1 Volumes of Solids

It is assumed at National 5 level maths that you are familiar with calculating areas for a range of shapes and composite shapes. It is important that you remember area formulas as they may be needed in exam questions and they will not appear on the formula sheet. The box below lists some of the formulae that you should know.

Area of a rectangle or square	$=$ length $\times$ breadth
Area of a triangle	$= \frac{1}{2}$ base $\times$ height
Area of a circle	$= \pi r^2$
Area of a kite with diagonals d_1 and d_2	$= \frac{1}{2} d_1 \times d_2$

Volumes of Cuboids and Cylinders

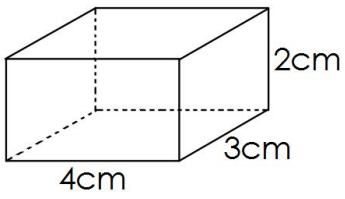
For National 5 maths, it is assumed that you know the basic formulae for volumes of cubes and cuboids. That is :

Volume of a cuboid $=$ length $\times$ breadth $\times$ height

Volume of a Cylinder (height h) $= \pi r^2 h$

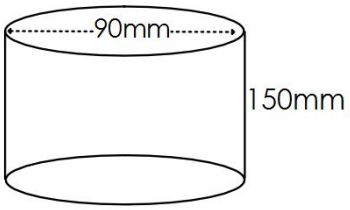
Examples

1. Find the volume of the cuboid shown below.



$$\begin{aligned} \text{Volume of cuboid} &= lbh \\ &= 4 \times 3 \times 2 \\ &= 24cm^3. \end{aligned}$$

2. Find the volume of the cylinder shown below.



The cylinder has radius $90 \div 2 = 45$mm.

$$\begin{aligned} \text{Volume of Cylinder} &= \pi r^2 h \\ &= \pi \times 45^2 \times 150 \\ &= 954258.768mm^3 \end{aligned}$$

For the following solids, the volume formulae are provided on the formula sheet but you must learn how to use them correctly.

Volume of a Sphere

The volume of a sphere of radius, r, is given by the formula:

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

Examples

1. Caluclate the volume of a sphere of radius 5cm.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \pi \times 5^3 \\ &= \frac{4}{3} \times \pi \times 125 \\ &= 523.599 cm^3 (3d.p.) \end{aligned}$$

2. Calculate the volume of a sphere with diameter 8cm.

   If the diameter = 8cm, then the radius = 8 ÷ 2 = 4cm.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \pi \times 4^3 \\ &= \frac{4}{3} \times \pi \times 64 \\ &= 268.083 cm^3 (3d.p.) \end{aligned}$$

3. The Earth has a radius of approximately 3959 miles. Calculate the volume of the Earth. Round your answer to 3 significant figures.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \pi \times 3959^3 \\ &= 259\,923\,241\,563.71 \\ &= 260\,000\,000\,000 \text{ miles}^3 (3s.f.) \end{aligned}$$

4. Find the volume of a hemisphere with diameter 20m.

   A hemisphere is half of a sphere. So, to find the volume of a hemisphere, find the volume of a sphere and then half your answer.

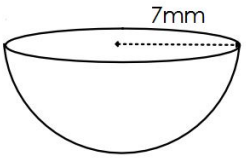
   As diameter is 20m, the radius must be 10m.

$$\begin{aligned} \text{Volume of Sphere} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \pi \times 10^3 \\ &= \frac{4}{3} \times \pi \times 1000 \\ &= 4188.79cm^3(3d.p.) \end{aligned}$$

   Therefore,

$$\text{Volume of hemisphere} = 4188.79 \div 2 = 2094.40m^3(2d.p.).$$

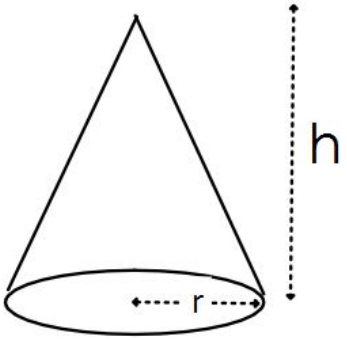
5. Find the volume of this hemisphere.



$$\begin{aligned} \text{Volume of Sphere} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \pi \times 7^3 \\ &= 1436.755cm^3 \\ \text{Volume of Hemisphere} &= 1436.755 \div 2 = 718.38cm^3 \end{aligned}$$

Volume of a Cone

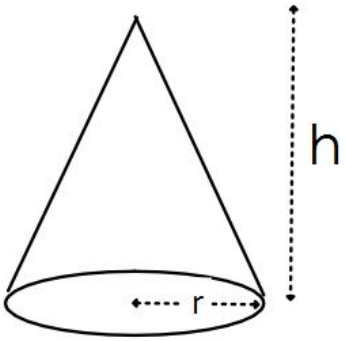
The volume of a cone of radius, r and height, h, is given by the formula:



$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

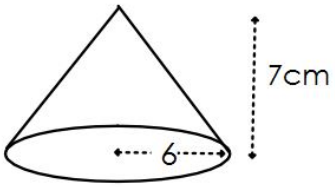
Examples

1. Calculate the volume of a cone of radius 6cm and height 7cm.



$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \pi \times 6^2 \times 7 \\ &= 263.894 cm^3 \end{aligned}$$

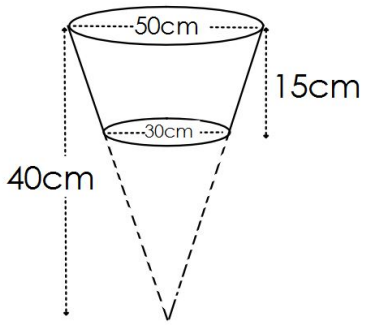
2. A manufacturer designs a new ice cream cone designed to be filled with 60ml of ice cream. The design is shown below. Will this design satisfy the requirements?
(Remember that 1ml = 1cm^3.)



$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \pi \times 3^2 \times 6 \\ &= 56.55 cm^3 \\ &= 56.55 ml \end{aligned}$$

As the cone only holds 56.55ml, 3.45ml short of 60ml, this cone design will not meet the requirements of the manufacturer.

3. A garden planter is designed as shown. Calculate the volume of the planter. Give your answer in litres.



The planter is made up of a large cone with a smaller cone cut off the bottom. To calculate the volume of the planter, the volumes of the two cones in the diagram must be calculated.

The larger cone has radius $50 \div 2 = 25$cm and height 40cm.

$$\begin{aligned} \text{Volume of Large Cone} &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \pi \times 25^2 \times 40 \\ &= 26179.939 cm^3 \end{aligned}$$

The small cone has height $40-15 = 25$cm and radius $30 \div 2 = 15$cm.

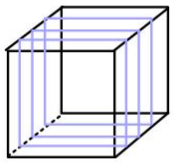
$$\begin{aligned} \text{Volume of Small Cone} &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \pi \times 15^2 \times 25 \\ &= 5890.486 cm^3 \end{aligned}$$

Therefore,

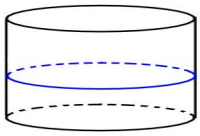
$$\begin{aligned} \text{Volume of Planter} &= 26179.939 - 5890.486 \\ &= 20289.453 cm^3 = 20289.453 ml = 20.289 litres. \end{aligned}$$

Volume of a Prism

A prism is a 3D shape with a uniform cross section. This just means that if a prism were to be cut into slices, each slice would be identical in shape and size. For example, a cube is a prism with a square cross section,



and a cylinder is a prism with a circle as cross section.

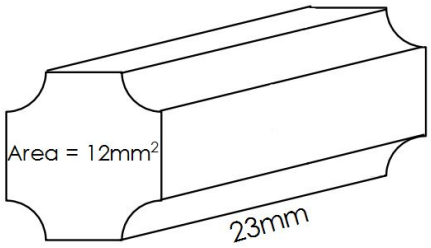


To find the volume of a prism, calculate the area of the cross section and multiply by the height of the prism.

> Volume of a prism = area of cross section × height

Examples

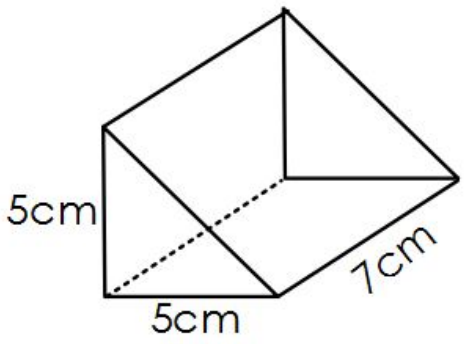
1. Calculate the volume of the prism shown below.



This is a prism with a cross sectional area of 12mm^2. Therefore,

$$\begin{aligned} \text{Volume of prism} &= \text{cross section area} \times \text{height} \\ &= 12 \times 23 \\ &= 276\text{cm}^3. \end{aligned}$$

2. Calculate the volume of the prism shown below.



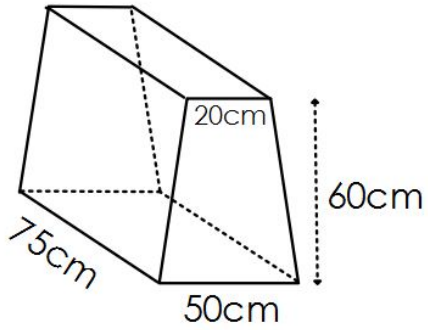
This is a triangular prism with a triangular cross section.

$$\begin{aligned} \text{Cross Section Area} &= \frac{1}{2}\text{base} \times \text{height} \\ &= \frac{1}{2} \times 5 \times 5 \\ &= 12.5cm^2. \end{aligned}$$

Now that the cross section area has been calculated, the formula can be applied to find the area of the triangular prism.

$$\begin{aligned} \text{Volume of triangular prism} &= \text{cross section area} \times \text{height} \\ &= 12.5 \times 7 \\ &= 87.5cm^3. \end{aligned}$$

3. Calculate the volume of the storage unit shown below. Give your answer in litres.



This storage unit has a trapezoidal cross section. The area of a trapezium is calculated using the formula

$$\text{Area of a trapezium} = \frac{1}{2}(a+b)h$$

where a and b are the parallel sides and h is the distance between them. So,

$$\begin{aligned} \text{Area of trapezium} &= \frac{1}{2}(a+b)h \\ &= \frac{1}{2}(50+20)\times 60 \\ &= \frac{1}{2}70\times 60 \\ &= 2100cm^2. \end{aligned}$$

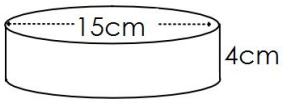
Therefore,

$$\begin{aligned} \text{Volume of storage unit} &= \text{cross section area} \times \text{height} \\ &= 2100\times 75 \\ &= 157500cm^3 \\ &= 157500ml = 157.5 \text{ litres.} \end{aligned}$$

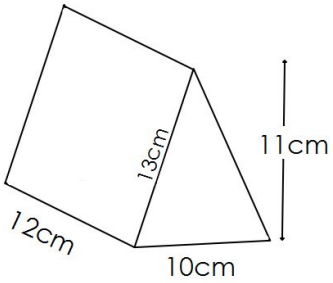
1.1 Volumes Practice Questions

1. Find the volumes of:
   (a) a sphere with radius 9.2mm.
   (b) a cylinder with radius 7.3cm and height 4.2cm.
   (c) a cuboid with dimensions 4cm,12cm and 18.4cm.
   (d) a cone with radius 15mm and height 36mm.
   (e) a hemisphere with radius 12cm.

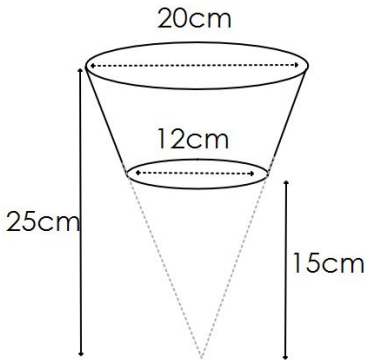
2. Find the volumes of:
   (a)



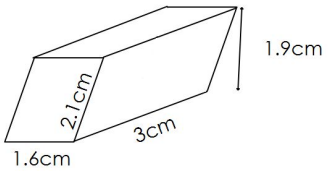
   (b)



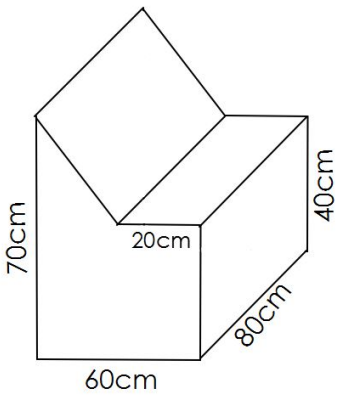
(c)


20cm
12cm
25cm
15cm

(d)


1.9cm
2.1cm
3cm
1.6cm

(e)


70cm
20cm
40cm
80cm
60cm