Solutions

1. hypotenuse = 5 (Pythagoras – or 3,4,5 triangle)

$\sin x = \frac{3}{5} \quad \cos x = \frac{4}{5}$

$\sin^2 x + \cos^2 x = \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = 1$

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2. *a* is the amplitude, using symmetry, top of wave is 5
Hence a = -5 (since sine wave is inverted)

b is number of waves in 360°
whole wave will take up 120°
so *b* = 3

a = -5, *b* = 3

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3. a) 1.30 pm is 1.5 hours after midnight,
put t = 1.5 into the formula

$D = 12.5 + 9.5\sin(30 \times 1.5)$

Depth = 19.217… = 19.2 metres (1 dp)

b) Maximum depth is when sine is maximum (= 1)
Max depth = 12.5 + 9.5 = 22 metres

Minimum depth is when sine is minimum (= -1)
Min depth = 12.5 - 9.5 = 3 metres

Maximum difference is 22 – 3 = 19 metres.

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4. $y = k\sin ax°$ — *k* = 3 (amplitude)
a = 2 (number of waves in 360°)

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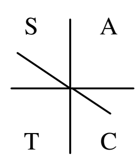
5. $y = a\cos bx°$ — *a* = 3 (amplitude)
b = 2 (number of waves in 360°)

Solving Equations

1. $3\tan x + 5 = 0 \rightarrow \tan x = -\frac{5}{3}$



$x = \tan^{-1}\frac{5}{3}$ acute $x = 59.03...°$

$x = 180 - 59 = 121°$ *or* $x = 360 - 59 = 301°$

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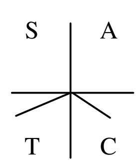
2. $2 + 3\sin x = 0 \rightarrow \sin x = -\frac{2}{3}$



$x = \sin^{-1}\frac{2}{3}$ acute $x = 41.81..°$

$x = 180 + 42 = 222°$ *or* $x = 360 - 42 = 318°$

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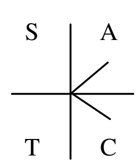
3. $7\cos x - 2 = 0 \rightarrow \cos x = \frac{2}{7}$



$x = \cos^{-1}\frac{2}{7}$ acute $x = 73.398..°$

$x = 73°$ *or* $x = 360 - 73 = 287°$

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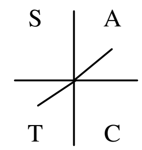
4. $5\tan x - 9 = 0 \rightarrow \tan x = \frac{9}{5}$



$x = \tan^{-1}\frac{9}{5}$ acute $x = 60.945..°$

$x = 61°$ *or* $x = 180 + 61 = 241°$

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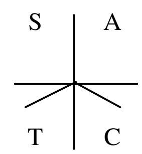
5. $5\sin x + 2 = 0 \rightarrow \sin x = -\frac{2}{5}$



$x = \sin^{-1}\frac{2}{5}$ acute $x = 23.578..°$

$x = 180 + 24 = 204°$ *or* $x = 360 - 24 = 336°$

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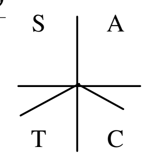
6. $\tan 40 = 2\sin x + 1 \rightarrow \sin x = -\frac{0.1609}{2}$



$x = \sin^{-1}\frac{0.1609}{2}$ acute $x = 4.614..°$

$x = 180 + 5 = 185°$ *or* $x = 360 - 5 = 355°$

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7. $2\tan 24 = \tan q \rightarrow \tan q = 0.8905$

$x = \tan^{-1} 0.8905$ acute $x = 41.683..°$

q is an acute angle, so q = 42° (2 sf)

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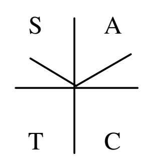
8. Solve $y = \sin x$ and $y = 0.4 \rightarrow \sin x = 0.4$

$x = \sin^{-1} 0.4$ acute $x = 23.578..°$



$x = 24°$ *or* $x = 180 - 24 = 156°$

A is (24°, 0.4) and B is (156°, 0.4)

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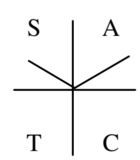
9. a) amplitude = 3, so a = 3

since max is at 90°, there is 1 wave in 360°

hence b = 1

b) $3\sin x = 2 \rightarrow \sin x = \frac{2}{3}$



$x = \sin^{-1}\frac{2}{3}$ acute $x = 41.81..°$

$x = 42°$ *or* $x = 180 - 42 = 138°$

P is (42°, 2) and Q is (138°, 2)

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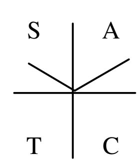
10. a) S is (90°, 1)

b) $\sin x = 0.5$



$x = \sin^{-1} 0.5$ acute $x = 30°$

$x = 30°$ *or* $x = 180 - 30 = 150°$

T is (30°, 0.5) and P is (150°, 0.5)

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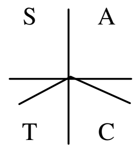
11. a) A is (90°, 0)

b) $\cos x = -0.5$



$x = \cos^{-1} 0.5$ acute $x = 60°$

$x = 180 + 60 = 240°$ *or* $x = 360 - 60 = 300°$

B is (240°, -0.5) and C is (300°, -0.5)

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12. a) Maximum value of H is when cosine is maximum (= 1)

h = 1.9 + 0.3 = 2.2 metres

b) After 8 seconds

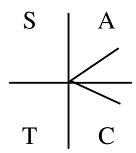
$h = 1.9 + 0.3\cos(30 \times 8) \rightarrow 1.75$ metres

c) put h = 2.05 in equation

$2.05 = 1.9 + 0.3\cos(30t) \rightarrow 0.15 = 0.3\cos 30t$

$\rightarrow \cos 30t = 0.5 \rightarrow 30t = \cos^{-1} 0.5$

acute $30t = 60°$, 300, …..



so, 30t = 60

t = 2 seconds – first time

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13. a) Put t = 10 for October into formula.

$V = 1 + 0.5\cos(30 \times 10) \rightarrow 1.25$ million gallons

b) t can only take whole number values of 1 to 12

$V(1) = 1 + 0.5\cos(30 \times 1) \rightarrow 1.43$

$V(2) = 1 + 0.5\cos(30 \times 2) \rightarrow 1.25$

$V(3) = 1 + 0.5\cos(30 \times 3) \rightarrow 1$

$V(4) = 1 + 0.5\cos(30 \times 4) \rightarrow 0.75$

$V(5) = 1 + 0.5\cos(30 \times 5) \rightarrow 0.567$

$V(6) = 1 + 0.5\cos(30 \times 6) \rightarrow 0.5$

$V(7) = 1 + 0.5\cos(30 \times 7) \rightarrow 0.567$

$V(8) = 1 + 0.5\cos(30 \times 8) \rightarrow 0.75$

$V(9) = 1 + 0.5\cos(30 \times 9) \rightarrow 1$

$V(10)$ OK see part (a)

$V(11) = 1 + 0.5\cos(30 \times 11) \rightarrow 1.433$

$V(12) = 1 + 0.5\cos(30 \times 12) \rightarrow 1.5$

Council will need to consider water rationing in June.

See next column for an alternative solution:

Alternative solution: (Fuller understanding required)

For t = 1, 2, 3 and 10, 11, 12 the cosine is positive (in 1st and 4th quadrants)

For t = 4, 5, 6 and 7, 8, 9 the cosine is negative.

the minimum value will occur when cos = -1
i..e cos 180°, then t = 6.

Hence V = 1 – 0.5 = 0.5

So, rationing needs to be considered in June

Now look at t = 5 and t = 7,
Work out V for these and you find
For May (t = 5) V= 0.567
For July (t = 7) V= 0.567 (symmetrical)

These are over critical level of 0.55 million gallons

So rationing only needs to be considered in June.
