

# Trig Identities

## N5 Maths Exam Questions

$$\sin^2 x + \cos^2 x = 1$$

$$\tan x = \frac{\sin x}{\cos x}$$

Source: 2019 P2 Q17 N5 Maths

(1)

Expand and simplify

$$(\sin x^\circ + \cos x^\circ)^2.$$

Show your working.

Answer:  $1 + 2\sin x \cos x$

Source: 2018 P1 Q18 N5 Maths

(2)

Express  $\sin x^\circ \cos x^\circ \tan x^\circ$  in its simplest form.

Show your working.

Answer:  $\sin^2 x$

Source: 2016 P1 Q11 N5 Maths

(3)

Simplify

$$\tan^2 x^\circ \cos^2 x^\circ.$$

Show your working.

Answer:  $\sin^2 x$

Source: Practice Paper A P2 Q9b N5 Maths

(4)

Show that

$$\tan x \cos x = \sin x.$$

Answer:

Prove using  $\tan x = \frac{\sin x}{\cos x}$  on the LHS and cancelling down to make the same as the RHS

## Trig Identities Worksheet

1. Show that  $\tan x \cos x = \sin x$

2. Show that  $\frac{\sin x}{\tan x} = \cos x$

3. Show that  $\frac{\tan x}{\sin x} = \frac{1}{\cos x}$

4. Show that  $\frac{\sin^2 x}{\tan x} = \sin x \cos x$

5. Show that  $\frac{1 - \cos^2 A}{\cos^2 A} = \tan^2 A$

6. Show that  $\frac{1 - \sin^2 A}{\cos^2 A} = 1$

7. Show that  $(\cos x + \sin x)^2 = 1 + 2\sin x \cos x$

8. Show that  $(\cos x + \sin x)(\cos x - \sin x) + 2\sin^2 x = 1$

9. Show that  $\sin^3 x + \sin x \cos^2 x = \sin x$

10. Show that  $\cos^2 x \sin^2 x + \cos^4 x = \cos^2 x$

## Worked Solutions

1.  $\tan x \cos x = \sin x$

LHS: Substitute  $\tan x = \frac{\sin x}{\cos x}$ :  $\frac{\sin x}{\cos x} \times \cos x$

Cancel the  $\cos x$  diagonally:  $= \sin x$

2.  $\frac{\sin x}{\tan x} = \cos x$

LHS: Substitute  $\tan x = \frac{\sin x}{\cos x}$ :  $\frac{\sin x}{\frac{\sin x}{\cos x}}$

$$= \frac{\sin x}{1} \div \frac{\sin x}{\cos x}$$

Cancel the  $\sin x$  diagonally:  $= \frac{\sin x}{1} \times \frac{\cos x}{\sin x}$

$$= \cos x$$

3.  $\frac{\tan x}{\sin x} = \frac{1}{\cos x}$

LHS Substitute  $\tan x = \frac{\sin x}{\cos x}$ :  $\frac{\frac{\sin x}{\cos x}}{\sin x}$

$$= \frac{\sin x}{\cos x} \div \frac{\sin x}{1}$$

Cancel the  $\sin x$  diagonally:  $= \frac{\sin x}{\cos x} \times \frac{1}{\sin x}$

$$= \frac{1}{\cos x}$$

$$4. \quad \frac{\sin^2 x}{\tan x} = \sin x \cos x$$

$$\text{LHS Substitute } \tan x = \frac{\sin x}{\cos x}:$$

$$\frac{\sin^2 x}{\frac{\sin x}{\cos x}}$$

$$= \frac{\sin^2 x}{1} \div \frac{\sin x}{\cos x}$$

$$= \frac{\sin^2 x}{1} \times \frac{\cos x}{\sin x}$$

$$\text{Cancel the } \sin x \text{ diagonally:} \quad = \sin x \cos x$$

$$5. \quad \frac{1 - \cos^2 A}{\cos^2 A} = \tan^2 A$$

$$\sin^2 A + \cos^2 A = 1$$

$$\text{Re-arrange:} \quad \sin^2 A = 1 - \cos^2 A \text{ --- (1)}$$

$$\text{Substitute (1) into the top line, LHS:} \quad \frac{1 - \cos^2 A}{\cos^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$\text{Since } \tan x = \frac{\sin x}{\cos x}:$$

$$= \tan^2 A$$

$$6. \quad \frac{1 - \sin^2 A}{\cos^2 A} = 1$$

$$\sin^2 A + \cos^2 A = 1$$

Re-arrange:  $\cos^2 A = 1 - \sin^2 A \text{ --- (1)}$

Substitute (1) into the top line, LHS:  $\frac{1 - \sin^2 A}{\cos^2 A}$

$$= \frac{\cos^2 A}{\cos^2 A}$$

$$= 1$$

$$7. \quad (\cos x + \sin x)^2 = 1 + 2\sin x \cos x$$

LHS:  $(\cos x + \sin x)^2$

$$= (\cos x + \sin x)(\cos x + \sin x)$$

$$= \cos^2 x + \cos x \sin x + \sin x \cos x + \sin^2 x$$

$$= \cos^2 x + 2\sin x \cos x + \sin^2 x$$

$$= 1 + 2\sin x \cos x \text{ (since } \sin^2 x + \cos^2 x = 1)$$

$$8. \quad (\cos x + \sin x)(\cos x - \sin x) + 2\sin^2 x = 1$$

LHS:  $(\cos x + \sin x)(\cos x - \sin x) + 2\sin^2 x$

Multiply out brackets:  $= \cos x(\cos x - \sin x) + \sin x(\cos x - \sin x) + 2\sin^2 x$

$$= \cos^2 x - \cos x \sin x + \sin x \cos x - \sin^2 x + 2\sin^2 x$$

Simplify:  $= \cos^2 x - \sin^2 x + 2\sin^2 x$

Simplify again:  $= \cos^2 x + \sin^2 x$

$$= 1 \quad \text{(since } \sin^2 x + \cos^2 x = 1)$$

$$9. \sin^3 x + \sin x \cos^2 x = \sin x$$

LHS, factorise:

$$\sin x (\sin^2 x + \cos^2 x)$$

Substitute:  $\sin^2 A + \cos^2 A = 1$

$$= \sin x (1)$$

$$= \sin x$$

$$10. \cos^2 x \sin^2 x + \cos^4 x = \cos^2 x$$

LHS, factorise:

$$\cos^2 x (\sin^2 x + \cos^2 x)$$

Substitute:  $\sin^2 A + \cos^2 A = 1$

$$= \cos^2 x (1)$$

$$= \cos^2 x$$