# Maths and Numeracy Methodology Document 

St Columba's High School
St Andrew's Primary School
St Joseph's Primary School
St Ninian's Primary School

## Introduction

This is the 1st Edition of the Numeracy and Maths methodology document that has been jointly produced by the Maths Department in St Columba's High School and our feeder primary schools, St Andrew's, St Joseph's, and St Ninian's.

The aim of this document is to promote the use of common methodologies in numeracy and mathematics across the cluster to improve the educational experience of pupils. We hope that this helps to improve the confidence and fluency in key mathematical/numerical skills that they need across the curriculum.

Our primary schools have been actively involved in ongoing discussions and development so that this document reflects joint Primary and High school approaches and methodologies. It is hoped that this awareness raising will result in all teachers adopting the same approaches thus improving consistency of teaching across the curriculum.

This document is also made available to parents/carers and pupils so they can refer to it. Several videos will be available as the document gets transferred onto sways.

Hopefully this document will help you, or at least encourage you to consult with the Maths Department. The process is intended to be a collegiate one and your comments, contributions and questions are always welcome.

Thank you
The Maths Department

## Contents

Number and Number Processes

- Place Value
- Addition and Subtraction
- Multiplying
- Dividing

Estimation and Rounding

- Nearest whole number
- Decimal Places
- Significant Figures

Fractions, Decimals and Percentages

- Fraction of a quantity
- Percentage calculations (non-calculator)
- Percentage calculations (calculator)
- Simplifying/equivalent fractions

Money
Expressions and Equations
Time

- Writing time in 12 hour and 24 hour
- Calculating Time Intervals


## Number and Number Processes

## Place Value

Place Value is extremely important in Maths and Numeracy.
Ideally when introducing decimal place value, we would use the following headings

$$
\begin{array}{llllllll}
\text { Th } & \mathrm{H} & \mathrm{~T} & \mathrm{U} & \cdot & \frac{1}{10} & \frac{1}{100} & \frac{1}{1000}
\end{array}
$$

This allows pupils to link together decimals and fractions.

## Addition and Subtraction

Some pupils can perform calculations in their heads but it also important to note that at some point pupils must learn to write down their working correctly. Showing working allows pupils to share their thought process, especially when calculations can get trickier.

## Examples

1. $456+213$

H T U add units first, then tens then hundreds
456
(work right to left)
$+213$
669

Pupils can also use partitioning
$400+50+6+200+10+3$ (add hundreds, tens and units separately)
$=600+60+9$
$=669$
2. $456+364$

H T U add units first, then tens then hundreds
456 (work right to left)

| 364 |
| ---: |
| 820 |
| 101 |

carry digits either above or below the line
3. When adding or subtracting decimals, we encourage pupils to lay out their working appropriately.

H T U $\cdot \frac{1}{10} \quad$ line the decimal points up.
$325 \cdot 6$
$+364 \cdot 3$
$689 \cdot 9$

## Multiplying

There are a variety of methods we can use for multiplying. It is important that pupils use the method they are most comfortable with and that they are getting the correct answer!

## Multiplying by multiples of 10, 100 and 1000

When multiplying by 10/100 avoid language such as "add a zero"
The decimal point is fixed. The decimal point doesn't move. Only the digits move.

To multiply by 10 every digit is moved one place to the left.
To multiply by 100 every digit is moved two places to the left.
To multiply by 1000 every digit is moved three places to the left.
(a) $351 \times 10$
(b) $34.67 \times 100$


| 3 | 5 | 1 | 0 |
| :--- | :--- | :--- | :--- |

Move one place to the left

$$
\begin{array}{rlllll}
\text { Th } & \mathrm{H} & \mathrm{~T} & \mathrm{U} \cdot & \frac{1}{10} & \frac{1}{100} \\
& 3 & 4 \cdot & 6
\end{array}
$$

$$
3467
$$

Move two places to the left

Some pupils have great difficulty grasping the process of moving digits and find it easier to "move the decimal point".

It was agreed (Primary and High school) that teachers should use their professional judgement as to when the "moving point" should be resorted to.

## Partitioning Numbers

"chunk it up"
Example:
$23 \times 5$
$20 \times 3=60$
$5 \times 3=15$
$60+15=75$
"Box Method"

|  | 20 | 3 |
| :--- | :--- | :--- |
| 5 | 100 | 15 | $100+15=115$

23
X 5
115

The box method links to multiplying out brackets in $4^{\text {th }}$ level/National 4/National 5

Long Multiplication
$23 \times 14$

|  | 20 | 3 |
| :---: | :--- | :--- |
| 10 | 200 | 30 |
| 4 | 80 | 12 |

$$
200+80+30+12=322
$$

23
X 14
$92<-23 \times 4$
$+230<-23 \times 10$
322 <-2 added together

## Dividing

## Dividing by multiples of 10,100 and 1000

To divide by 10 every digit is movẹd one place to the right.
To divide by 100 every digit is moved two places to the right.
To divide by 1000 every digit is moved three places to the right
(a) $351 \div 10$
(b) $34.67 \div 100$
Th H T U • $\frac{1}{10}$
351

Th H T U • $\frac{1}{10} \frac{1}{100} \frac{1}{1000} \frac{1}{10000}$
$34 \cdot 67$
$\xrightarrow[\text { Move two places to the right }]{0 \cdot 3}$
(c) $45 \div 50$

Most pupils when dividing by 50 will divide by 5 , and then by 10 .

$$
\begin{aligned}
& 45 \div 5=9 \\
& 9 \div 10=0 \cdot 9 \\
& \text { Or vice versa. }
\end{aligned}
$$

Towards the end of Second Level and in Third level, pupils are discouraged from getting "remainders". Instead, they should look to extend the "bus stop" and make a decimal number.

## Examples

$$
\begin{aligned}
& 5 \longdiv { 8 r ^ { 3 } } \\
& <- \text { discourage this during } 2^{\text {nd }} \text { level } \\
& 5 \longdiv { 8 . 6 } \\
& \text { <- Majority of pupils should be able to } \\
& \text { get this answer in secondary }
\end{aligned}
$$

It is a good idea to link this to money as money is to 2 decimal places.

$$
£ 43 \div 5=£ 8 \cdot 60
$$

## Negative Numbers

During $2^{\text {nd }}$ level, pupils are expected to be able to answer questions involving negative numbers in context. Such as using thermometers, dealing with money etc.

As they progress into $3^{\text {rd }}$ level, the questions become more challenging and the maths department introduce a number of the concepts visually. We use double sided counters as well as encouraging pupils to write down any working.

Visual in jotters ( + and - cancel each other out)
$\operatorname{eg}(5+(-3)=2$
(2) $-13+8=-5$


# Multiples and Factors (3 $3^{\text {rd }}$ level) <br> Multiples: 

Pupils should have a good grasp of their times tables.

## Factors:

The maths department encourage pupils to write down the factors in pairs. This is easier and means that they are unlikely to miss any out. This helps link to factorising in National 5.
Factors of 12: 12 factors of 15
26
35
34

## Estimating and Rounding

Pupils are introduced to estimating and rounding in early primary by using a variety of concrete materials. Activities such as estimating the number of items in a jar allow pupils to estimate and then count and check their answers.

Rounding Rules/Strategies:
5 or more, raise the score 4 or less, let it rest
50 or more, raise the score 49 or less, let it rest
500 or more, raise the score 499 or less, let it rest

Using a number line or a sloping number line to visualise the numbers. Round 27 to the nearest 10 .

Answer is 30 .


## e.g. Round 256 to the nearest 10:



## Round 256 to the nearest 100:



Estimate the answer to the following sum:
$378+243=$

Children are allowed to choose rounding to nearest 10 or 100 to help with their estimate. Children should discuss which would allow for a better estimate and why. Challenge children to work out exact answer (Rounding and adjusting/Compensating strategy).

$$
\begin{aligned}
& 378+243= \\
& 380^{+2}+240^{-3}=620 \\
& \text { Challenge- } \\
& 620-2+3=621
\end{aligned}
$$

Resources:

100 squares, Numeracy Blueprint boards, Number Lines, Counters, Place Value Grids, Heinemann Active Cards/Activities (Jump up, crouch down), Estimation game (http://www.mathsisfun.com/numbers/estimation-game.php), Textbooks

## Rounding to a given number of Decimal Places

Pupils are introduced to rounding to a given number of decimal places in primary school and extend that knowledge to up to 3 decimal places in secondary. This builds on their previous knowledge of rounding to the nearest 10,100 etc.

## Examples

1. Round the following to 1 decimal place.
a) $12 \cdot 342$ we only look at the digit after the $1^{\text {st }}$ decimal place which is 4 .
$=12 \cdot 3$
b) 154.687 we only look at the digit after the $1^{\text {st }}$ decimal place which is 8 .
$=154 \cdot 7$
c) 1999.99999
$=2000 \cdot 0$
we only look at the digit after the $1^{\text {st }}$ decimal place, which is 9 , however this will affect the next numbers etc
2. Round the following to 2 decimal places.
a) $12 \cdot 1342$ we only look at the digit after the 2 nd decimal place which is 4 .
$=12 \cdot 13$
b) $154 \cdot 3687$ we only look at the digit after the 2 nd decimal place which is 8 .
$=154 \cdot 37$
c) 1999.999999
$=2000 \cdot 00$
we only look at the digit after the 2 nd decimal place, which is 9 , however this will affect the next numbers etc
3. Round the following to 3 decimal places.
a) $12 \cdot 12342$ we only look at the digit after the 3 rd decimal place which is 4 .
$=12 \cdot 123$
b) 154.35687 we only look at the digit after the 3 rd decimal place which is 8 .
$=154 \cdot 357$
c) 1999.9999999
$=2000 \cdot 000$
we only look at the digit after the 3rd decimal place, which is 9 , however this will affect the next numbers etc

When performing calculations involving money it is important that pupils round their answer (when appropriate) to 2 decimal places.

Eg $£ 19 \div 2=£ 9 \cdot 50$ (the zero must be included in the final answer)
An answer of $9 \cdot 5$ will not be accepted at SQA level.

## Rounding to Significant Figures (4 ${ }^{\text {th }}$ Level)

Rounding to a given number of significant figures is introduced at the beginning of $2^{\text {nd }}$ year to most pupils.

The rounding rules are the same, but zeros can be complicated!!
If a zero is only used to show the position of the decimal point than they are NOT significant.

Examples

1. 403 has 3 significant figures
2. $4 \cdot 03$ has 3 significant figures
3. $40 \cdot 3$ has 3 significant figures
4. $4 \cdot 030$ has 4 significant figures
5. $0 \cdot 403$ has 3 significant figures
6. $\quad 0 \cdot 4030$ has 4 significant figures
(1 $1^{\text {st }}$ zero positions the decimal point but the last shows accuracy)

## Fractions, Decimals and Percentages

Pupils find working with fractions, decimals and percentages easier if they have a good grasp of their times tables.

## $1^{\text {st }}$ Level

Identifying Fractions- Within known shapes
This leads on to numbers- Fractions of an amount (halves, quarters and thirds)
Concrete and pictorial materials- Grouping (Extension of multiplication and division strategies- children can make the connection)

Counting in halves, quarters and thirds- empty number lines, oral counting.

## Calculating a fraction of a quantity

$2^{\text {nd }} / 3^{\text {rd }}$ level

Pupils are introduced to calculating a fraction of a quantity with "barmodelling" and using concrete materials such as counters.


This is then moved on to showing the calculation.
"Divide by the denominator, multiply by the numerator"
$=12 \div 4=3$
$=3 \times 3$
$=9$
"Divide by the bottom times by the top"

In $3^{\text {rd }}$ level, we move this on to working with decimals, converting units (eg m into cm ) but keeping the same strategy.

## Example

Calculate $\frac{3}{5}$ of $2 m$
convert 2 m to 200 cm
$=200 \div 5=40$
$=40 \times 3$
$=120 \mathrm{~cm}$
units are important!!

## Calculating a Percentage- Non Calculator Methods

## Common Percentages

Pupils will know the "common percentages"
$50 \%=\frac{1}{2} \quad 25 \%=\frac{1}{4} \quad 75 \%=\frac{3}{4} \quad 33 \frac{1}{3} \%=\frac{1}{3} \quad 66 \frac{2}{3} \%=\frac{2}{3}$

This then allows pupils to perform calculations and link back to "calculating fraction of a quantity"

## Example

$25 \%$ of $£ 120$
$=\frac{1}{4}$ of $£ 120$
$=£ 30$

Calculating 10\%
(divide 235 by 10 then multiply by 7)
Depending on the question will depend on how much working will need to be shown!
$10 \%$ of $£ 450$
= £45
$30 \%$ of $£ 450$
$10 \%$ of $£ 450=£ 45 \quad$ (divide 450 by 10 then multiply by 3 )
$£ 45 \times 3=£ 135$
$70 \%$ of 235
$10 \%$ of $235=2 \cdot 35$
$2 \cdot 35 \times 7=16 \cdot 45$
(divide 450 by 10)

The same method applies for $20 \%, 40 \%, 60 \%, 80 \%$ and $90 \%$.

## Calculating 1\%

$1 \%$ of 140
(divide 140 by 100)
$=1 \cdot 4$
$3 \%$ of 235
$1 \%$ of $235=2 \cdot 35 \quad$ (divide by 100 then multiply by 3 )
$2 \cdot 35 \times 3=7 \cdot 05$

## Calculating a Percentage- Calculator Method

$13 \%$ of 250 g
$1 \%=2 \cdot 5$
$2.5 \times 13=32 g$
(divide by 100 then multiply by 13)
Remember to show working even though you are using a calculator!

## Simplifying/Equivalent Fractions

Pupils are introduced to equivalent and simplifying fractions using concrete materials. (Every school in the cluster has a set of Rainbow Fractions that are used for staff training.)

## Equivalent Fractions

The picture on the left shows the use of rainbow fractions which allows pupils to visualise equivalent fractions. The picture on the right demonstrates the "abstract" approach.


## Simplifying Fractions

Pupils in primary school are encouraged to write down the factors of the numerator (top number) and the denominator (bottom number). This strategy helps pupils to be able to simplify the fraction fully. A skill that is required for SQA exams.


## Mixed Numbers and Improper Fractions

Pupils are introduced to mixed numbers and improper fractions by a visual representation. This is then moved onto being able to convert between the two without drawing a diagram or using concrete materials.


## Money

Money is not covered until pupils are confident in addition and subtraction.
Pupils are not exposed to physical money as much these days due to contactless methods/ online shopping/ apple pay etc which has had an impact on pupils knowledge and understanding.
$1^{\text {st }}$ Level
In first level pupils work up to $£ 1 / £ 10$. Working with buying items and receiving change. A lot of practical activities involving identifying coins, paying for items, making amounts with the coins etc
$2^{\text {nd }}$ Level
Pupils recap a lot of $1^{\text {st }}$ level work before moving on.
Budgeting- costs and values of things. Comparing prices in different shops etc. Planning events/holiday/party.

Conversion of pence to pounds and vice versa - recap of knowledge of multiplying/dividing by 100.

Addition and subtraction of decimals- using mental strategies.
Profit and Loss - using calculations weekly/monthly. Link to percentages as well.

Look at different strategies for costings eg buy one get one free. Comparison of pounds and pence. (decimals and place value).

Debit/credit cards- make pupils aware of what they are.
Bank statements look at balance/overdraft.
$3^{\text {rd }}$ Level

Recap of knowledge of primary. Embedded in S1 course.
Word problems involving money- link to
rounding/calculations/percentages/decimals/ratio
There are a number of lessons in the PSE/Citizenship courses - personal finance.

S1: Ethics of Spending
S2: credit and debt
S3: budgeting

## S4: After School Finance

## S5: Risk and Reward

## Expressions and Equations

$1^{\text {st }}$ level
During $1^{\text {st }}$ level pupils use a variety of resources around active learning. These include:
visual and concrete materials counting back and front
blue print boards
number squares/lines
counters etc for number sentences
10 frames
Heinemann active cards
tuff trays (adult initiated)
Heinemann active
Number squares for pupils struggling
Place Value (link)
Quick practice before written
Basic sums replace letters with number - number facts
single operation function machine input/output
balancing scales
$2^{\text {nd }}$ and $3^{\text {rd }}$ level

Pupils find this topic easier if they have a good grasp of their times tables and are familiar with writing their working down.

At the start of S1, pupils are introduced to simplifying expressions. There are many ways we introduce the using a picture, or verbally talking through examples.

## Example 1:

Simplify:
$3 a+2 b+4 a$
$=7 a+2 b$

We use the phrase "like terms" when simplifying an expression.

After we complete negative numbers (approx. October S1) we then introduce simplifying with negatives.

## Example 2:

$4 c+5 d-6 c-7 d$
$=-2 \mathrm{c}-2 \mathrm{~d}$

## Evaluating Expressions

Setting out of solutions is important.
Work should be clear.
We expect pupils to use the following steps when answering questions:

- Expression
- Substitution
- any working that may be required
- Answer

The equal symbols should be in a vertical line, one below the other.

$$
=
$$

$=$
$=$

## Example 1

Evaluate the following when $a=4 b=3$ and $c=2$
a) $a+b+c$
b) $2 a+3 b-c$
c) $a b c$
$=4+3+2$
$=2 \times 4+3 \times 3-2$
$=4 \times 3 \times 2$
$=9$
$=8+9-2$
$=24$
$=15$

## Equations

Across the Cluster we use the "balancing" approach to solve equations. Pupils are first introduced in Primary 7 to the balancing method and it is then explored in more detail in S1.

This approach extends to use in all areas. For example, with Pythagoras, Trigonometry, and in Changing the subject of a formula.

Setting out of solutions is important.
Work should be clear.
We expect pupils to use the following steps when answering questions:

- Formula/Equation
- Substitution
- any working that may be required
- Answer with units (if required)

The equal symbols should be in a vertical line, one below the other.
$=$
$=$
=

## Example 1: $2^{\text {nd }}$ Level



$$
x+1=5
$$

$$
x=4
$$

## Example 2: $2^{\text {nd }}$ level



$$
\begin{aligned}
& \underline{2 x}=\underline{4} \\
& 2 \\
& x=2
\end{aligned} \quad \text { divide both sides by } 2
$$

## Example 1: $3^{\text {rd }}$ Level



| $2 x+$1 <br> -1 | $=$5 <br> -1 | Subtract 1 from both sides |  |
| ---: | :--- | :--- | :--- |
| $2 x$ |  |  |  |
| $\frac{2 x}{2}$ |  |  |  |
| $x$ |  |  |  |
|  |  |  |  |

## Example 2: $3^{\text {rd }}$ Level



The = signs should form a vertical line to clearly delineate the Left Hand Side and Right Hand Side of the equation.

## Example 1: $4^{\text {th }}$ Level (Pythagoras)

## Calculating the Hypotenuse



Always draw diagram:
Write down:

$$
\begin{aligned}
x^{2} & =6^{2}+8^{2} \\
& =36+64 \\
& =100 \\
x & =\sqrt{ } 100 \\
x & =10 \mathrm{~cm}
\end{aligned}
$$

## Formulae

Maths would expect the following when using formulae.

3 3rd
If $l=5 \mathrm{~cm}$ and $b=4 \mathrm{~cm}$, calculate the area, $A$.

| $A=l b$ | formula |
| :--- | :--- |
| $A=5 \times 4$ | substitute |
| $A=20 \mathrm{~cm}^{2}$ | calculation and units |

Find the volume of the cuboid, $I=4 \mathrm{~cm}, \mathrm{~b}=5 \mathrm{~cm}$ and $\mathrm{h}=3 \mathrm{~cm}$.

$$
\begin{aligned}
V & =l b h \\
& =4 \times 5 \times 3 \\
& =60 \mathrm{~cm}^{3}
\end{aligned}
$$

A more advanced formula used in National 5
(cosine rule)
Calculate a if $b=5 \mathrm{~cm}, c=4 \mathrm{~cm}$ and $\angle A=60^{\circ}$
$a^{2}=b^{2}+c^{2}-2 b c \cos A \quad$ formula
$a^{2}=5^{2}+4^{2}-2 \times 5 \times 4 \cos 60^{\circ} \quad$ substitute
$a^{2}=21 \quad$ calculation
$a=4.58 \mathrm{~cm} \quad$ answer with units
This is also an example of the minimum amount of working specified by the SQA.

## Inequalities

The above method can also be used for solving inequalities.

## 2018 P2 National 5

$3 x<6(x-1)-12$
$3 x<6 x-6-12$
multiply out brackets
$3 x<6 x-18$
$-3 x \quad-3 x$
$3 x<-18$
$x>6$
gathered together "like terms"
answer

## Expanding Brackets

When expanding brackets, we introduce using the grid method. This method is introduced in primary when multiplying (partioning numbers).

## Example

$4(x+3)$
$=4 x+12$

|  | $x$ | +3 |
| :---: | :---: | :---: |
| 4 | $4 x$ | +12 |

We can then expand this for double brackets and beyond.

$$
(x+2)(x+3)
$$

$=x^{2}+3 x+2 x+6$

|  | $x$ | +2 |
| :---: | :---: | :---: |
| $x$ | $x^{2}$ | $+2 x$ |
| +3 | $+3 x$ | +6 |

$=x^{2}+5 x+6$
$(x+2)\left(x^{2}+3 x-4\right)$
$=x^{3}+3 x^{2}-4 x+2 x^{2}+6 x-8$

|  | $x^{2}$ | $+3 x$ | -4 |
| :---: | :---: | :---: | :---: |
| $x$ | $x^{3}$ | $+3 x^{2}$ | $-4 x$ |
| +2 | $+2 x^{2}$ | $+6 x$ | -8 |

$=x^{3}+5 x^{2}+2 x-8$

## Time

The following list gives an idea of when we expect pupils to cover the different aspects of time during $1^{\text {st }}$ level.

Using 12 hour clock and link to 24 hour notation.
Plan and organise for events throughout day
Daily routine
Approximate Guideline
P2 : O'clock half hour
P3: Quarter past and quarter to
P4: 5 min intervals
Pupils are introduced to these through active learning. Eg ordering times, playing BINGO

Pupils do struggle with DIGITAL clock for "twenty to" etc

During $2^{\text {nd }}$ level pupils are introduced to the following:
Convert to and from analogue- digital
Convert between Seconds/minutes/hours/days/weeks/months
Convert between 12 and 24 hour time

12 hour clock times should be written with am/pm and dot between hours and minutes
e.g. 3.12am
10.15am
7.35pm

24 hour clock times should be written with four numbers
e.g. 0312
1015
1935

Reading and Interpreting Timetables

## Calculating duration of time intervals

Use horizontal line broken into stages of time. Stages are broken into minutes and/or hours depending on duration.


But pupils can decide on stages e.g.
$0915 \longrightarrow 0930 \longrightarrow 1000 \longrightarrow 1500 \longrightarrow 1534$
$15 \mathrm{~min}+30 \mathrm{~min}+5 \mathrm{hrs}+34 \mathrm{mins}=5 \mathrm{hrs} 79 \mathrm{mins}=6 \mathrm{hrs} 19 \mathrm{mins}$

