Outcome 4 HOMEWORK

1. The sum of the first n terms of an arithmetic series is $n(n+5)$.

   Find the first three terms of the series.

2. a) In an arithmetic series of 9 terms, the first term is 5 and the last is 23.

   Find the sum of the 9 terms.

   b) Find the sum of the first seven terms of a geometric series that has an eighth term of $\frac{2}{3}$ and a fifth term of 18.

3. For each of the following geometric series state whether a sum to infinity exists, and if so find it.

   a) $84-42+21-10\frac{1}{2}+5\frac{1}{4}-\ldots$

   b) $8+12+18+27+40\frac{1}{2}+\ldots$

   c) $64-16+4-1+\frac{1}{4}-\ldots$

4. The 3rd term of an arithmetic series is 17 and the 7th term is 33.

   Find a formula for the sum of n terms and find the sum of 20 terms.

5. Find the values of x for which $x-6$, $2x$ and $8x+20$ are consecutive terms of a geometric series.

6. Find a) $\sum_{k=11}^{20}(2k+1)$ b) $\sum_{k=1}^{\infty}\frac{1}{2^{k-1}}$

7. Expand the following as geometric series and state the necessary condition on x for each series to be valid.

   a) $\frac{1}{1+x}$ b) $\frac{1}{4-x}$ c) $\frac{1}{3+x}$

8. By first expressing $\dfrac{1}{x^2+3x+2}$ in partial fractions, use the fact that for $-1<r<1$

$$\frac{1}{1-r}=1+r+r^2+\ldots=\sum_{r=0}^{\infty} r^k$$

to express $\dfrac{1}{x^2+3x+2}$ in the form $\sum_{k=0}^{\infty} a_k x^k$.

Give an expression for the coefficient a_k, $k=0,1,2,\ldots$, and state the real values of x for which the series is valid.

9. Find $\sum_{k=1}^{n} k$ and $\sum_{k=1}^{n}(2k-1)$.

For all positive integral values of n, the sum of the first n terms of a series is $3n^2+2n$.

Find the nth term in its simplest form.

10. The population of a colony of insects increases in such a way that if it is N at the beginning of a week, then at the end of the week it is $a+bN$, where a and b are constants and $0<b<1$.

a) Starting from the beginning of the week when the population is N, write down an expression for the population at the end of one, two, three and four weeks.

   Show that at the end of n consecutive weeks the population is

$$a\left(\frac{1-b^n}{1-b}\right)+b^n N.$$

b) When a = 2000 and b = 0.2, it is known that the population takes about 4 weeks to increase from N to $2N$.

   Estimate a value for N from this information.