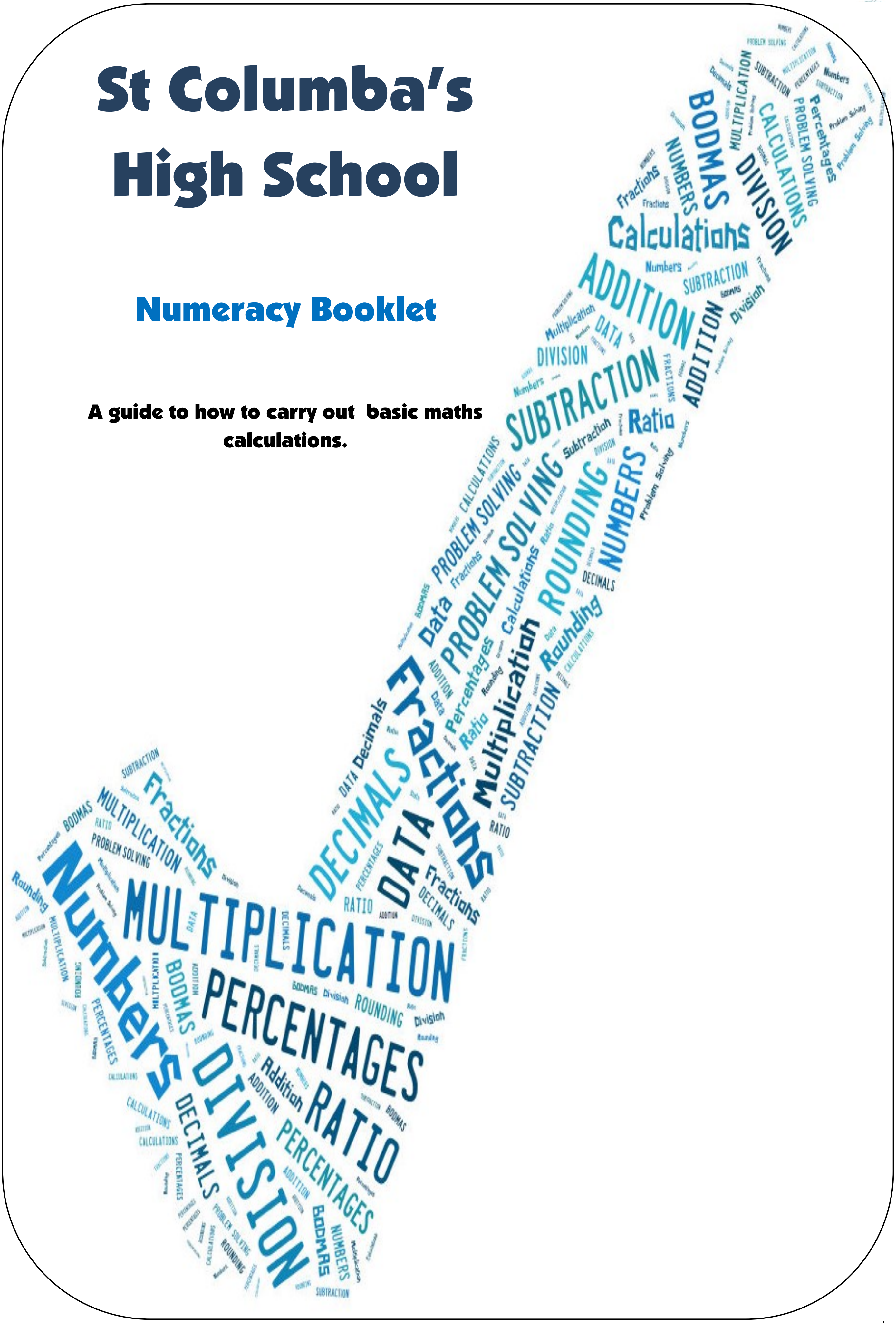
St Columba's High School

Numeracy Booklet

A guide to how to carry out basic maths calculations.



INTRODUCTION

Numeracy skills are an important part of a young persons learning. These basic maths skills will be encountered in various places in the school curriculum in many different departments. Numeracy is also vital in day to day life through shopping, dealing effectively with money and budgeting.

This booklet aims to show how these skills are taught and the methods that are used. This should allow pupils to experience a consistent approach across the curriculum and will allow parents to see the methods that their children will be using.

Throughout the booklet, there are links to relevant online videos explaining concepts in more detail. These are accessed via a QR reader (these can be downloaded to a mobile phone or tablet) and scanned to give instant access to the link.

There is an online maths dictionary for kids which provides a quick reference source for definitions and quick explanations of many maths topics. Use this link to go direct to the site.

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Written Methods for Addition

Words that often suggest that you need to do an addition calculation include:

Total Altogether Sum Increase **And** **More** ***Plus***

When addition calculations can't be carried out mentally, a written method can be used.

An addition sum is set out vertically. The digits must be lined up correctly one below the other. We always work from right to left. First, add the units, then the tens, hundreds, thousands etc.

Carrying digits, when necessary, are usually carried above the line.

If the addition involves decimals, make sure that the decimal points are lined up.

EXAMPLES

1.Find 456 + 213

$$\begin{array}{cccc} & H & T & U \\ & 4 & 5 & 6 \\ + & 2 & 1 & 3 \\ \hline & 6 & 6 & 9 \\ \hline \end{array}$$

2. Find 456 + 364

$$\begin{array}{cccc} & H & T & U \\ & 4 & 5 & 6 \\ + & 3_1 & 6_1 & 4 \\ \hline & 8 & 2 & 0 \\ \hline \end{array}$$

3. Find 325.6 + 364.3

$$\begin{array}{cccccc} & H & T & U & \cdot & t \\ & 3 & 2 & 5 & \cdot & 6 \\ + & 3 & 6 & 4 & \cdot & 3 \\ \hline & 6 & 8 & 9 & \cdot & 9 \\ \hline \end{array}$$

4. Find 12.4 + 13.21

$$\begin{array}{cccccc} & 1 & 2 & . & 4 & 0 \\ + & 1_1 & 3_1 & . & 2 & 1 \\ \hline & 3 & 2 & . & 2 & 1 \\ \hline \end{array}$$

add in a zero so there are the same number of digits after the point.

BASIC SUBTRACTION

Written Methods for Subtraction

When subtraction calculations can't be carried out mentally, a written method can be used.

Like addition, a subtraction sum is set out vertically. The digits must be lined up correctly one below the other.

Always work from right to left. Subtract the units, then the tens, hundreds, thousands etc. Subtractions must always be top number minus bottom number. It may sometimes be necessary to exchange (borrow) from another column to do this (see Example 2 below).

If the subtraction involves decimals, make sure that the decimal points are lined up in the working and the answer.

EXAMPLES

1.Find 3471—2140

$$\begin{array}{r} 3\,4\,7\,1 \\ -\quad 2\,1\,4\,0 \\ \hline 1\,3\,3\,1 \\ \hline \end{array}$$

2.Find 2556—1418

6 is smaller than 8 so we must borrow one ten from the next column.

$$\begin{array}{r} 2\;\;5\;\;\overset{4}{\cancel{5}}\;\;{}^{1}6 \\ -\;\;1\;\;4\;\;1\;\;8 \\ \hline 1\;\;1\;\;3\;\;8 \\ \hline \end{array}$$

3.Find 4000—356

we must exchange from the 4 thousands and work back to the units column.

We now exchange from this column to the tens column

and finally from the tens column to the units column.

$$\begin{array}{r} \overset{3}{\cancel{4}}\;\;{}^{1}\overset{9}{\cancel{0}}\;\;{}^{1}\overset{9}{\cancel{0}}\;\;{}^{1}0 \\ -\quad 3\;\;5\;\;6 \\ \hline 3\;\;6\;\;4\;\;4 \\ \hline \end{array}$$

Mental Methods for Addition and Subtraction

The first method for carrying out addition calculations is to see if you can do it mentally (in your head) without writing down working.

This is really useful if you are in a shop and need to quickly work out a total to pay or what change you should get.

There are various methods to try. A few examples are shown below.

EXAMPLES

1. Find 34 + 29

Method 1	round 29 to 30. 34 + 30 = 64 So 34 + 29 = 63
Method 2	Add 30 + 20 = 50 Then 4 + 9 = 13 So 34+ 29 = 50 + 13 = 63

1. Find 57 - 32

Method 1	round 32 to 30. 57 - 30 = 27 So 57 - 32 = 27 –2 = 25
Method 2	Do 50 - 30 = 20 Then 7 - 2= 5 So 57 - 32 = 20 + 5 = 25

Method 3 Count up. To go from 32 to 40 is 8. From 40 to 57 is another 17. So the difference is 17 + 8 = 25.

BASIC MULTIPLICATION

Multiplication is another important basic skill. Good knowledge of times tables helps!

Words that show that you need to carry out a multiplication calculation include:

MULTIPLIED BY **Multiple** *Product*

Groups of Lots of **Times**

When using a written method for multiplication, the sum is set out vertically and worked from right to left.

Carrying digits, if any, are placed in the correct column of the sum.

EXAMPLES

1. Find 21 x 7

```
  2 1
x   7
-----
1 4 7
-----
```

next, do 7 x 2 first, answer goes here

do 7 x 1 first, answer goes here

2. Find 39 x 5

```
   3 9
x  ₄5
------
 1 9 5
------
```

Start with 5 x 9 = 45. Write the unit, 5, in units column. Carry the tens, 4, in the tens column.

Now do 5 x 3 = 15. Add on the extra 4 to get 19.

3. Find 246 x 6

```
    2 4 6
x   ₂ ₃ 6
---------
  1 4 7 6
---------
```

Start with 6 x 6 = 36. Write the unit, 6, in units column. Carry the tens, 3, in the tens column.

Now do 6 x 4 = 24. Add on the extra 3 to get 27. Write the 7 in the tens column and carry the 2.

Finally, 6 x 2 = 12. Add on the carried 2 to get 14.

BASIC DIVISION

Division is the opposite of multiplication. Once again, good knowledge of times tables helps!

Words that show that you need to carry out a division calculation include:

DIVIDED BY **Equal groups of** *Share*

Share equally divided into **Split between**

When using a written method for division, the sum is set out in a specific way and worked from left to right.

EXAMPLES

1. Find 5 ÷ 7

$$\begin{array}{r} 1\ \text{r}2 \\ 5\overline{)\,7} \end{array} \quad \text{or} \quad \begin{array}{r} 1\cdot4 \\ 5\overline{)\,7\cdot{}^{2}0} \end{array}$$

In most situations, remainders are not used in secondary maths as a solution to a calculation. It is more common for the answer to be written as a decimal. However remainders are useful as shown below.

2. Find 345 ÷ 5

$$\begin{array}{r} 0\ 6\ 9 \\ 5\overline{)\,3\ {}^{3}4\ {}^{4}5} \end{array}$$

45 divided by 5 = 9

34 divided by 5 = 6 remainder 4.
Write the 6 above and carry the 4.

3 divided by 5 doesn't go so write 0 above and carry the three over

3. Find 4783 ÷ 5

$$\begin{array}{r} 0\ 9\ 5\ 6\cdot6 \\ 5\overline{)\,4\ {}^{4}7\ {}^{2}8\ {}^{3}3\cdot{}^{3}0} \end{array}$$

When we divide 33 by 5, there is a remainder of 3.
We then add a decimal point and a 0 to continue finding the answer as a decimal.

ROUNDING

Numbers are often rounded to put them into a more convenient form. For example, a newspaper headline may say 3000 fans at football match. However, it is very unlikely that the actual attendance was exactly 3000—it would have been near to 3000 and has been rounded.

Rounding Rules.

In rounding, we use the rule:

5 or greater, round up

Less than 5, round down.

We can round to a given number of decimal places, to the nearest whole number or to the nearest 10, 100 or 1000.

When rounding, always take the context of the question into account.

EXAMPLES

1. **Round these to the nearest whole number**: Look at the first number after the decimal point . This value shows if you must round up or down.

(a) 3.4 → 3. As $4 < 5$, round down.

(b) 4.6 → 5. As $6 > 5$, round up.

2. **Round these to the nearest 10**: Look at the unit value. This is what shows if you need to round up or down.

(a) 34 → 30. As $4 < 5$, round down.

(b) 46 → 50. As $6 \geq 5$, round up.

3. **Round to one decimal place**: Look at the number in the second decimal place. This is the value that tells you whether to round up or down.

(a) 3.46 → 3.5. As $6 \geq 5$, round up.

(b) 46.539 → 46.5. As $3 < 5$, round down.

4. **Round these to two decimal places:** Look at the number in the third decimal place. This value shows if you must round up or down.

(a) 3.354 → 3. 35 As $4 < 5$,round down.

(b) 4.786 → 4.79. As $6 > 5$, round up.

ESTIMATING

It is often useful to estimate an answer to a calculation prior to carrying out the sum as this gives an idea of a sensible answer and helps to avoid mistakes.

When estimating, one simple method is to round digits in order to change the problem into a much easier calculation.

EXAMPLES

3294 people attended a football match. The ticket price was £11. What was the total value of the ticket sales?

The calculation is 3294 x 11.

To estimate, round 3294 → 3300

and 11 → 10.

The estimation calculation is

3300 x 10 = £33 000.

Therefore, when doing the actual calculation, the answer would be expected to be reasonably close to £33 000.

(The actual answer is £36 234)

Prizes were bought for 19 children attencding a party. Each prize cost £1.97.

How much did the prizes cost altogether?

The calculation is 19 x 1.97.

To estimate, round 19 → 20

and 1.97 → 2.

The estimation calculation is

20 x 2 = £40.

Therefore, when doing the actual calculation, the answer would be expected to be reasonably close to £40.

(The actual answer is £37.43)

ORDER OF OPERATIONS

In maths, calculations must be carried out in the correct order.

We use the memory aid

BODMAS

to help us remember this correct order.

B	brackets
O	of
D	divide
M	multiply
A	add
S	subtract

EXAMPLES

1. Find 5 + 6 x 2

Using BODMAS, we multiply first.

5+ 6 x 2 = 5 + 12 = 17.

2. Find 4 + 8 ÷ 4

Using BODMAS, we divide first.

4 + 8 ÷ 4 = 4 + 2 = 6.

3. Find 5 + 2 (10 - 3)

Brackets come first, so 10– 3 = 7

5 + 2 x 7 = 5 + 14 = 19.

4. **Find 15 + 6 x 2—7**

By BODMAS, multiply first

= 15 + 12—7

= 27—7 =20

5. Find 4 x 6 + 18 ÷ 2

Multiply first:

= 24 + 18 ÷ 2

Next divide:

= 24 + 9

= 33

FRACTIONS

Fractions represent parts of a whole number. They are written as shown below where the top number is the numerator and the bottom number is the denominator. (Quick memory aid **D**enominator = **D**own)

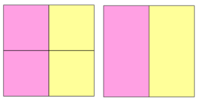
$$\frac{5}{7}$$

numerator

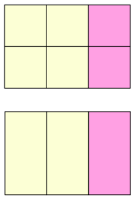
denominator

Equivalent Fractions

Equivalent fractions are fractions with different numbers in the numerator and denominator that represent the same proportion of a whole number.

Eg. 

$$\frac{2}{4} = \frac{1}{2}$$



$\frac{2}{6}$

$\frac{1}{3}$

$$\frac{2}{6} = \frac{1}{3}$$

Improper (or Top Heavy) Fractions

Improper fractions are fractions where the numerator is greater than the denominator. Eg. $\frac{13}{4}$

Mixed Numbers

A mixed number is a number with a whole number part and a fraction part. Eg. $5\frac{1}{2}$

SIMPLIFYING FRACTIONS

Simplifying Fractions

To simplify a fraction, we divide the numerator and denominator by the same number to find a smaller equivalent fraction.

The main step in simplifying fractions is to find the largest number that can divide into both the numerator and denominator.

$$\frac{4}{16} \underset{\div 4}{\overset{\div 4}{=}} \frac{1}{4}$$

EXAMPLES

1. $$\frac{3}{12} \underset{\div 3}{\overset{\div 3}{=}} \frac{1}{4}$$

2. $$\frac{10}{15} \underset{\div 5}{\overset{\div 5}{=}} \frac{2}{3}$$

3. $$\frac{63}{81} \underset{\div 5}{\overset{\div 5}{=}} \frac{7}{9}$$

4. If it isn't immediately clear what the biggest number to divide by is, we can simplify in a few steps. Keep going until there is no number that can divide both numerator and denominator.

$$\frac{84}{128} \underset{\div 2}{\overset{\div 2}{=}} \frac{42}{64} \underset{\div 2}{\overset{\div 2}{=}} \frac{21}{32}$$

ADDING FRACTIONS

Fractions can only be added or subtracted if the denominators are the same. This is called having a " common denominator ".

If the fractions have the same denominator, then we just add the numerators together to get the solution.

Method 1 - Common denominator.

Choose a common denominator. Here, we have chosen 4.

$$\frac{1^{\times 2}}{2_{\times 2}} + \frac{1}{4}$$

$$= \frac{2}{4} + \frac{1}{4}$$

so $^1/_2$ becomes $^2/_4$

$$= \frac{3}{4}$$

Method 2 - Smile and Kiss

Draw a kiss. Multiply along these lines. This gives the numerator.

Draw a smile. Multiply the denominators together. This gives the new denominator.

$$\frac{2}{3} + \frac{1}{7}$$

$$= \frac{2\times7 + 3\times1}{21}$$

$$= \frac{14+3}{21}$$

$$= \frac{17}{21}$$

EXAMPLES

Find $\frac{3}{10} + \frac{2}{5}$

$$\frac{3}{10} + \frac{2}{5}$$

$$= \frac{3}{10} + \frac{4}{10}$$

$$= \frac{7}{10}$$

Find $\frac{1}{5} + \frac{3}{8}$

$$\frac{1}{5} + \frac{3}{8}$$

$$= \frac{1\times8 + 5\times3}{40}$$

$$= \frac{8+15}{40}$$

$$= \frac{23}{40}$$

SUBTRACTING FRACTIONS

Fractions are subtracted in the same way as adding.

Once again, there are two methods we can use.

Method 1 - Common denominator.

Choose a common denominator.
Here, we have chosen 4.

$$\frac{1}{2}\,{}^{\times 2}_{\times 2} - \frac{1}{4}$$

$$= \frac{2}{4} - \frac{1}{4}$$

so $^{1}/_{2}$ becomes $^{2}/_{4}$

$$= \frac{1}{4}$$

Method 2 - Smile and Kiss

Draw a kiss.
Multiply along these lines.
This gives the numerator.

Draw a smile.
Multiply the denominators together. This gives the new denominator.

$$\frac{2}{3} - \frac{1}{7}$$

$$= \frac{2\times7 - 3\times1}{21}$$

$$= \frac{14-3}{21}$$

$$= \frac{11}{21}$$

EXAMPLES

Find $\frac{7}{10} - \frac{2}{5}$

$$\frac{7}{10} - \frac{2}{5}$$

$$= \frac{7}{10} - \frac{4}{10}$$

$$= \frac{3}{10}$$

Find $\frac{7}{12} - \frac{1}{5}$

$$\frac{7}{12} - \frac{1}{5}$$

$$= \frac{7\times5 - 12\times1}{60}$$

$$= \frac{35-12}{60}$$

$$= \frac{23}{60}$$

MULTIPLYING AND DIVIVING FRACTIONS

Multiplying fractions

This is a very simple process.

The method is to multiply the numerators together and then multiply the denominators together. Simplify your answer if you can.

Example 1.

$$\frac{3}{8} \times \frac{2}{5}$$

$$= \frac{3 \times 2}{8 \times 5}$$

$$= \frac{6}{40}$$

$$= \frac{3}{20}$$

Example 2.

$$\frac{2}{3} \times \frac{1}{4}$$

$$= \frac{2 \times 1}{3 \times 4}$$

$$= \frac{2}{12}$$

$$= \frac{1}{6}$$

Dividing Fractions

The process is similar to multiplying fractions however there is one extra step. To divide two fractions, the second fraction is turned upside down so that the numerator becomes the denominator. Then the fractions are multiplied as before.

Example 1.

$$\frac{3}{8} \div \frac{2}{5}$$

$$= \frac{3}{8} \times \frac{5}{2}$$

$$= \frac{3 \times 5}{8 \times 2}$$

$$= \frac{15}{16}$$

Example 2.

$$\frac{2}{9} \div \frac{3}{5}$$

$$= \frac{2}{9} \times \frac{5}{3}$$

$$= \frac{2 \times 5}{9 \times 3}$$

$$= \frac{10}{27}$$

DECIMALS

The definition of a decimal is a fraction which has a denominator of 10 or 100 or 1000 etc.

For example $\frac{3}{10}, \frac{34}{100}, \frac{5}{100}$ are decimal fractions and these are written as the decimal numbers 0.3, 0.34, 0.05 respectively.

When doing addition or subtraction calculations involving decimals, it is important to remember that the number of digits after the decimal point must be the same for both numbers. If this is not the case, extra zeros are added.

When multiplying two decimal numbers, remember that the total number of digits after the decimal point in the question will be equal to the number of digits after the decimal point in the answer.

EXAMPLES

1. Find 2·5 + 0·587 + 4·63

Working
$$\begin{array}{r} 2{\cdot}500 \\ 0.587 \\ +\ 4.630 \\ \hline 7.717 \\ \hline \end{array}$$

Extra zeros are needed as there must be the same number of digits after the decimal point.

2. Find 0·53 x 5000

2. 0·53 x 1000 x 5

Split the 5000 into 1000 x 5.

= 530 x 5

= 2650

3. Find 48 ÷ 6000

= 48 ÷ 6 ÷ 1000

= 8 ÷ 1000 = 0·008

Multiply both parts by 1000.

4. Find 2·4 ÷ 0·006 = 2400 ÷ 6 = 400

5. 0·004 x 0·7 We know that 4 x 7 = 28.

So 0·004 x 0·7 = 0·0028.

There are four digits after the decimal point in the question so there should be four digits after the decimal point in the answer.

PERCENTAGES USING A CALCULATOR

A percentage is a rate or proportion out of 100.

The percentage symbol is %.

Do not use the percentage key on a calculator! That is not the easiest way!

By using percentages, it is possible to compare results. For example, if a pupil got 16 out of 30 in a French test and 18 out of 35 in a Maths test, percentages can be used to show which subject had the best result by changing the marks into percentages. The French mark is 53% and the Maths mark is 51% so we can see that this pupil did better in French.

Percentages are used in a variety of situations, for example sales in shops,

EXAMPLES

Method 1 - Convert the percentage to a decimal and multiply.

1. Find 34% of 72

   34% = 0.34 as a decimal

   Calculation is 0.34 x 72 = 24.48

2. Find 79% of 36

   79% = 0.79 as a decimal

   Calculation is 0.79 x 36 = 28.44

3. Find 3% of 12

   3% = 0.03 as a decimal

   Calculation is 0.03 x 12 = 0.36

Method 2 - Divide by 100 on the calculator to change to a decimal and then multiply.

1. Find 27% of 63

   Calculation is $\frac{27}{100} \times 63$ = 17.01

2. Find 7% of 342

   Calculation is $\frac{7}{100} \times 342$ = 23.94

3. Find 85% of 1203

   Calculation is $\frac{85}{100} \times 1203$ = 1022.55

Often, percentages are worked out without a calculator. To help with this, pupils should know how to calculate basic percentages using the fraction equivalents shown below.

Percentage	Fraction	Calculation
1%	$^{1}/_{100}$	÷ 100
5%	$^{1}/_{20}$	÷ 20
10%	$^{1}/_{10}$	÷ 10
20%	$^{1}/_{5}$	÷ 5
25%	$^{1}/_{4}$	÷ 4
33 $^{1}/_{3}$ %	$^{1}/_{3}$	÷ 3
50%	$^{1}/_{2}$	÷ 2
66 $^{2}/_{3}$%	$^{2}/_{3}$	÷ 3, x2
75%	$^{3}/_{4}$	÷4, x3

These base percentages can be used in various combinations to find harder percentages.

For example.,

To find 17% of a quantity, find 10% + 5% + 1% + 1%.

21% = 20% + 1%

49% = 50% - 1% etc.

EXAMPLES

Find the following percentages of 240.

1%	= 240 ÷ 100	= 2.4	
5%	= 240 ÷ 20	= 12	
10%	= 240 ÷ 10	=24	
20%	= 240 ÷ 5	= 48	
25%	= 240 ÷ 4	= 60	
33 $^{1}/_{3}$ %	= 240 ÷ 3	= 80	
50%	= 240 ÷ 2	= 120	
66 $^{2}/_{3}$%	= 240 ÷ 3, x2	= 80 x 2	= 160
75%	= 240 ÷ 4, x 3	= 60 x 3	= 180

Find the following percentages of 360.

1%	= 360÷ 100	= 3.6	
2%	= 2 x 1%	= 2 x 3.6	= 7.2
10%	= 360÷ 10	= 36	
5%	= 1/2 of 10%	= 36 ÷ 2	= 18
15%	= 10% + 5%	= 36 + 18	= 54
16%	= 15% + 1%	= 54 + 3.6	=57.6
33 $^{1}/_{3}$ %	= 360 ÷ 3	= 120	
80%	= 8 x 10%	= 8 x 36	= 288

MONEY

Pupils should realize that money calculations must be rounded to 2 decimal places as this corresponds to rounding to the nearest penny.

Best Buys - Pupils should be able to compare prices to identify the best value for money.

Wages and Salaries - Pupils will learn that people earn money in all sorts of ways, e.g. hourly, weekly, monthly or yearly (salary). (Remember: 52 weeks per year, 12 months in a year and "annual" means yearly.)

Pupils should also be able to calculate overtime rates of pay including double time (normal rate x 2) and time and a half (normal rate x 1.5). They will also learn about commission and bonuses.

They should know that **Net Pay = Gross pay – Deductions** where deductions are anything taken off the total wage eg tax, national insurance, pension.

Foreign Exchange - Pupils should be able to use exchange rates to find the value of money in different currencies.

Foreign Money = Number of Pounds X Exchange Rate   *Number of Pounds = Foreign Money ÷ Exchange Rate*

EXAMPLES

1. *Juicy Jam Company sells jams in 100g and 250g jars. The price for the 100g jar is £1.32.*

The price for the 250g jar is £2.89. Which jar is the best value?

Find the cost per gram for both jars of jam.

100g costs £1.32 = 132p

Price per gram = 132 ÷ 100 = **1.32p.**

250g costs £2.89 =289p

Price per gram = 289 ÷ 250 = **1.156p.**

As the larger jar has the lowest price per gram, it is better value.

2. An exchange rate was £1 = € 1.37.

(a) How many Euros would I get for £230?

Euros = 230 x 1.37 = **€315.10**

(b) How many pounds would I get for € 460?

Pounds = 460 ÷ 1.37 = **£335.77**

3. *If a salary is £19760 per annum, what is the weekly wage?*

£19760 ÷ 52 = **£380**

4. *If a wage was £249 for working 30 hours, what was the hourly rate of pay?*

Hourly rate = £249 ÷ 30 = **£8.30**

5. *A job offers a basic wage of £14.50 per hour. Overtime pay is paid at double time.*

What would the pay be for 7 hours overtime?

Overtime = 7 X (2 X £14.50)= **£203**

6. *An employee earns 13% commission on each sale made. How much is she paid for sales totalling £3000?*

Commission = 13% of £3000

= 13 ÷ 100 X 3000 = **£390**

7. *Blair has a gross pay of £26000 per annum. He pays £4892 in deductions. Calculate his annual net pay.*

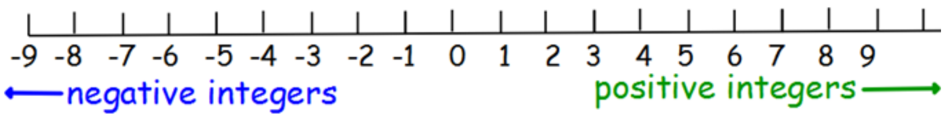
Net pay = £26000 - £4892 = **£21108**

NEGATIVE NUMBERS

Negative Numbers are numbers below zero. They are shown using a "-" sign in front of the number. Eg -4 is read "negative four" or "minus 4". This means that –4 is four below zero.

They are commonly seen in contexts such as temperatures and bank balances.

Negative Integers are negative whole numbers - that just means that they are not fractions or decimal fractions. So, –2 is a negative integer but –2.4 is not.

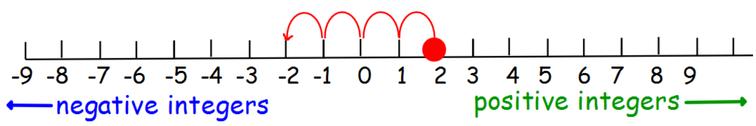


EXAMPLES

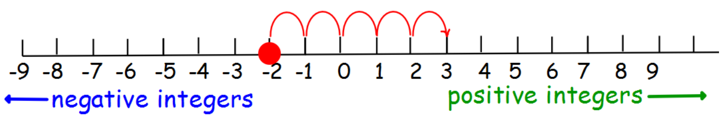
Think of adding and subtracting integers as moving along the number line.

Move in the positive direction if adding and in the negative direction if subtracting.

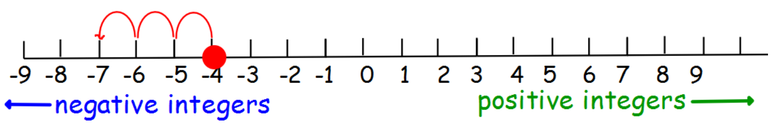
1. 2 – 4 = - 2



2. –2 + 5 = 3

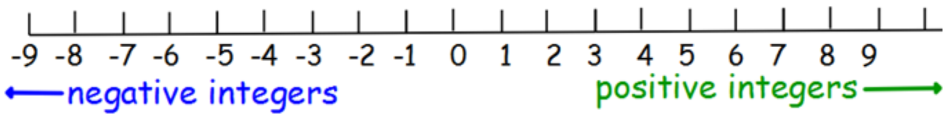


3. -4 - 3 = -7



There are a few rules to follow in order to carry out calculations involving negative numbers. We start here with adding and subtracting.

Adding and Subtracting Negative Numbers



Adding a negative number is the same as taking away.

Subtracting a negative number is the same as adding!!!

EXAMPLES

$$2 + (-3) = 2 - 3 = -1$$

$$4 + (-1) = 4 - 1 = 3$$

$$-1 + (-7) = -1 - 7 = -8$$

$$2 - (-3) = 2 + 3 = 5$$

$$4 - (-1) = 4 + 1 = 5$$

$$-1 - (-7) = -1 + 7 = 6$$

MULTIPLYING NEGATIVE NUMBERS

When multiplying negative numbers, there are similar rules to follow. Remember these rules!

We use a quick way of writing negative and positive.

Negative is written "-ve" and Positive is written "+ ve".

The rules are:

$$+\text{ve} \times -\text{ve} = -\text{ve}$$
$$-\text{ve} \times +\text{ve} = -\text{ve}$$
$$-\text{ve} \times -\text{ve} = +\text{ve}$$
$$+\text{ve} \times +\text{ve} = +\text{ve}$$

So, one negative number times one positive number gives a negative answer. If both numbers are positive or both are negative, the answer will be positive.

EXAMPLES

$$2 \times (-4) = -8$$

$$3 \times (-5) = -15$$

$$(-2) \times 7 = -14$$

$$(-5) \times 6 = -30$$

$$(-2) \times (-4) = 8$$

$$(-3) \times (-5) = 15$$

$$(-2) \times (-7) = 14$$

$$(-5) \times (-6) = 30$$

DIVIDING NEGATIVE NUMBERS

When multiplying negative numbers, there are similar rules to follow. Remember these rules!

We use a quick way of writing negative and positive.

Negative is written "-ve" and Positive is written "+ ve".

The rules are:

$$-\text{ve} \div +\text{ve} = -\text{ve}$$
$$+\text{ve} \div -\text{ve} = -\text{ve}$$
$$-\text{ve} \div -\text{ve} = +\text{ve}$$
$$+\text{ve} \div +\text{ve} = +\text{ve}$$

So, one negative number times one positive number gives a negative answer. If both numbers are positive or both are negative, the answer will be positive.

EXAMPLES

$$(-4) \div 2 = -2$$

$$(-9) \div 3 = -3$$

$$12 \div (-3) = -4$$

$$50 \div (-5) = -10$$

$$(-4) \div (-2) = 2$$

$$(-9) \div (-3) = 3$$

$$(-12) \div (-3) = 4$$

$$50 \div 5 = 10$$

TIME

Pupils should be able to convert times between 24 hour and 12 hour time.

To convert between 12 hour and 24 hour, add 12 hours on to any time after noon and remove the "pm". So, for example,

3:40pm = 1540 hours

To convert a time before noon, simply add a zero in front of any single digit time and leave any double digit time as it is (without the "am" part). Therefore,

8:27am = 0827 hours and 11:56am = 1156 hours.

Midnight = 0000 hours, Noon = 1200 hours

Note that, if a question is given in 24 hour clock time the answer must also be given 24 hour clock time. Similarly, a question with times in the 12 hour clock should be answered in the same format making sure that am and pm are used correctly.

EXAMPLES

Change these times to 24 hour time.

(a)	3:45am	=	0345hours
(b)	4:27pm	=	1627hours
(c)	7:21pm	=	1921hours
(d)	2:25am	=	0225 hours
(e)	11:42pm	=	2342 hours
(f)	00:05am	=	0005 hours
(g)	12:25pm	=	1225hours
(h)	1:14pm	=	13:14hours

Change these times to 12 hour time.

(a)	22:15hours	=	10:15pm
(b)	1146hours	=	11:46am
(c)	0514hours	=	5:14am
(d)	0234hours	=	2:34am
(e)	2222hours	=	10:22pm
(f)	1345hours	=	1:45pm
(g)	2359hours	=	11:59pm
(h)	0002hours	=	12:02am

CONVERTING TIMES

Converting hours and minutes to decimal hours

In many calculations, times must be changed from hours and minutes to a decimal value and then back again in the final answer.

Pupils should be aware of the basics:

Minutes	Hours
15	0.25
30	0.5
45	0.75

A common mistake made by pupils is to say that 4 hours 30 minutes is equal to 4·3hrs or thinking that 0·1 hours is 10 minutes.

To change hours and minutes into a decimal, first take the minutes time. As there are 60 minutes in an hour, divide the minutes by 60 to change it into a decimal part of an hour. Then add this to the number of hours.

To change decimal hours to hours and minutes, take the decimal part and multiply by 60 to find the minutes time.

EXAMPLES

Change these times into hours.

1. 3hours 24minutes 20÷60 = 0.4

   So, 3hours 24minutes = 3.4 hours

2. 2hours 18minutes 18÷60 = 0.3

   So, 2hours 18minutes = 2.3 hours

3. 1hour 57minutes 57÷60 = 0.95

   So, 1hours 57minutes = 1.95 hours

4. 5 hours 17 minutes 17÷60=0.2833..

   So, 5 hours 17 minutes = 5.28 hours

Change these times into hours and minutes.

1. 3.6 hours 0.6 x 60 = 36

   So, 3.6 hours = 3 hours 36 minutes.

2. 4.7 hours 0.7 x 60 = 42

   So 4.7 hours = 4 hours 42 minutes

3. 2.9 hours 0.9 x 60 = 54

   So 2.9 hours = 2 hours 54 minutes

4. 1.643 hours 0.643 x 60 = 38.58

   So 1.643 hours = 1 hour 39 minutes

CALCULATING TIME

Calculating time intervals is an important skill. For example, a film started at 7:25pm and finished at 9:22pm. How long does the film last?

7:25pm → 9:22pm

If pupils find this difficult to work out mentally, they can use a line diagra, and split the time up into easy to manage parts.

7:25pm 8:00pm 9:00pm 9:22pm

35 minutes 1 hour 22 minutes

From 7:25pm until 8pm is 35 minutes. From 8pm until 9pm is one hour and from 9pm until 9:22pm is 22 minutes. To find the total time taken, add these all together to find the total time taken.

1 hour + 35 minutes + 22 minutes = 1 hour 57 minutes.

EXAMPLES

How long is it between 11:15am and 2:37pm?

11:15am 12:00pm 2:00pm 2:37pm

45 minutes 2 hour 37 minutes

2 hours + 45 minutes + 37 minutes = 2 hours 82 minutes.

But, there are 60 minutes in one hour so 82 minutes = 60 minutes + 22 minutes = I hour 22 minutes.

= 2 hours + 1 hour 22 minutes

= 3 hours 22 minutes

SPEED, DISTANCE AND TIME

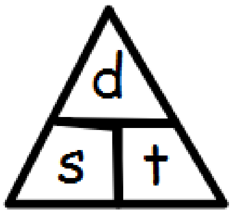
Time problems often occur in the context of speed, distance and time calculations. There are three formulae to learn for speed, distance and time calculations:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

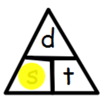
$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

The following triangle is useful as a memory aid for these formulae. Cover up the quantity you wish to find and the triangle shows the formula to use. Remember to put the letters into the triangle in alphabetical order. If the letters are beside each other, multiply. If the letters are above and below each other, then divide.



EXAMPLES

1. Find the speed of a person travelling 8 miles in 3 hours.

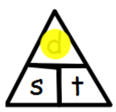


$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{8}{3}$$

$$= 2.67\text{mph}$$

2. Find the distance travelled by a car going at 70mph for 3 hours.

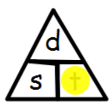


$$\text{distance} = \text{speed} \times \text{time}$$

$$= 70 \times 3.$$

$$= 210\text{miles}$$

3. Find the time taken by a bus travelling at 50 mph to cover 20 miles.



$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{20}{50}$$

$$= 0.4\text{hours.}$$

However, 0.4 hours = 0.4 x 60 minutes

= 24 minutes.

The bus journey takes 24 minutes.

CONVERTING UNITS OF MEASURE

Commonly used units of metric measure include:

Length: millimeter (mm) centimeter (cm) metre (m) kilometer (km)

Mass: milligrams (mg) grams (g) kilograms (kg)

Volume: millilitre (ml) litre (l)

Pupils should be able to convert between different units of measures.

Length

conversion	sum
mm to cm	÷10
cm to m	÷100
m to km	÷1000
km to m	X 1000
m to cm	X 100
cm to mm	X 10

Mass

conversion	sum
mg to g	÷1000
g to kg	÷1000
kg to g	x1000
g to mg	X 10000

Volume

conversion	sum
ml to l	÷10
l to ml	÷100

EXAMPLES

For each length, change to the given unit.

(a) 40mm into cm 40÷10 = 4cm

(b) 25 mm to cm 25÷10 = 2.5cm

(c) 3cm to mm 3x10 =30mm

(d) 4.3cm into mm 4.3 x 10= 43mm

(e) 120cm to m 120÷100 = 1.2m

(f) 3m to cm 3 x100=300cm

(g) 5.6m to cm 5.6x100= 560cm

(h) 4km to m 4 x1000 =4000m

(i) 4.2km to m 4.2x1000=4200m

(j) 3400m to km 3400÷1000=3.4km

(k) 2050m to km 2050÷1000=2.05km

For each weight, change to the given unit.

(a) 3000g into kg 3000÷1000 = 3kg

(b) 4200g into kg 4200÷1000 = 4.2kg

(c) 5kg to g 5 x 1000 = 5000g

(d) 6.7kg to g 6.7 x 1000 = 6700g

(e) 4.02kg to g 4.02 x 1000 = 4020g

For each volume, change to the given unit.

(a) 3litres in ml 3 x 1000 = 3000ml

(b) 4.5litres in ml 4.5 x 1000 = 4500ml

(c) 2000ml in litres 2000÷1000 = 2l

(d) 1500ml in litres 1500÷1000 = 1.5l

BASIC ALGEBRA

Algebra is an important skill in maths. A good understanding of basic algebra is essential for numerous maths topics.

Expressions - an expression is a mathematical sentence. It can be made up of letters and numbers. Eg $3x + 4y - 2$. Each part of the expression is called a "term".

Simplifying Expressions - to simplify an expression, all the similar terms are collected together. Eg, $4x + 5y + 2 + 3x + 5$

$= 7x + 5y + 7$

This is called "collecting like terms".

EXAMPLES

Simplify the following expressions:

(a) $4x + 6y - 2x + 3y + 2$

$= 4x - 2x + 6y + 3y + 2$

$= 2x + 9y + 2$

(b) $7a + 5 - 2a + 3b - 6 - 4b$

$= 7a - 2a + 3b - 4b + 5 - 6$

$= 5a - b - 1$

(c) $12p + 16 - 2q + 34p + 2q$

$= 12p + 34p - 2q + 2q + 16$

$= 46p + 16$

SOLVING EQUATIONS

Equations - an equation is a statement showing that two things are equal to each other. Eg $4x + 2 = 3x + 1$.

To solve an equation, each side must be balanced to ensure that they remain equal. This means that whatever mathematical operation is carried out on one side must also be carried out on the other side of the equation. The aim is to leave only the letter term on the left and the numbers on the right.

Examples

$$x + 3 = 5$$
$$-3 \qquad -3$$
$$x = 2$$

$$4x = 20$$
$$\div 4 \qquad \div 4$$
$$x = 5$$

EXAMPLES

Solve for x.

(a)

$$2x + 4 = 12$$
$$-4 \qquad -4$$
$$2x = 8$$
$$\div 2 \qquad \div 2$$
$$x = 4$$

(b)

$$3x - 5 = 10$$
$$+5 \qquad +5$$
$$3x = 15$$
$$\div 3 \qquad \div 3$$
$$x = 5$$

(c)

$$5x - 2 = 3x + 6$$
$$-3x \qquad -3x$$
$$5x - 3x - 2 = 6$$

collect like terms

$$2x - 2 = 6$$
$$+2 \qquad +2$$
$$2x = 6 + 2$$

collect like terms

$$2x = 8$$
$$\div 2 \qquad \div 2$$
$$x = 4$$

DRAWING GRAPHS

Pupils should always use a sharp pencil and ruler. Remember these rules:

•give the graph a title

•label the axes

•label the vertical axes on the lines (not on the spaces)

•plot points on a line graph neatly (using a cross or a dot)

•if asked to draw a line of best fit then the line should have the same number of points above the line as below it.

•if necessary, make use of a jagged line to show that the lower part of the graph has been missed out

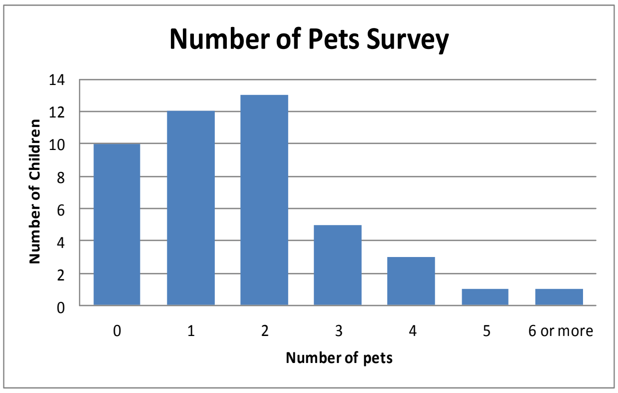
•label all the sections or include a key when drawing a pie chart

EXAMPLES

Bar Graphs

These are commonly used to display data pictorially. Important points and information can be easily read from the graph. They can be used for continuous or discrete data.

Remember that there must be spaces between the bars and each bar must be of the same width. Axes must be labelled.

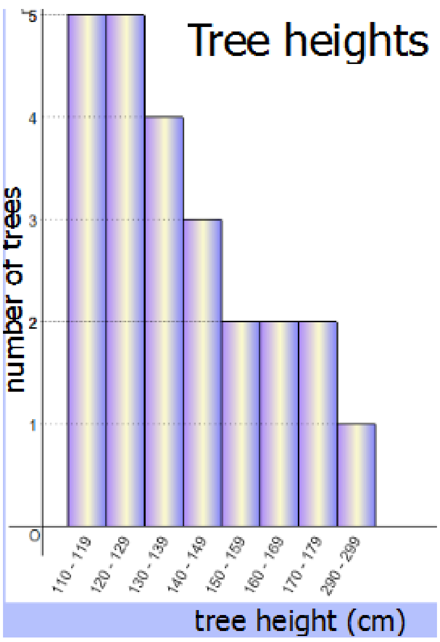


Histograms

Histograms are only used for continuous data. In a histogram, the bars are together as the data is continuous.

Note that the bars can be different widths.

Histograms are usually used with grouped data.

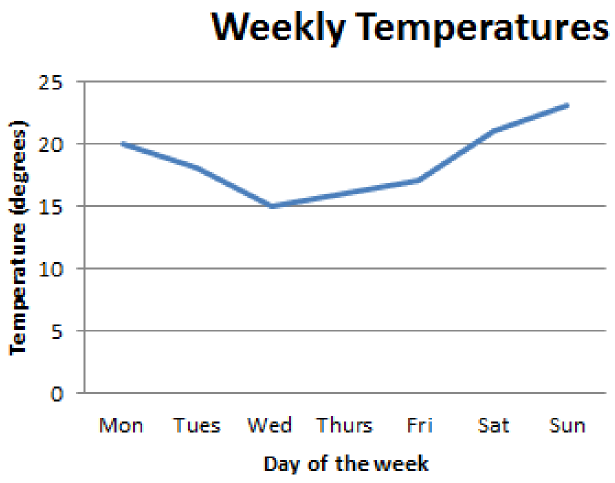


LINE GRAPHS AND PIE CHARTS

Line graphs are often used to show a trend over a number of days or hours.

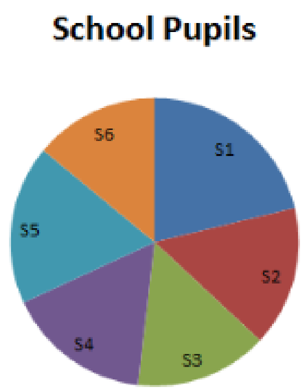
Line graphs are plotted as a series of points. These points are then joined with straight lines. The ends of the line graph do not have to join to the axes.

They should always have a title and the axes should be labelled.



Pie Charts use different sectors of a circle to represent data.

This pie chart shows the number of pupils in each year group of a school.



Total Pupils = 571

It is clear from the chart that S1 has the most pupils as this section is largest.

We can calculate how many pupils are in S1 by measuring the angle at the centre of the circle for the S1 sector.

The angle here is 76^o.

As there are 360^o in a circle, this can be written as a fraction of a circle as $^{76}/_{360}$.

To calculate the number of pupils, multiply the fraction by the total number of pupils. $^{76}/_{360} \times 571 = 121$ pupils (rounded to the nearest whole number).

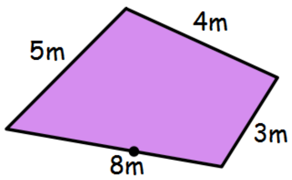
PERIMETER AND AREA

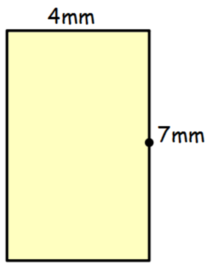
The perimeter of a 2D shape is the distance around the outside of the shape.

It is calculated by adding together all of the side lengths of the shape.

Example: The perimeter of this shape is

5 + 4 + 8+ 3 = 20cm.



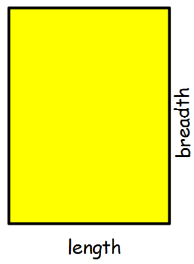


The perimeter of the rectangle is 4 + 7 + 4 + 7 = 22cm.

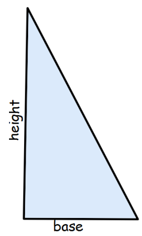
The area of a 2D shape is the amount of space that the shape takes up. It is measured in square units.

Some area formulae are:

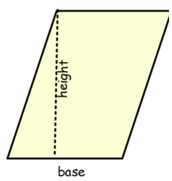
Area of a square or rectangle $\text{Area} = \text{length} \times \text{breadth}$



Area of a triangle $\text{Area} = \frac{1}{2}\text{base} \times \text{height}$



Area of a parallelogram $\text{Area} = \text{base} \times \text{height}$



PROBLEM SOLVING

Strategies for effective problem solving in maths

1. Look for **important words** in the question. Write them down. Make sure that I know how to do.

2. **Look for a pattern.** Is there anything that is happening over and over again. Can I use this to solve the problem?

3. **Have a go.**

Try to get an answer. Check that your answer makes sense!

4. Use a chart or a table. Will this help?

5. Will a drawing help. Can I sketch the problem?

6. **Work backwards.** Can I work from the end of the problem to the beginning? Will this help me get an answer?

7. Make the problem a bit easier. Change the numbers and solve the easy problem. Can the same method be used for the harder one?

8. *Can I use something to **make a model** of the problem? Will this help me solve it?*

9. **Think logically.**

Can I tell something about the answer immediately.

How do I know if an answer is correct or incorrect?

EXAMPLES