

X100/701

NATIONAL
QUALIFICATIONS
2008TUESDAY, 20 MAY
1.00 PM – 4.00 PMMATHEMATICS
ADVANCED HIGHER**Read carefully**

1. Calculators may be used in this paper.
2. Candidates should answer all questions.
3. **Full credit will be given only where the solution contains appropriate working.**

ST. LUKE'S HIGH SCHOOL

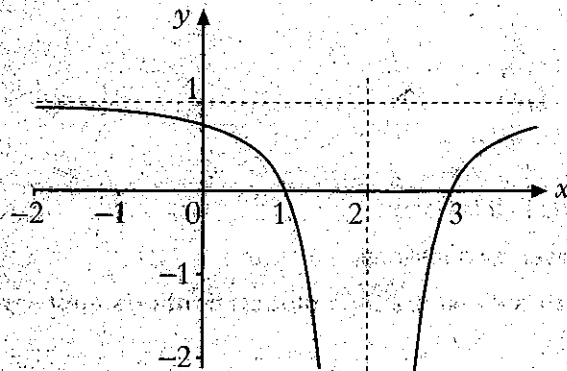
Answer all the questions.

1. The first term of an arithmetic sequence is 2 and the 20th term is 97. Obtain the sum of the first 50 terms. 4

2. (a) Differentiate $f(x) = \cos^{-1}(3x)$ where $-\frac{1}{3} < x < \frac{1}{3}$. 2

(b) Given $x = 2 \sec \theta$, $y = 3 \sin \theta$, use parametric differentiation to find $\frac{dy}{dx}$ in terms of θ . 3

3. Part of the graph $y = f(x)$ is shown below, where the dotted lines indicate asymptotes. Sketch the graph $y = -f(x + 1)$ showing its asymptotes. Write down the equations of the asymptotes. 4



4. Express $\frac{12x^2 + 20}{x(x^2 + 5)}$ in partial fractions. 3

Hence evaluate

$$\int_1^2 \frac{12x^2 + 20}{x(x^2 + 5)} dx. \quad \text{3}$$

5. A curve is defined by the equation $xy^2 + 3x^2y = 4$ for $x > 0$ and $y > 0$. 3

Use implicit differentiation to find $\frac{dy}{dx}$. 3

Hence find an equation of the tangent to the curve where $x = 1$. 3

6. Let the matrix $A = \begin{pmatrix} 1 & x \\ x & 4 \end{pmatrix}$.
- (a) Obtain the value(s) of x for which A is singular. 2
- (b) When $x = 2$, show that $A^2 = pA$ for some constant p .
Determine the value of q such that $A^4 = qA$. 3
7. Use integration by parts to obtain $\int 8x^2 \sin 4x \, dx$. 5
8. Write down and simplify the general term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{10}$. 3
Hence, or otherwise, obtain the term in x^{14} . 2
9. Write down the derivative of $\tan x$. 1
Show that $1 + \tan^2 x = \sec^2 x$. 1
Hence obtain $\int \tan^2 x \, dx$. 2
10. A body moves along a straight line with velocity $v = t^3 - 12t^2 + 32t$ at time t .
- (a) Obtain the value of its acceleration when $t = 0$. 1
- (b) At time $t = 0$, the body is at the origin O . Obtain a formula for the displacement of the body at time t . 2
Show that the body returns to O , and obtain the time, T , when this happens. 2
11. For each of the following statements, decide whether it is true or false and prove your conclusion.
- A For all natural numbers m , if m^2 is divisible by 4 then m is divisible by 4.
- B The cube of any odd integer p plus the square of any even integer q is always odd. 5
12. Obtain the first three non-zero terms in the Maclaurin expansion of $x \ln(2 + x)$. 3
Hence, or otherwise, deduce the first three non-zero terms in the Maclaurin expansion of $x \ln(2 - x)$. 2
Hence obtain the first **two** non-zero terms in the Maclaurin expansion of $x \ln(4 - x^2)$. 2
- [Throughout this question, it can be assumed that $-2 < x < 2$.]

[Turn over for Questions 13 to 16 on Page four

13. Obtain the general solution of the differential equation

$$\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = 2x^2. \quad 7$$

Given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = 1$, when $x = 0$, find the particular solution. 3

14. (a) Find an equation of the plane π_1 through the points $A(1, 1, 1)$, $B(2, -1, 1)$ and $C(0, 3, 3)$. 3

(b) The plane π_2 has equation $x + 3y - z = 2$.
Given that the point $(0, a, b)$ lies on both the planes π_1 and π_2 , find the values of a and b . Hence find an equation of the line of intersection of the planes π_1 and π_2 . 4

(c) Find the size of the acute angle between the planes π_1 and π_2 . 3

15. Let $f(x) = \frac{x}{\ln x}$ for $x > 1$.

(a) Derive expressions for $f'(x)$ and $f''(x)$, simplifying your answers. 2,2

(b) Obtain the coordinates and nature of the stationary point of the curve $y = f(x)$. 3

(c) Obtain the coordinates of the point of inflexion. 2

16. Given $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to write down an expression for z^k in terms of θ , where k is a positive integer.

Hence show that $\frac{1}{z^k} = \cos k\theta - i \sin k\theta$. 3

Deduce expressions for $\cos k\theta$ and $\sin k\theta$ in terms of z . 2

Show that $\cos^2 \theta \sin^2 \theta = -\frac{1}{16} \left(z^2 - \frac{1}{z^2} \right)^2$. 3

Hence show that $\cos^2 \theta \sin^2 \theta = a + b \cos 4\theta$, for suitable constants a and b . 2

[END OF QUESTION PAPER]