NATIONAL 5 MATHEMATICS

COURSE NOTES

RELATIONSHIPS

I  MATHS

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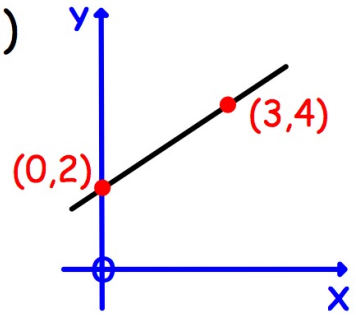
STRAIGHT LINE: EQUATION

Gradient m and Y-intercept C : $y = mx + C$

(1)



$m = \frac{4 - 2}{3 - 0} = \frac{2}{3}$

$C = 2$

$y = mx + C$

$y = \frac{2}{3}x + 2$

x3 to remove fraction

$3y = 2x + 6$

$0 = 2x - 3y + 6$

Can also be written in the form $Ax + By + C = 0$

$2x - 3y + 6 = 0$

(2) Find the gradient of the line $2x - 3y - 9 = 0$

rearrange to $y = mx + C$

$2x \quad - 9 = 3y$

$y = \frac{2}{3}x - 3$

$\frac{2}{3}x \quad - 3 = y$

$m = \frac{2}{3}$

(3) If point (k,4) lies on line $2x + 3y = 6$, find k.

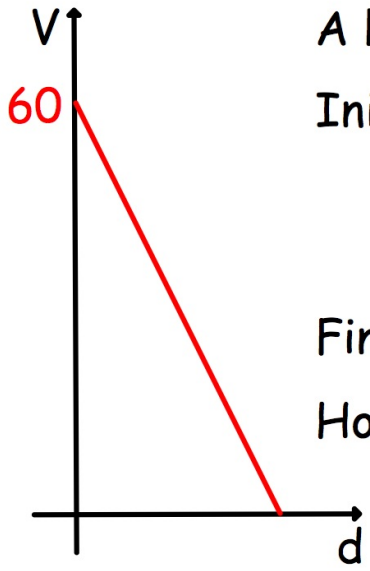
$x = k \ , y = 4$

$2 \times k + 3 \times 4 = 6$

$2k = -6$

$k = -3$

(4)



A leaking oil tank loses 2 litres each day.

Initially it contains 60 litres.

Find the equation of the line.

How many days until the tank empties ?

rate of loss

gradient m = -2

y-intercept C = 60

$y = mx + C$

$V = -2d + 60$

empty V = 0

$0 = -2d + 60$

$2d = 60$

$d = 30$

30 days

Gradient m and through point (a,b) : $y - b = m(x - a)$

(1) Equation of the line gradient $-^2/_3$, through P(-1,2).

P (a b / -1 , 2)
m = $-^2/_3$

$$y - b = m(x - a)$$

$$y - 2 = \frac{-2}{3}(x - (-1))$$ remove fraction before breaking brackets

$$3(y - 2) = -2(x + 1)$$ multiplied both sides by 3

$$3y - 6 = -2x - 2$$

$$\underline{\underline{3y = -2x + 4}}$$

(2) Equation of the line through P(2,1) and Q(3,-1).

P(x_2 2 , y_2 1)

Q(x_1 3 , y_1 -1)

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-1)}{2 - 3} = \frac{2}{-1} = -2$$

P(a b / 2 , 1)
m = -2

$$y - b = m(x - a)$$

$$y - 1 = -2(x - 2)$$ or can use Q(3,-1)

$$y - 1 = -2x + 4$$

$$\underline{\underline{y = -2x + 5}}$$

EQUATIONS and INEQUALITIES

(1) $2y + 17 = 3$ — subtract 17 from each side

$2y = -14$ — divide each side by 2

$y = -7$

(2) $5 - 3w = 17$ — subtract 5 from each side

$-3w = 12$ — divide each side by -3

$w = -4$

(3) $-3 - 2p = -11$ — add 3 to each side

$-2p = -8$ — divide each side by -2

$p = 4$

(4) $7x - 5 = 3x - 14$ — subtract 3x from each side

$4x - 5 = -14$ — add 5 to each side

$4x = -9$ — divide each side by 4

$x = {}^{-9}/_{4}$

$x = -{}^{9}/_{4}$

(5) $8 + 2n = 6 - 3n$ — add 3n to each side

$8 + 5n = 6$ — subtract 8 from each side

$5n = -2$ — divide each side by 5

$n = -{}^{2}/_{5}$

EQUATIONS WITH BRACKETS

First remove brackets

watch for negative multipliers

(1) $2(w + 6) = 5(w - 3)$

$2w + 12 = 5w - 15$

$12 = 3w - 15$

$27 = 3w$

$w = 9$

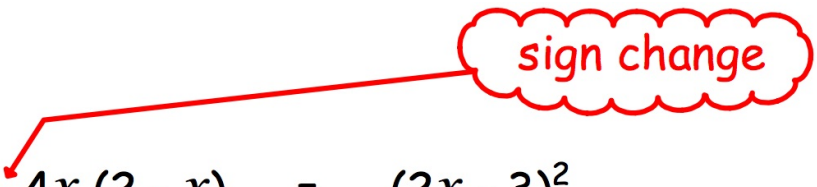
(2) $3y = 14 - 2(y - 3)$

$3y = 14 - 2y + 6$

$3y = 20 - 2y$

$5y = 20$

$y = 4$



(3) $6 - 4x(2 - x) = (2x - 3)^2$

$6 - 8x + 4x^2 = 4x^2 - 12x + 9$

$6 - 8x = -12x + 9$

$4x = 3$

$x = {}^3/_4$

EQUATIONS WITH FRACTIONS

First multiply both sides to remove fractions.

$$\frac{m}{2} - \frac{m-1}{3} = 2$$

$$\frac{3m}{6} - \frac{2(m-1)}{6} = \frac{12}{6}$$

write as fractions with a common denominator

$$3m - 2(m-1) = 12$$

multiply each side by 6

$$3m - 2m + 2 = 12$$

$$m + 2 = 12$$

$$m = 10$$

CROSS-MULTIPLICATION:

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

(1) $\frac{x}{2} = \frac{x+3}{5}$

$$5x = 2(x+3)$$

$$5x = 2x + 6$$

$$3x = 6$$

$$x = 2$$

(2) $\frac{5x}{x-1} = 4$

$$\frac{5x}{x-1} = \frac{4}{1}$$

$$5x = 4(x-1)$$

$$5x = 4x - 4$$

$$x = -4$$

NOTE: same equation

$\frac{1}{2}x = \frac{1}{5}(x+3)$

INEQUALITIES (INEQUATIONS)

Follow the same rules for equations, but if multiply or divide by a negative: reverse inequality sign.

no sign change

(i) $5x \geq 30$

$x \geq 6$

sign change

(ii) $-5x \geq 30$

$x \leq -6$

(iii) $5x \geq -30$

$x \geq -6$

(iv) $-5x \geq -30$

$x \leq 6$

(1) $7x - 2 \geq 3x - 11$

$4x - 2 \geq -11$

$4x \geq -9$

$x \geq -9/4$

no sign change

(2) $5 - 2a < 13$

$-2a < 8$

$+2a > -8$

$a > -4$

sign change

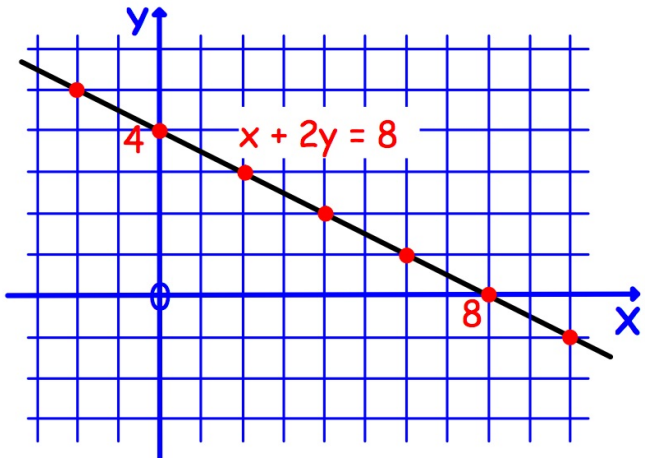
SIMULTANEOUS EQUATIONS

STRAIGHT LINES

The equation gives a rule connecting the x and y coordinates of any point on the line.

$x + 2y = 8$

x	-2	0	2	4	6	8	10
y	5	4	3	2	1	0	-1



SKETCHING

Plot two points and draw the line through them.

Prefer where the line meets the axes:

meets x-axis where $y = 0$

meets y-axis where $x = 0$

$x + 2y = 8$

$x = 0$; $0 + 2y = 8$

$y = 4$

plot point (0,4)

$x + 2y = 8$

$y = 0$; $x + 3 \times 0 = 8$

$x = 8$

plot point (8,0)

line through (0,4) and (8,0) already shown above.

SOLVE SYSTEM OF LINEAR EQUATIONS

GRAPHICAL SOLUTION:

Draw the two lines.
The point of intersection is the solution.

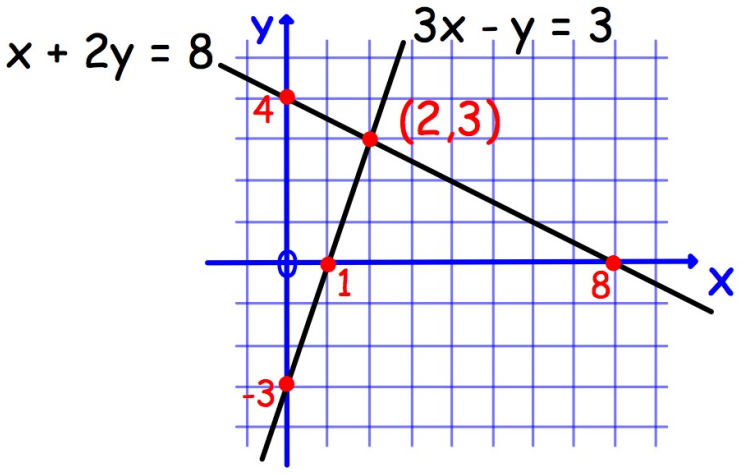
Solve GRAPHICALLY

$$x + 2y = 8$$
$$3x - y = 3$$

1st line:

$x + 2y = 8$

(0,4) (8,0)



2nd line:

$3x - y = 3$

$3x0 - y = 3$

$y = -3$

(0,-3)

$3x - y = 3$

$3x - 0 = 3$

$x = 1$

(1,0)

solution:

$x = 2 , y = 3$

ALGEBRAIC SOLUTION: SUBSTITUTION

(i) Rearrange one equation to y =
(if simple both equations, avoiding fractions)

(ii) Substitute for y in the other and solve for x.

(iii) Substitute x value in one equation to find y.

Solve ALGEBRAICALLY

$$3y + 2x = 11$$
$$y - x = 2$$

$y - x = 2$
rearrange $y = x + 2$ substitute

$$3y + 2x = 11$$
$$3(x + 2) + 2x = 11$$
$$3x + 6 + 2x = 11$$
$$5x = 5$$
$$x = 1$$

$$y = x + 2$$
$$= 1 + 2$$
$$y = 3$$

solution: $x = 1$, $y = 3$

check:

$3y + 2x = 11$

$3 \times 3 + 2 \times 1 = 11$ ✓

ALGEBRAIC SOLUTION: ELIMINATION

Multiples of the equations can be added or subtracted to eliminate one of the variables.

Solve ALGEBRAICALLY

$$5x + 2y = -1$$
$$2x + 3y = 4$$

$5x + 2y = -1$ $\times 3$

$2x + 3y = 4$ $\times 2$

$15x + 6y = -3$

$4x + 6y = 8$

subtract $11x = -11$

$x = -1$

$2x + 3y = 4$

$2\times(-1) + 3y = 4$

$-2 + 3y = 4$

$3y = 6$

$y = 2$

solution: $x = -1$, $y = 2$

check: $2x + 3y = 4$

$2\times(-1) + 3\times2 = 4$ ✓

$5x + 2y = -1$

$5\times(-1) + 2\times2 = -1$ ✓

PROBLEMS: MODELLING

(i) Introduce 2 letters for the 2 unknowns.

(ii) Form 2 equations (each with 2 letters) to model

(iii) Solve for the two letters.

(iv) Interpret the solution to answer the problem.

An adult ticket costs £2 more than a child's.
Three adults and two children costs £11.

Find the cost of an adult and a child's ticket.

child ticket £x $3y + 2x = 11$

adult ticket £y $y = x + 2$

solved previously by substitution

$x = 1$, $y = 3$

Costs: adult £3 , child £1

TRANSPOSE FORMULAE (Change of Subject)

Apply the rules for solving equations.

addition and subtraction: $x + a = b$ $\quad x = b - a$ $\qquad$ $x - a = b$ $\quad x = b + a$

multiplication and division: $\frac{x}{a} = b$ $\quad x = ab$ $\qquad$ $ax = b$ $\quad x = \frac{b}{a}$

powers and roots: $x^2 = a$ $\quad x = \sqrt{a}$ $\qquad$ $\sqrt{x} = a$ $\quad x = a^2$

r $\xrightarrow{\text{square}}$ r^2 $\xrightarrow{\times 3}$ $3r^2$ $\xrightarrow{+ p}$ $3r^2 + p$

$\sqrt{\frac{F - p}{3}}$ $\xleftarrow{\sqrt{\ }}$ $\frac{F - p}{3}$ $\xleftarrow{\div 3}$ $F - p$ $\xleftarrow{- p}$ F

Change the subject to r.

(1) $F = 3r^2 + p$

$F - p = 3r^2$ subtract p from each side

$\frac{F - p}{3} = r^2$ divide each side by 3

$\sqrt{\frac{F - p}{3}} = r$ square root of each side

$r = \sqrt{\frac{F - p}{3}}$

$$r \xrightarrow{\sqrt{\ }} \sqrt{r} \xrightarrow{-n} \sqrt{r} - n \xrightarrow{\div d} \frac{\sqrt{r} - n}{d}$$

$$(Wd + n)^2 \xleftarrow{\text{square}} Wd + n \xleftarrow{+n} Wd \xleftarrow{\times d} W$$

(2) $W = \dfrac{\sqrt{r} - n}{d}$

$Wd = \sqrt{r} - n$ multiply each side by d

$Wd + n = \sqrt{r}$ add n to each side

$(Wd + n)^2 = r$ square each side

$r = (Wd + n)^2$

target term is NEGATIVE, change its side

(3) $g = a(v - ar)$

$g = av - a^2r$

$g + a^2r = av$

$a^2r = av - g$

$r = \dfrac{av - g}{a^2}$

two target terms, factorise

(4) $h = \dfrac{\pi r + d}{r}$

$hr = \pi r + d$

$hr - \pi r = d$

$r(h - \pi) = d$

$r = \dfrac{d}{h - \pi}$