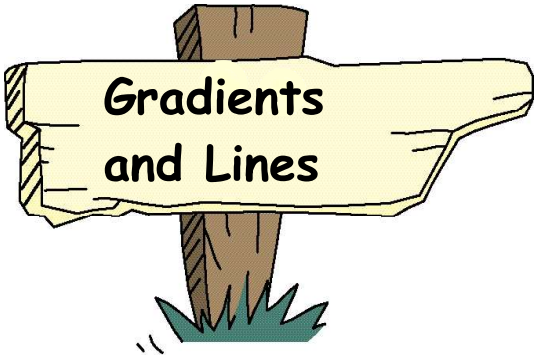
Chapter 6

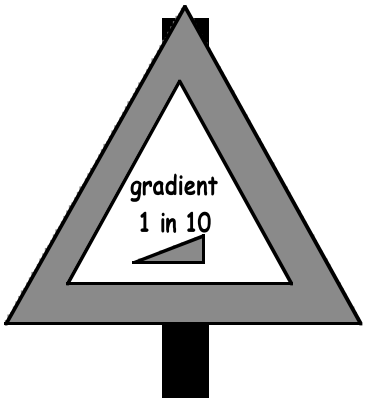


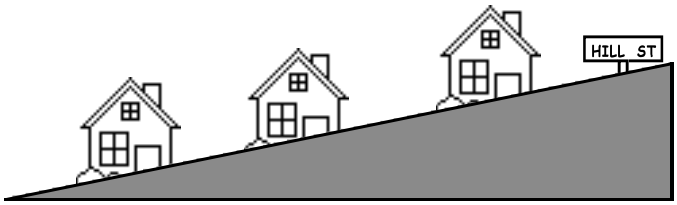
Gradients

We can measure how steep a hill or road is
or how steeply a ladder is resting against a wall.

This is called the slope or the **GRADIENT** of the hill or ladder.

The gradient of a hill is usually written as a fraction.
(*it can be given as a decimal or as a percentage*)

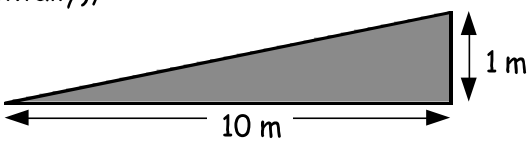




Hill Street has a gradient of 1 in 10.
This is written as

gradient = $\frac{1}{10}$

This means that for every 10 metres moved across (horizontally),
the road rises by 1 metre up the way (vertically).

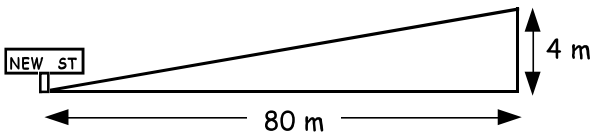


How to calculate the gradient of a hill.

Example :- New Street rises by 4 metres.
It is 80 metres (horizontally)
from the top end to the bottom.

Gradient = 4 metres in 80 metres

$= \frac{4}{80} \overset{\div 4}{\div 4} = \boxed{\frac{1}{20}}$



[Can you see that $\frac{1}{20}$ is **smaller** than $\frac{1}{10}$]

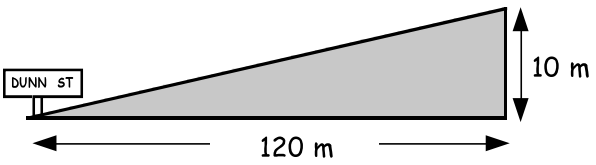
—> this means New Street is **less** steep than Hill Street.

Definition :- Gradient = $\frac{\text{vertical distance}}{\text{horizontal distance}}$

LEARN

Exercise 1 (You MAY use a calculator)

1. Look at this picture of Dunn Street.



(a) Calculate the gradient like this :-

Copy :-

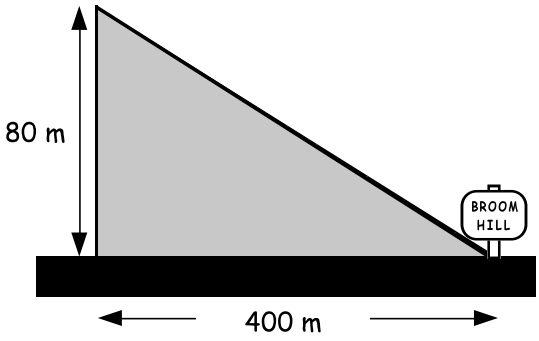
$$\text{Gradient} = \frac{\text{vertical distance}}{\text{horizontal distance}}$$

$$\Rightarrow \text{grad} = \frac{10}{120} \Rightarrow \text{grad} = \boxed{\frac{?}{?}}$$ (simplify the fraction $\frac{10}{120}$)

(b) Compare the gradient of Dunn Street with that of Hill Street and New Street (from the previous page).

Which is the :- (i) steepest ?
(ii) least steep ?

2.



Look at the sketch of Broom Hill.

Calculate the gradient of the hill.

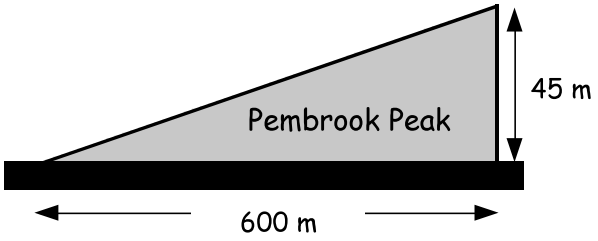
Copy :-

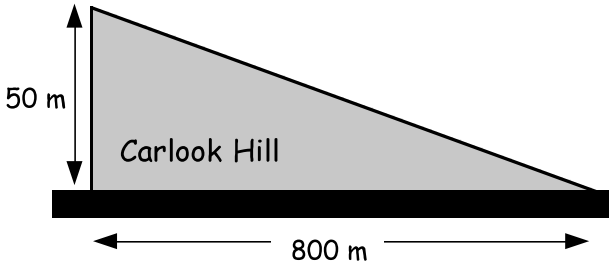
$$\text{Gradient} = \frac{\text{vertical distance}}{\text{horizontal distance}}$$

$$\Rightarrow \text{grad} = \frac{80}{\ldots\ldots} = \boxed{\frac{?}{?}}$$

(simplify the fraction)

3. Shown below are two small hills.





<u>Here is how we can find which one is steeper</u>.

Copy :-

(a) Pembrook Peak – Gradient = $\frac{\text{vert}}{\text{horiz}} = \frac{45}{600}$ (can this be simplified ?)

(b) Find the gradient of Carlook Hill the same way.

$$\text{Gradient} = \frac{\text{vert}}{\text{horiz}} = \frac{50}{800} = \boxed{\quad}$$

cont'd

(c) It is NOT very easy to look at these 2 fractions and say which one is bigger.
To do this you :-

CHANGE THE FRACTIONS —> DECIMALS

(i) Pembrook – Gradient = $\frac{45}{600}$ = 45 ÷ 600 (Remember) = 0·075

(ii) Carlook – Gradient = $\frac{50}{800}$ = 50 ÷ ..... = 0·.....

(iii) Which is the bigger ? (which hill is steeper ?)

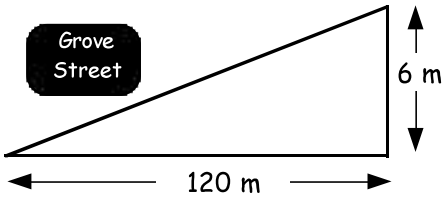
4. Shown below are the side views of 4 roads.

(i) Write down the gradient of each hill (as a fraction).

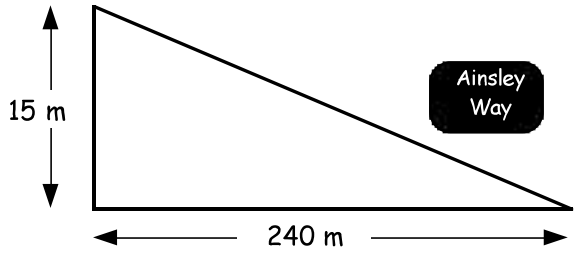
(ii) Change each fraction to a decimal (see question 3c).

(iii) Write the 4 hills in order, steepest first.

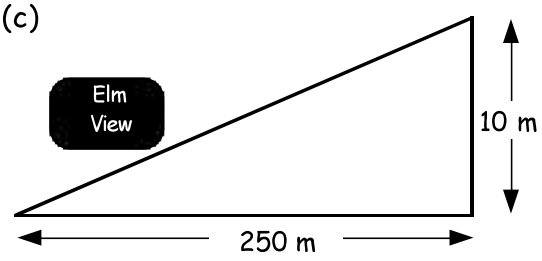
(a)



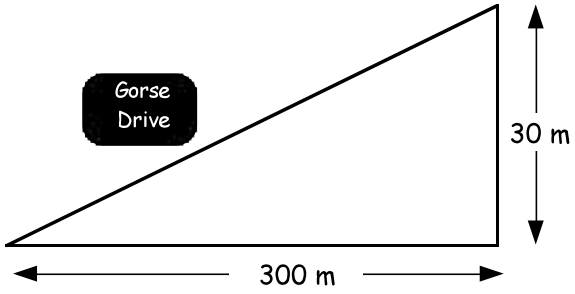
(b)



(c)



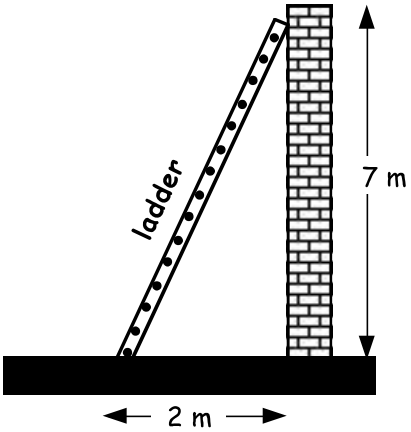
(d)



Set down like this :- Grove Street —> Grad = $\frac{\text{vert}}{\text{horiz}}$ = $\frac{6}{120}$ = (6 ÷ 120) = 0·.....

5. This picture shows a ladder placed against a wall.

You can measure how steep the ladder is (the gradient) in the same way you worked out the gradient of the hills and roads.



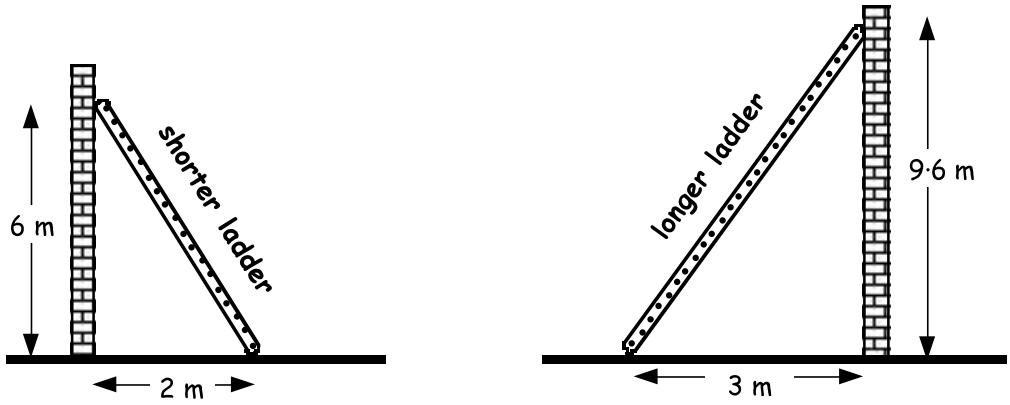
Copy :- Gradient = $\frac{\text{vert}}{\text{horiz}}$

=> Grad = $\frac{7}{2}$ = (7 ÷ 2) = 3·....

(This is a much bigger gradient (3·5) than any of the hills measured in Question 4).

This means the ladder is resting <u>quite steeply</u> against the wall.

6. A window cleaner uses two ladders in his job.



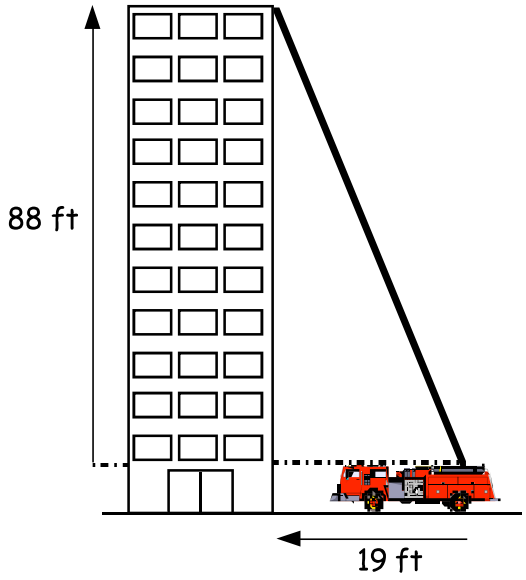
(a) Calculate the gradient of the shorter ladder.

Grad = $\frac{\text{vert}}{\text{horiz}} = \frac{6}{\ldots} = \boxed{\frac{?}{?}}$

(b) Calculate the gradient of the longer ladder.

(c) Which ladder lies at a steeper angle to the ground ?

7.

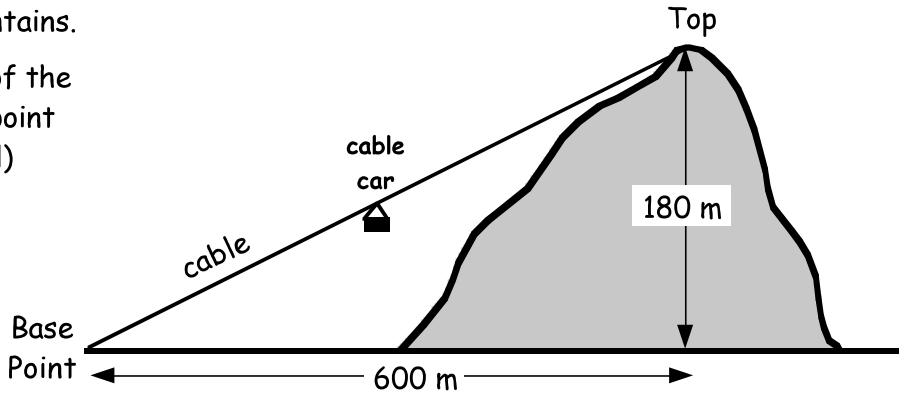


The fire engine used its extended ladder to rescue someone from the top of this building.

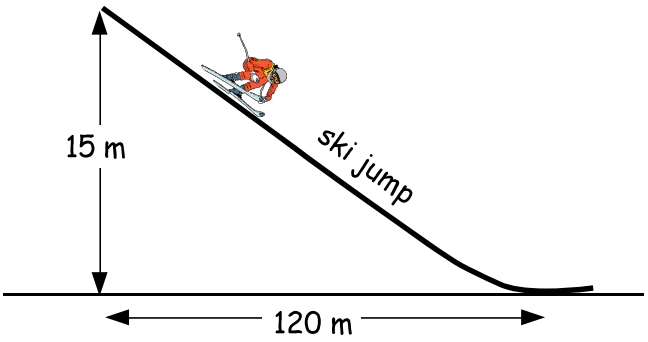
Calculate the gradient of the ladder. (as a decimal)

8. This is a picture of a cable car on the Craighorne mountains.

Calculate the gradient of the cable joining the base point to the top. (as a decimal)

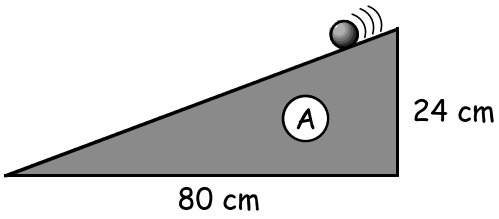


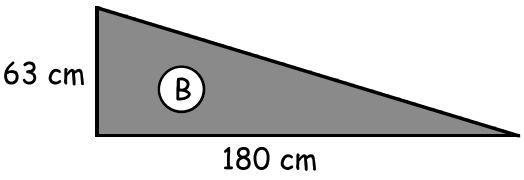
9. Calculate the gradient of this ski jump.

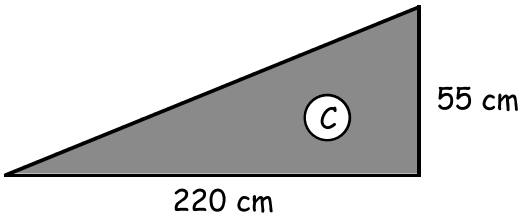


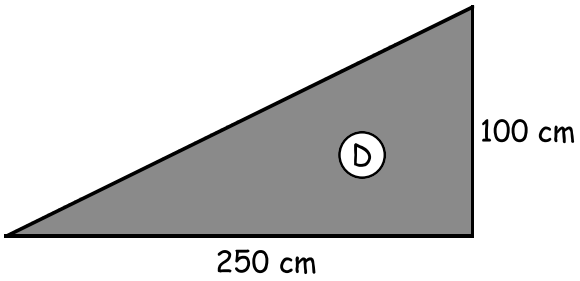
10. The steeper the slope – the faster a ball-bearing will run down it.

Calculate the gradient of each of the following slopes and say which is steepest and which is "shallowest".









Lines (Equations)

Reminder – coordinates

This is called a coordinate diagram.
(or a Cartesian Diagram)

The 2 main lines are called axes.

The horizontal one is the *x*-axis.
The vertical one is the *y*-axis.

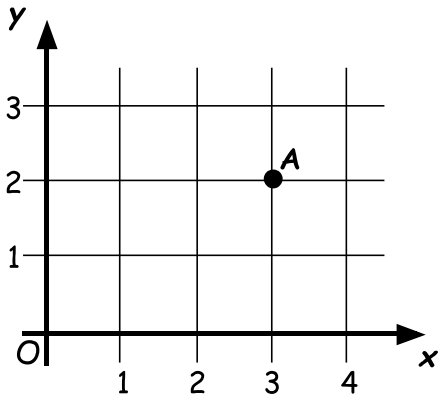
The point where they meet is the **ORIGIN**.

The point A is "3 to the right and 2 up" from the origin.

We say it has coordinates A (3 , 2)

horizontal or *x*-coord

vertical or *y*-coord



Ex 3

1. a. 10·8 cm b. 17·9 cm
   c. 10·7 m d. 20·4 mm
   e. 66·2 cm f. 4·2 m
2. x must be smaller than the hypotenuse
3. 19·6 = t must be smaller than 23
4. 6·3 cm
5. 146 cm
6. 250 m
7. 40·3 cm
8. a. 48 ft b. 40 ft c. 8 ft
9. a. 2500 + 1400 + 2865 = 6765 m
10. 2·67 m

Review 5 — Algebra

1. a. 8a b. 7f
   c. 9t d. 13d + 3
   e. 4w – 5 f. 2y + 11
   g. 13m + 8n h. u
   i. 2g + 7h j. 4a + 1
   k. t l. $5x^2$
   m. p^2 n. $5a^2 + 7a$
   o. 0
2. a. 5x + 5 b. 6x – 24
   c. 3x – 6 d. 7x + 56
   e. 2a + 2b f. 5p – 5q
   g. 6x + 8 h. 8w – 4
   i. 10x + 10y + 20z j. 6a – 9b + 3c
   k. 15p – 10q + 20r l. 7a + 14b – 35
   m. 3t – 4s n. 3a + 2b + c
   o. 3a + 2b – 4c
3. a. 4x + 10 b. 7y
   c. 9t + 6 d. 6m + n
   e. 4x + 14 f. 3x + 1
   g. 6x + 1 h. 11x + 8y
   i. 11g + 2h
4. a. 7 b. 15 c. 10 d. 9
   e. 3·5 f. 7 g. 6 h. 5
   i. 5 j. 8 k. 6 l. 3·5
   m. 7 n. 10 o. 6 p. 0
   q. 11 r. 3

Answers to Chapter 6

Ex 1

1. a. $^1/_{12}$ b. since $^1/_{12} > {}^1/_{20}$, it is steeper
2. $^1/_5$
3. a. $^3/_{40}$ b. $^1/_{16}$ c. 0·075, 0·0625
   Pembroke is the steeper
4. a. $^6/_{120} = 0{\cdot}05$ b. $^{15}/_{240} = 0{\cdot}0625$
   c. $^{10}/_{250} = 0{\cdot}04$ d. $^{30}/_{300} = 0{\cdot}10$
   Gorse – Ainsley – Grove – Elm
5. 3·5
6. a. $^6/_2 = 3$ b. $^{9{\cdot}6}/_3 = 3{\cdot}2$ c. longer
7. 4·63
8. 0·3
9. 0·125
10. A – 0·3, B – 0·35, C – 0·25, D – 0·4
    –> steepest = D, shallowest = C

Ex 2

1. a. y = 2 x x coordinate
   b. yes (each time)
   c. a = 10 d. b = 16
2. a. (0,0) (1,3) (2,6) (3,9) b. * = 3
   c. y = 3 x x or y = 3x d. * = 12
3. a. (0,0) (1,1) (2,2) (3,3) (4,4) (5,5) (6,6)
   b. y = 1 x x
   c. y = x
4. a. (0,0) (2,1) (4,2) (6,3)
   b. works c. $y = {}^1/_2x$
5. i) a. (0,0) (3,1) (6,2) , $y = {}^1/_3x$
   ii) a. (0,0) (1,4) (2,8) , y = 4x

Ex 3

1. a. 0,2,4,6
   b. (0,0) (1,2) (2,4) (3,6)
   c. See diagram
2. a. 1,3,5,7
   b. (1,1) (3,3) (5,5) (7,7)
   c. See diagram
3. a. 0,1,2,3
   b. (0,0) (2,1) (4,2) (6,3)
   c. See diagram
4. a. 0,3,6,9
   b. (0,0) (1,3) (2,6) (3,9)
   c. See diagram
5. a. 0,1,2,3
   b. (0,0) (4,1) (8,2) (12,3)
   c. See diagram
6. a. 0,4,8,12
   b. (0,0) (1,4) (2,8) (3,12)
   c. See diagram
7. a. 0,6,12,18
   b. (0,0) (1,6) (2,12) (3,18)
   c. See diagram
8. a. 0,3,6,9
   b. (0,0) (2,3) (4,6) (6,9)
   c. See diagram
9. a. 0,10,20,30
   b. (0,0) (1,10) (2,20) (3,30)
   c. See diagram

Ex 4

1. a. 3,5,7,9
   b. (0,3) (1,5) (2,7) (3,9)
   c. See diagram
2. a. 2,5,8,11
   b. (0,2) (1,5) (2,8) (3,11)
   c. See diagram
3. a. –2,0,2,4
   b. (0,–2) (1,0) (2,2) (3,4)
   c. See diagram
4. a. 4,5,6,7
   b. (1,4) (2,5) (3,6) (4,7)
   c. See diagram
5. a. (0,1) (1,2) (2,5) (3,8) – graph
   b. (0,–1) (1,5) (2,9) (3,13) – graph
   c. (0,–2) (2,0) (4,2) (6,4) – graph
   d. (0,–3) (1,2) (2,7) (3,12) – graph
6. a. 2,3,4,5
   b. (0,2) (2,3) (4,4) (6,5)
   c. See diagram
7. a. –6 b. –12 c. –5 d. –20
   e. 1 f. 2 g. –6 h. –2
   i. –11 j. 2 k. –7 l. –5
8. a. –5,–3,–1,1,3
   b. (–2,–5) (–1,–3) (0,–1) (1,1) (2,3)
   c. See diagram
9. a. (–1,0) (0,2) (1,4) (2,6) – see diagram
   b. (–1,–2) (1,4) (3,10) (5,16) – see diagram
   c. (–2,2) (0,4) (2,6) (4,8) – see diagram
   d. (–4,–1) (–2,0) (0,1) (2,2) – see diagram

Review 6 — Areas

1. a. 13 cm^2 b. 20 cm^2
2. a. 77 cm^2 b. 800 mm^2 c. 66 m^2
3. a. 20 cm^2 b. 10 cm^2
4. a. 33 cm^2 b. 105 cm^2 c. 26 cm^2
5. a. 154 cm^2 b. 210 cm^2

Non Calculator Exercise No. 3

1. a. 11134 b. 2725 c. 1389000 d. 2952
   e. 60 f. 81 g. 11 h. 65
2. a. 4·15 b. 54·311 c. 43·88 d. 58·29
   e. 10258 f. 0·01122
3. a. 1085 g b. 2·36 m c. 9650 ml
4. a. 12 b. 6
5. a. $6^1/_2$ b. $3^3/_5$ c. $7^1/_4$ d. $10^2/_5$
   e. 8 f. 7
6. a. $^3/_5$ b. $^3/_4$ c. $^7/_8$
7. a. 0·86 b. 0·04 c. 0·172
8. a. £35 b. 18 kg c. 142 d. 60p
   e. 36p f. 39p
9. 32 males
10. a. –10, –5, –1, 0, 7 b. –42, –17, 35, 44
11. a. 15 b. – 11 c. – 6 d. – 65
    e. –27 f. – 13 g. 10 h. –2
    i. 0 j. –40 k. – 10
12. –£15
13. a. 2·05 pm b. 11·55 am
    c. 2·10 am
14. a. $1^1/_2$ b. $3^1/_4$ c. $^1/_5$
15. a. 30 b. 12
16. 16·55, 18·8, 21·81, 22·4

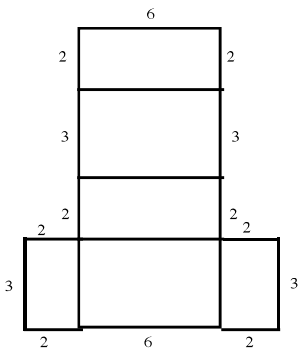
Answers to Chapter 7

Ex 1

1. Total = 10 + 10 + 15 + 15 + 6 + 6 = 62 cm^2
2. a. = 15 + 15 + 18 + 18 + 30 + 30 = 126 cm^2
   b. = 40 + 40 + 20 + 20 + 8 + 8 = 136 cm^2
   c. = 18 + 18 + 18 + 18 + 9 + 9 = 90 cm^2
   d. = 72 + 72 + 6 + 6 + 3 + 3 = 162 cm^2
3. a. same b. 25 cm^2 c. no d. 150 cm^2
4. a. 96 cm^2 b. 54 cm^2
   c. 600 cm^2 d. 6 cm^2
   e. 294 cm^2
5. 35 + 35 + 21 + 21 + 15 + 15 = 142 m^2
6. 240 + 120 + 120 + 200 + 200 = 880 cm^2
7. a. = 165 + 150 + 150 + 110 + 110 = 685 cm^2
   b. = 64 + 120 + 120 + 120 + 120 = 544 cm^2
   c. = 400 + 50 + 50 + 50 + 50 = 600 cm^2
8. a. 110 cm^2 b. 9 cm^2
   c. 74 cm^2 d. O.K.
9. a. 10 x 7 =70 cm^2 b. 4 cm^2
   c. 54 cm^2
10. a. 6 b. 12 c. 8 d. 1
11. a. 24 b. 24 c. 8 d. 8
12. 24 + 32 + 40 + 6 + 6 = 108 cm^2
13. a. = 70 + 42 + 56 + 24 + 24 = 216 cm^2
    b. = 130 + 50 + 120 + 30 + 30 = 360 cm^2

Ex 2

1. a. See net



   b/c. = 18 + 18 + 12 + 12 + 6 + 6 = 72 cm^2
2. a. See net
   b. = 12 + 12 + 12 + 12 + 9 + 9 = 66 cm^2
3. a. See net
   b. 9 x 6 = 54 cm^2
4. a. 96 cm^2 b. 64 cm^2
   c. 88 cm^2 d. 148 cm^2
5. a/b. See net
   c/d. = 30 + 24 + 18 + 6 + 6 = 84 cm^2

Review 7 — Wages/Salaries

1. £20104·80
2. £225
3. a. £12·40 b. £9·30
4. £74·40
5. a. £183·60 b. £32·40
   c. £216
6. a. £495 b. £16995
7. £252
8. £357·40
9. £800·28 – £217·69 = £582·59