# Volumes of Solids

You should know from National 4 how to calculate the volume of a **prism**. At National 5 level, you also need to be able to calculate the volume of a **pyramid**. Throughout this topic remember that:

- all volume questions must have answered in cubic units (e.g. $m^3$, $cm^3$, $inches^3$)
- you should always state your unrounded answer before rounding (see page 6)

**Formula.** This formula is **not** given on the National 5 Mathematics exam paper.

**Volume of a Prism:** $V = Ah$

Volume = Area of cross section × Height

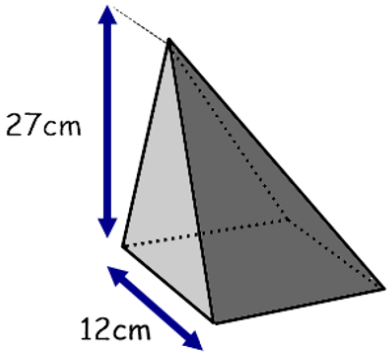
**Formula.** This formula is given on the National 5 Mathematics exam paper.

**Volume of a Pyramid:** $V = \frac{1}{3}Ah$

$\text{Volume} = \frac{1}{3}\text{Area of Base} \times \text{Height}$

Example 1 – Pyramid

**The diagram shows a pyramid with height 27cm and a square base with sides of length 12cm. Calculate the volume of the pyramid.**




**Solution**

The area of a square is given by the formula $A = L^2$, so the area of the base of this pyramid is $12^2 = 144cm^2$

Therefore the volume of the whole pyramid is

$$V = \frac{1}{3}Ah$$
$$= 144 \times 27 \div 3$$
$$= \underline{\underline{1296cm^3}}$$

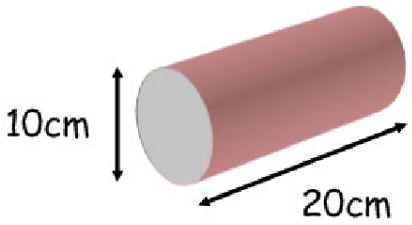
Special cases of prisms and pyramids are when the cross-sectional area of the prism is a circle (in which case you have a cylinder) or when the base of a pyramid is a circle (giving a cone). In these cases, we can adapt the earlier formulae to give us a quicker formula:

**Formula.** This formula is **not** given on the National 5 Mathematics exam paper.

**Volume of a Cylinder:** $V = \pi r^2 h$

Example 2 – cylinder

**Work out the volume of this cylinder. Round your answer to 2 significant figures.**




**Solution**

Diameter is 10cm so radius is 5cm

$$V = \pi r^2 h$$
$$= \pi \times 5^2 \times 20 \quad (\text{or } \pi \times 5 \times 5 \times 20)$$
$$= 1570 \cdot 796327....$$
$$= \underline{1600cm^3 \text{ (2 s.f.)}}$$

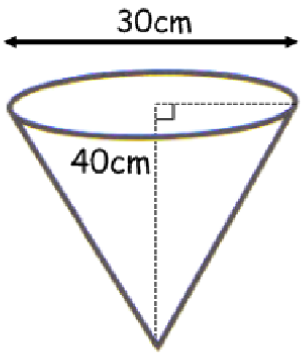
**Formula.** This formula is given on the National 5 Mathematics exam paper.

**Volume of a Cone:** $V = \frac{1}{3}\pi r^2 h$

In the formula for the volume of a cone, the 'height' $h$ refers to the **perpendicular** height (the one that goes straight up) and not any sloping heights.

Example 3 – cone

**Calculate the volume of this cone. Round your answer to 3 significant figures.**




**Solution**

Diameter is 30cm so radius is 15cm

$$V = \frac{1}{3}\pi r^2 h$$

$$= \pi \times 15^2 \times 40 \div 3 \quad (\text{ or } 1 \div 3 \times \pi \times 15^2 \times 40)$$

$$= 9424 \cdot 777961....$$

$$= \underline{9420\text{cm}^3} \text{ (3 s.f.)}$$

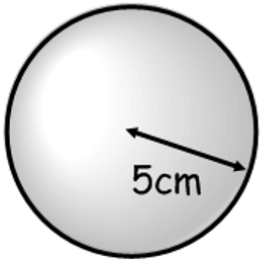
You are also expected to know how to calculate the volume of a sphere

**Formula.** This formula is given on the National 5 Mathematics exam paper.

**Volume of a Sphere:** $V = \frac{4}{3}\pi r^3$

Example 4 – sphere

**Calculate the volume of this sphere. Round your answer to 1 significant figure.**




**Solution**

Radius is 5cm

$$V = \frac{4}{3}\pi r^3$$

$$= \pi \times 5^3 \div 3 \times 4 \quad (\text{ or } 4 \div 3 \times \pi \times 5 \times 5 \times 5)$$

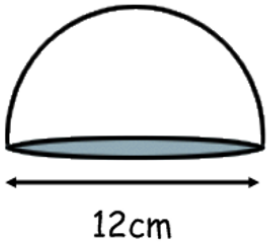
$$= 523 \cdot 5987756....$$

$$= \underline{500\text{cm}^3 \text{ (1 s.f.)}}$$

**Definition:** A **hemisphere** is half of a sphere.

Example 5 – hemisphere

**Calculate the volume of this hemisphere. Round your answer to 4 significant figures.**




**Solution**

Diameter is 12cm so radius is 6cm

$$V = \frac{4}{3}\pi r^3 \div 2$$

$$V = \pi \times 6^3 \div 3 \times 4 \div 2$$

$$V = 452 \cdot 3893421....$$

$$\underline{V = 452 \cdot 4\text{cm}^3 \text{ (4 s.f.)}}$$

## Composite Shapes

In the exam, you may be expected to deal with a shape formed from more than one other shape joined together. If the diagram is confusing you and you are not sure what the shape in the question is, then read the question carefully.

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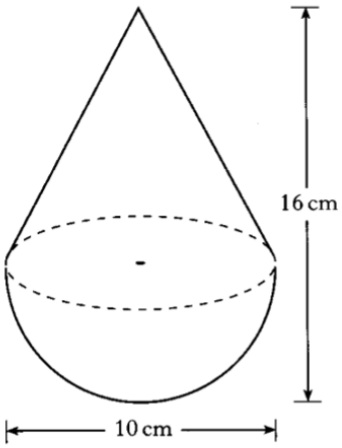
Example (2006 Intermediate 2 Paper 2)

**A child's toy is in the shape of a hemisphere with a cone on top, as shown in the diagram. The toy is 10 centimetres wide and 16 centimetres high. Calculate the volume of the toy. Give your answer correct to 2 significant figures.**




**Solution**

The cone and the hemisphere have the same radius, 5cm.

The 16cm line in the picture is made of the height of the cone plus the radius of the sphere.
Height of cone + 5cm = 16cm
i.e. height of cone is 16 – 5 = 11cm.

Volume of cone:

$$V = \frac{1}{3}\pi r^2 h$$
$$= \pi \times 5^2 \times 11 \div 3$$
$$= 287 \cdot 98...$$

Volume of hemisphere

$$V = \frac{4}{3}\pi r^3 \div 2$$
$$= \pi \times 5^3 \div 3 \times 4 \div 2$$
$$= 261 \cdot 8...$$

Total volume = 287·98 + 261·8 = 549·7… = 550cm³ (2 s.f.)

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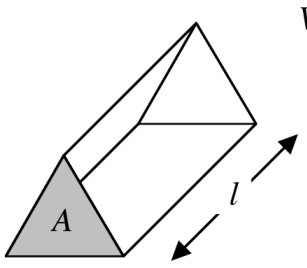
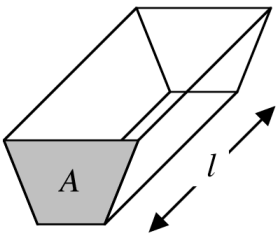
## CHAPTER 5: VOLUMES OF SOLIDS

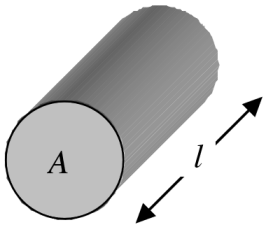
PRISMS a solid with the same cross-section throughout its length.
length $l$ is at right-angles to the area A.



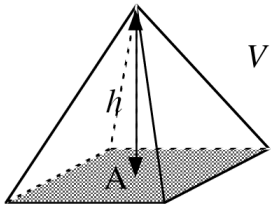

$V = Al$



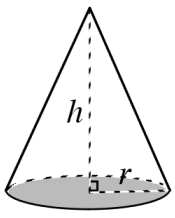

*cylinder* $V = \pi r^2 h$




PYRAMIDS

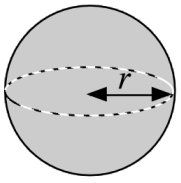



$V = \frac{1}{3}Ah$




*cone*
$V = \frac{1}{3}\pi r^2 h$

SPHERE




$V = \frac{4}{3}\pi r^3$