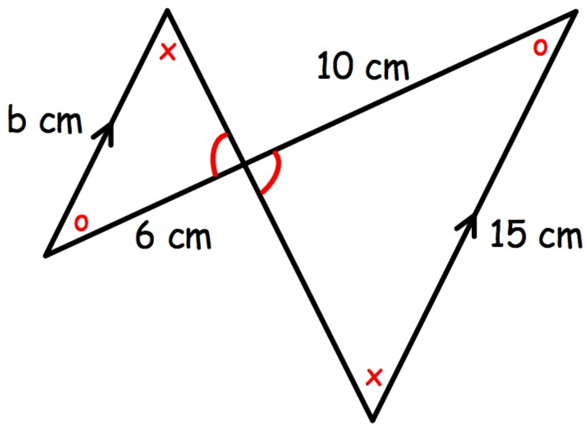
$SF = {}^{12}/_{8} = {}^{3}/_{2}$

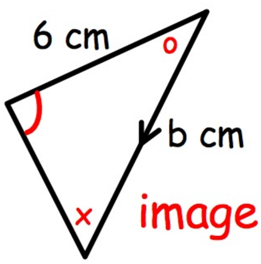
$a = 6 \times {}^{3}/_{2}$

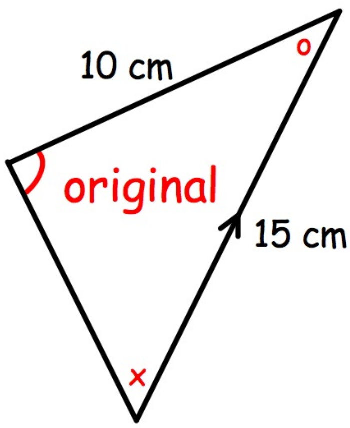
$= 6 \div 2 \times 3$

$= 9$

(4) Find b.







$SF = {}^{i}/_{o}$

$SF = {}^{6}/_{10} = {}^{3}/_{5}$

$b = 15 \times {}^{3}/_{5}$

$= 15 \div 5 \times 3$

$= 9$

SIMILAR SHAPES: AREA and VOLUME

$$\text{length SF} = \frac{a}{b}$$

2D shape: scale both dimensions, length and breadth.

$$\text{area SF} = a/b \times a/b = \frac{a^2}{b^2}$$

3D shape: scale length, breadth and height.

$$\text{volume SF} = a/b \times a/b \times a/b = \frac{a^3}{b^3}$$

AREA and VOLUME: WORKING BACKWARDS

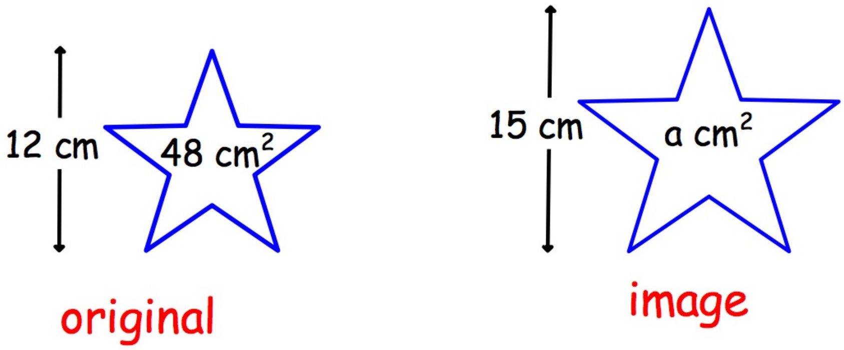
$$\text{length SF} = \frac{a}{b}$$

$$\text{area SF} = \frac{a^2}{b^2} \qquad \sqrt{\text{area SF}} = \text{length SF}$$

$$\text{volume SF} = \frac{a^3}{b^3} \qquad \sqrt[3]{\text{volume SF}} = \text{length SF}$$

AREA: ENLARGE/REDUCE

One shape is an enlargement of the other.



length SF = $^{15}/_{12}$ = $^{5}/_{4}$

area SF = $^{5}/_{4} \times {}^{5}/_{4} = {}^{25}/_{16}$

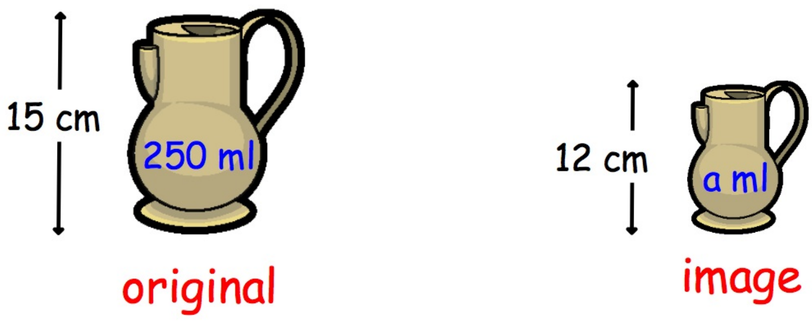
a = $48 \times {}^{25}/_{16}$

= $48 \div 16 \times 25$

= 75

VOLUME: ENLARGE/REDUCE

One shape is a reduction of the other.



length SF = $^{12}/_{15}$ = $^{4}/_{5}$

vol SF = $^{4}/_{5} \times {}^{4}/_{5} \times {}^{4}/_{5} = {}^{64}/_{125}$

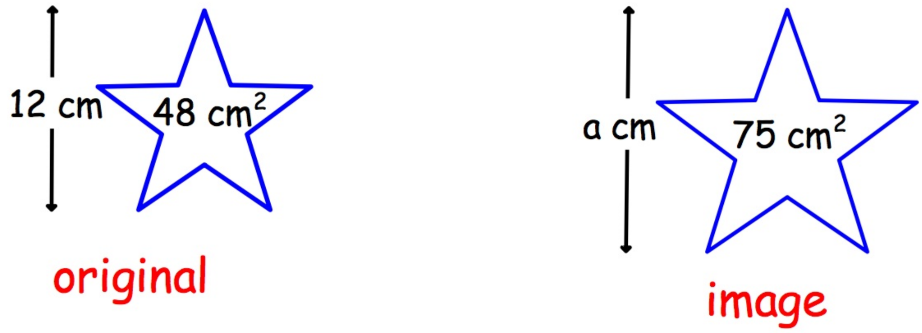
a = $250 \times {}^{64}/_{125}$

= $250 \div 125 \times 64$

= 128

AREA: WORKING BACKWARDS

One shape is an enlargement of the other.



area SF = $^{75}/_{48}$ = $^{25}/_{16}$

length SF = $\sqrt{^{25}/_{16}}$ = $^{5}/_{4}$

a = $12 \times {}^{5}/_{4}$

= $12 \div 4 \times 5$

= 15

VOLUME: WORKING BACKWARDS

One shape is a reduction of the other.

volume SF = $^{128}/_{250}$ = $^{64}/_{125}$

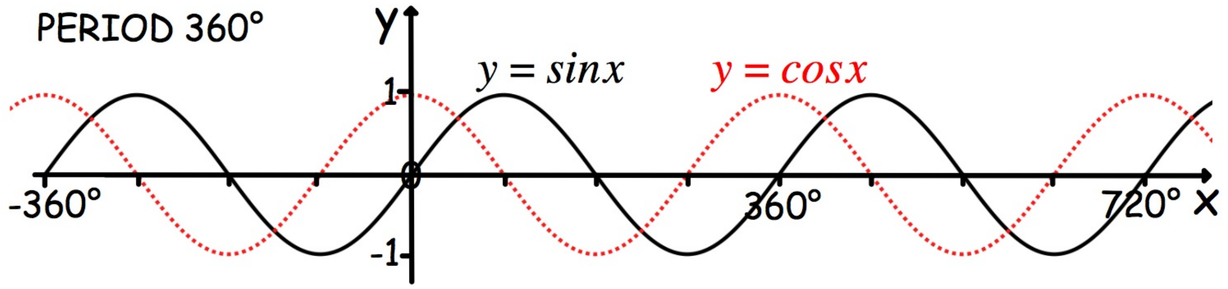
length SF = $\sqrt[3]{^{64}/_{125}}$ = $^{4}/_{5}$

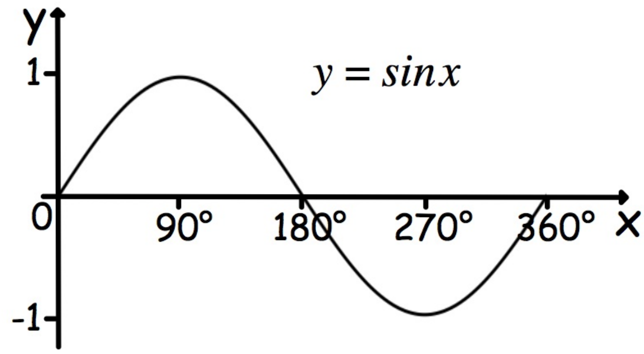
b = $15 \times {}^{4}/_{5}$

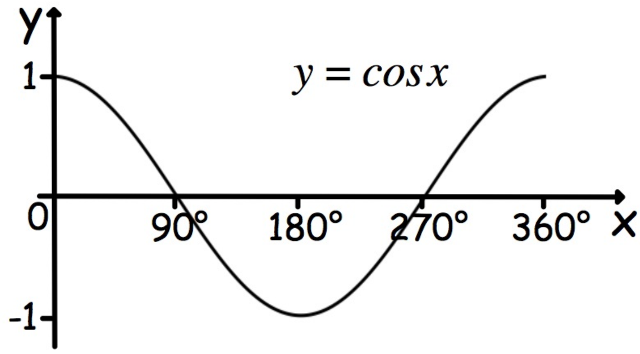
= $15 \div 5 \times 4$

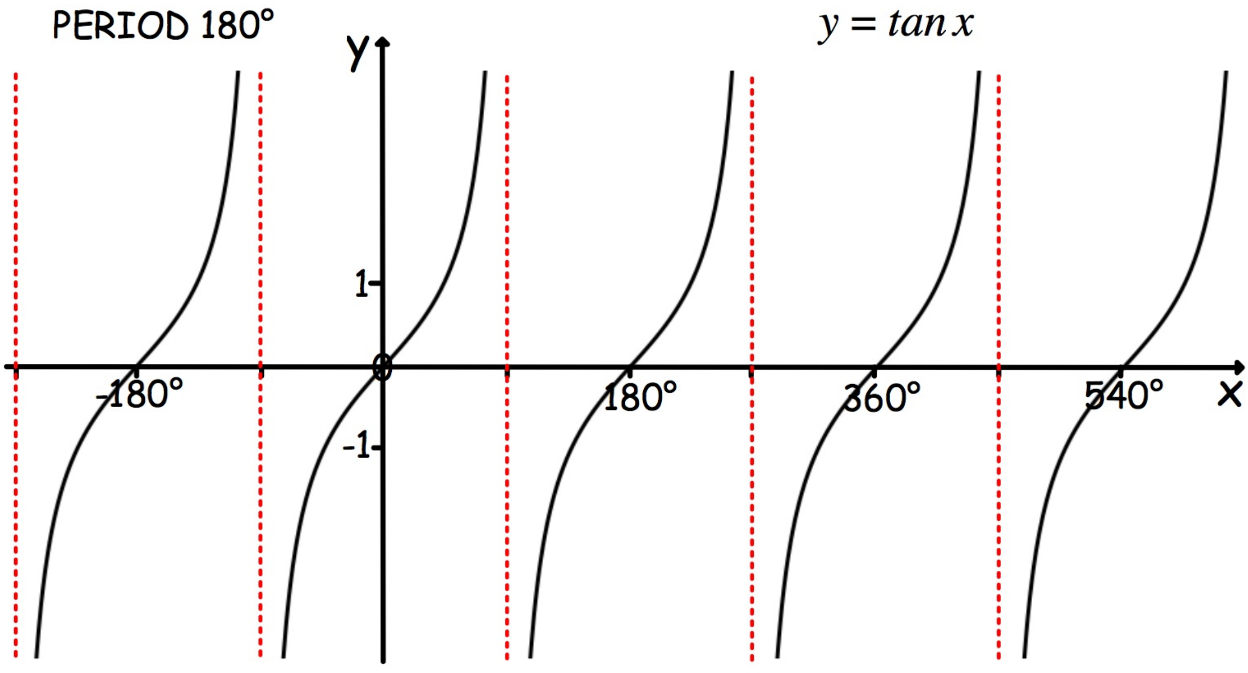
= 12

TRIGONOMETRY: GRAPHS





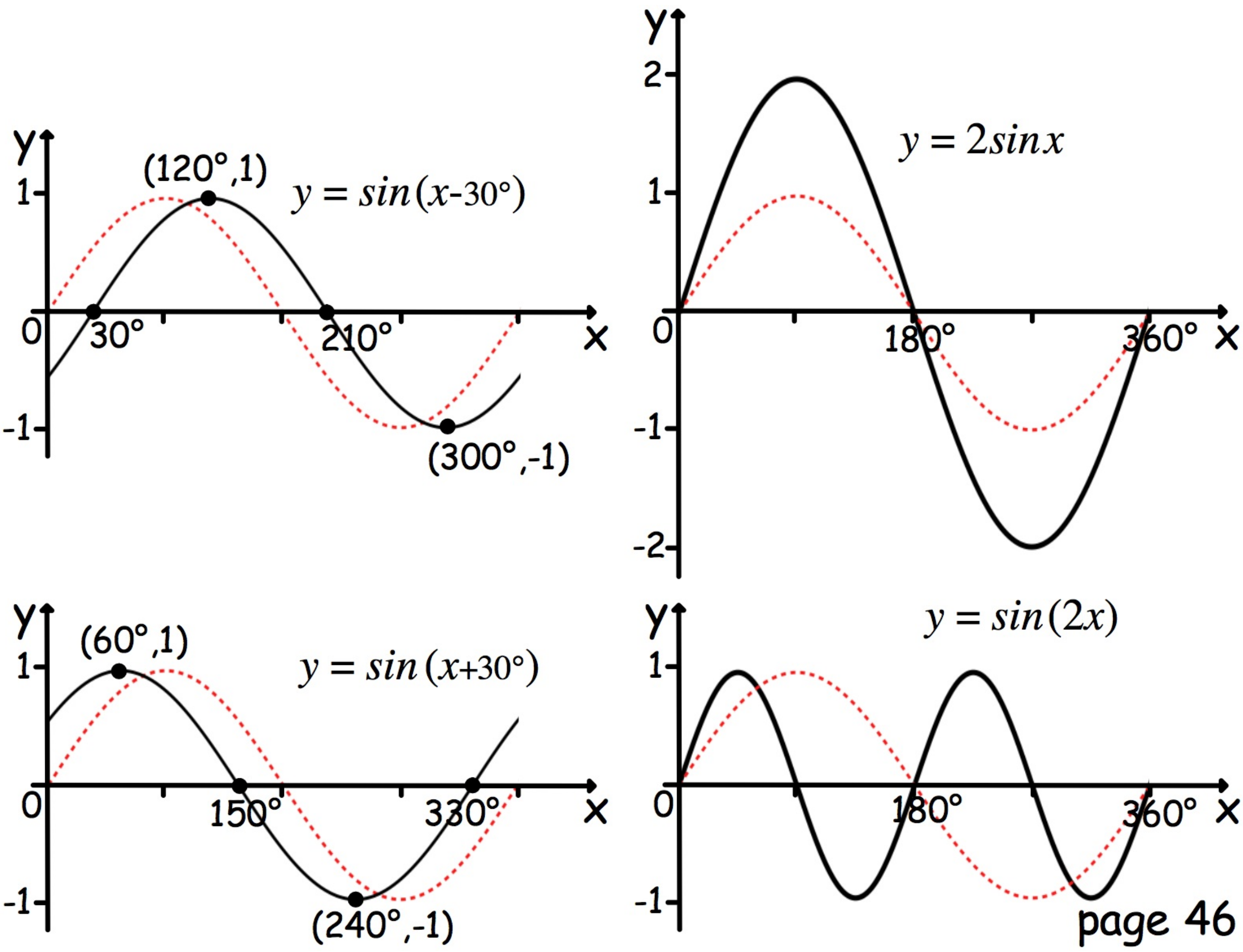




TRANSFORMATIONS

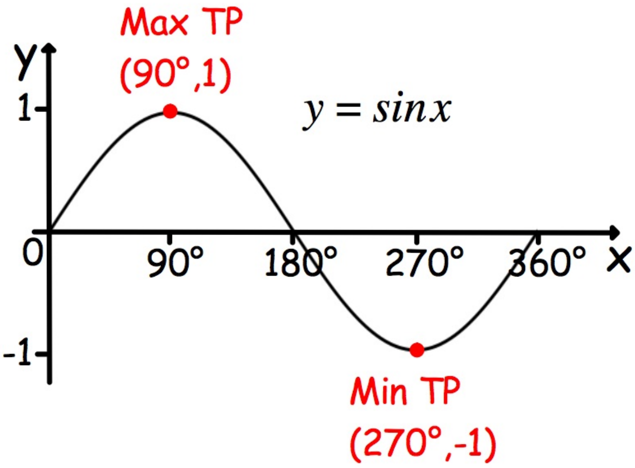
$y = a\sin x$	stretch a units vertically
$y = \sin(bx)$	period 360° ÷ b
$y = \sin(x + c)$	shift -c° horizontally
$y = \sin x + d$	shift d units vertically

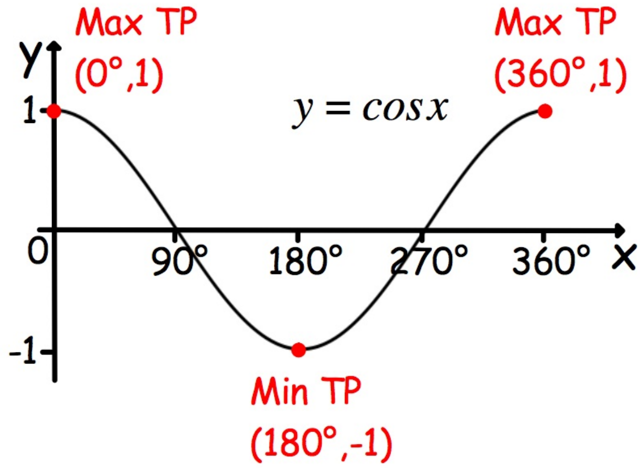
similarly for $y = \cos x$

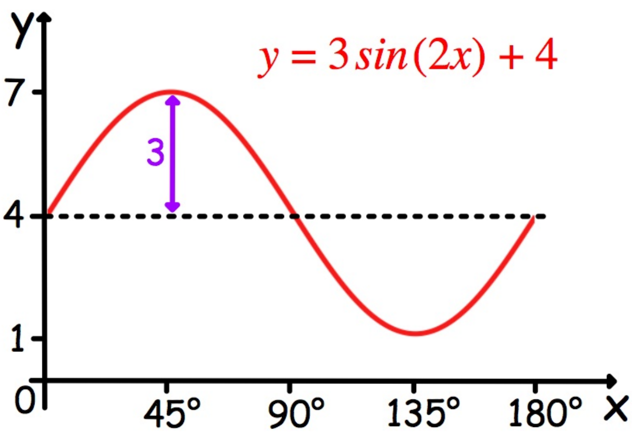


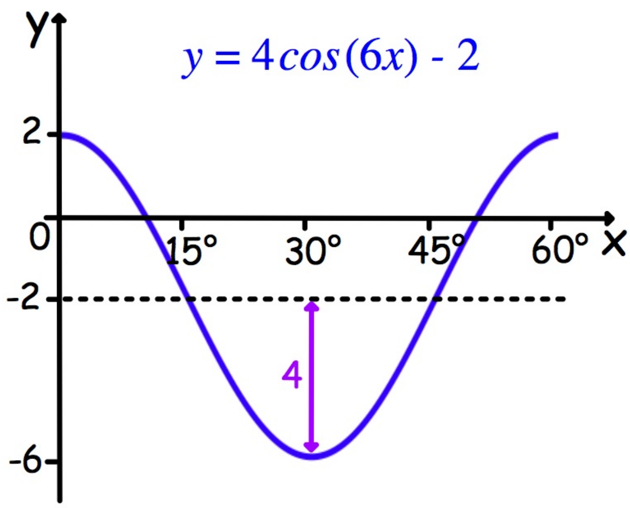
MAXIMUM and MINIMUM VALUES

Turning Points:









(90°, 1) → ÷2, ×3 + 4 → (45°, 7) MAX. TP

(270°, -1) → ÷2, ×3 + 4 → (135°, 1) MIN. TP

(0°, 1) → ÷6, ×4 - 2 → (0°, 2) MAX. TP

(180°, -1) → ÷6, ×4 - 2 → (30°, -6) MIN. TP

(1) $5\sin(2x - 30)° + 3$, $0 \le x \le 180$

MAXIMUM

$5\sin 90° + 3$
$= 5 \times 1 + 3$
$= 8$

$2x - 30 = 90$
$2x = 120$
$x = 60$

MINIMUM

$5\sin 270° + 3$
$= 5 \times (-1) + 3$
$= -2$

$2x - 30 = 270$
$2x = 300$
$x = 150$

MAX (60 , 8) and MIN (150 , -2)

(2) $5\cos(2x - 30)° + 3$, $0 \le x \le 180$

MAXIMUM

$5\cos 0° + 3$
$= 5 \times 1 + 3$
$= 8$

$2x - 30 = 0$
$2x = 30$
$x = 15$

MINIMUM

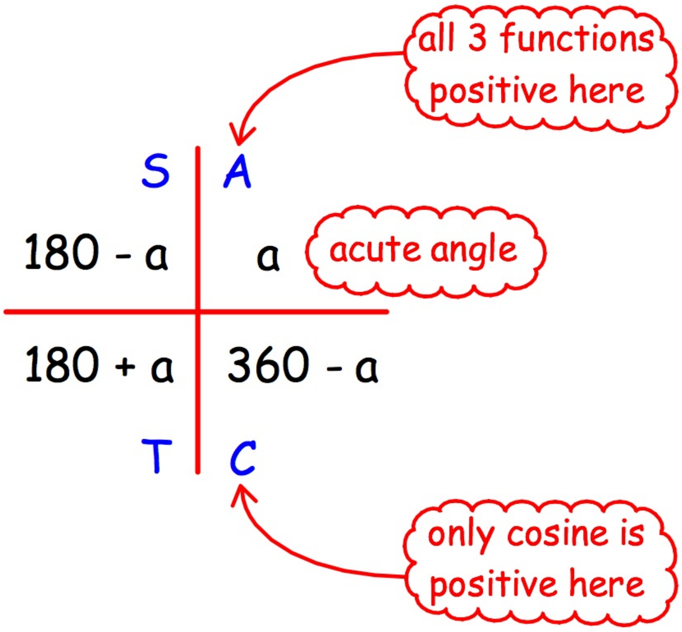
$5\cos 180° + 3$
$= 5 \times (-1) + 3$
$= -2$

$2x - 30 = 180$
$2x = 210$
$x = 105$

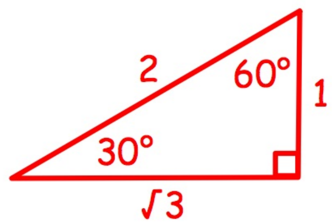
MAX (15 , 8) and MIN (105 , -2)

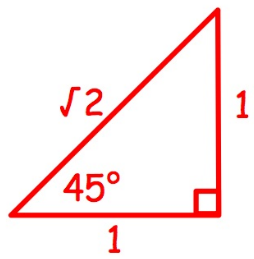
TRIGONOMETRY: EQUATIONS

CAST shows where functions are POSITIVE.
A function has the same value for 4 related angles.



EXACT VALUES



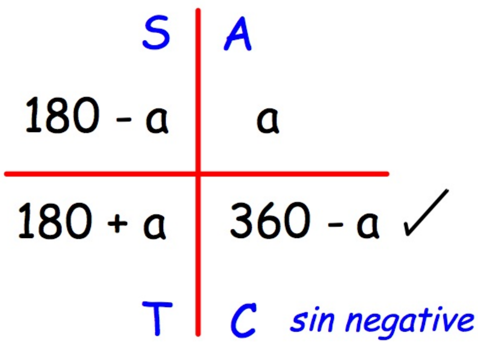


$\sin 300°$

$= \sin(360 - 60)°$

$= -\sin 60°$

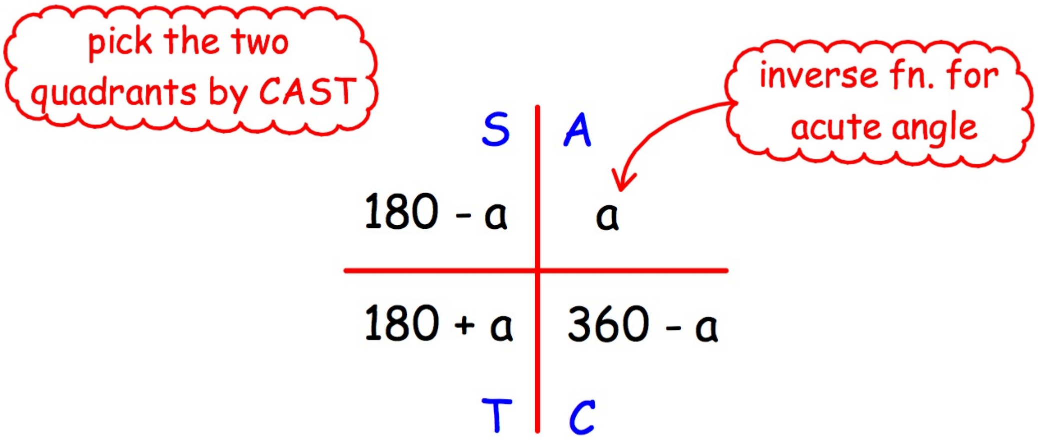
$= -\sqrt{3}/2$



SOLVING TRIG. EQUATION:

(i) Isolate trig. function.

(ii) Solve using CAST.

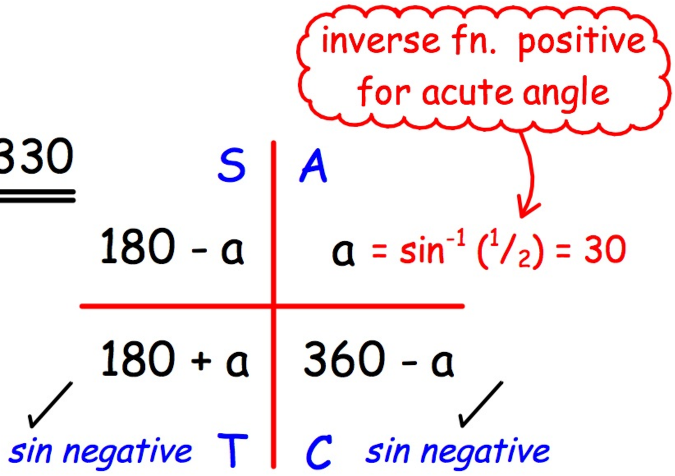


(1) $2\sin x° + 6 = 5$, $0 \leq x \leq 360$

$2\sin x° = -1$

$\sin x° = -{}^{1}/_{2}$

$\underline{\underline{x = 210 , 330}}$



Hence solve

(2) $2\sin 3x° + 6 = 5$, $0 \leq x \leq 120$

$3x = 210 , 330$

$\underline{\underline{x = 70 , 110}}$

TRIGONOMETRIC IDENTITIES

$\sin^2 A + \cos^2 A = 1$ $\qquad\qquad$ $\tan A = \dfrac{\sin A}{\cos A}$

can rearrange

$\cos^2 A = 1 - \sin^2 A$

$\sin^2 A = 1 - \cos^2 A$

(1) Show $(1 - \sin a)(1 + \sin a) = \cos^2 a$

$$\begin{aligned}(1 - \sin a)(1 + \sin a) &= 1 + \sin a - \sin a - \sin^2 a \\ &= 1 - \sin^2 a \\ &= \underline{\underline{\cos^2 a}}\end{aligned}$$

(2) Show $\cos^2 a\,(1 + \tan^2 a) = 1$

$$\begin{aligned}\cos^2 a\,(1 + \tan^2 a) &= \cos^2 a\left(1 + \frac{\sin^2 a}{\cos^2 a}\right) \\ &= \cos^2 a + \sin^2 a \\ &= \underline{\underline{1}}\end{aligned}$$