Essential Skills
National 5 Maths

Content:

G. Rennie

Brannock High School

Essential Skills 1

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Multiplying Brackets involving Indices (Non Calculator)

Multiply out and simplify:

1. $x^{\frac{1}{4}}(x^{\frac{3}{4}} + x^{-\frac{1}{4}})$

2. $x^{\frac{2}{7}}(x^{\frac{1}{7}} - x^{-\frac{2}{7}})$

3. $3x^{\frac{1}{5}}(2x^{\frac{3}{5}} - x^{-\frac{1}{5}})$

4. $4x^{\frac{2}{3}}(3x^{\frac{4}{3}} + 2x^{-\frac{2}{3}})$

5. $5x^{\frac{1}{2}}(x^{\frac{5}{2}} + x^{-\frac{1}{2}})$

6. $x^{\frac{2}{3}}(x^{\frac{1}{2}} - x^{-\frac{2}{3}})$

7. $a^{\frac{1}{4}}(a^{\frac{3}{2}} - a^{-\frac{1}{4}})$

8. $b^{\frac{2}{3}}(3b^{\frac{1}{4}} + b^{-\frac{2}{3}})$

9. $6c^{\frac{1}{8}}(c^{\frac{3}{4}} + 2c^{-\frac{1}{8}})$

10. $x^{\frac{1}{2}}(x^{-\frac{7}{2}} - x^{-\frac{1}{2}})$

APPLYING QUESTION

(a) Multiply out and simplify $x^{\frac{1}{4}}(x^{\frac{1}{2}} + x^{-\frac{1}{4}})$

(b) **Hence**, evaluate when $x = 16$

Essential Skills 2

The questions in this series of worksheets appear frequently.

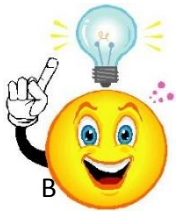
These are the GIFTS you must take to succeed

Completing the Square (Non Calculator)

Write the following in the form $(x + a)^2 + b$ and state the coordinates of the turning point.

1. $x^2 + 8x - 3$
2. $x^2 - 6x - 1$
3. $x^2 + 12x + 20$
4. $x^2 - 18x$
5. $x^2 - 2x + 7$
6. $x^2 + 10x + 13$
7. $x^2 + 4x - 9$
8. $x^2 - 6x + 6$
9. $x^2 + 14x - 25$
10. $x^2 - 4x + 1$

APPLYING QUESTION



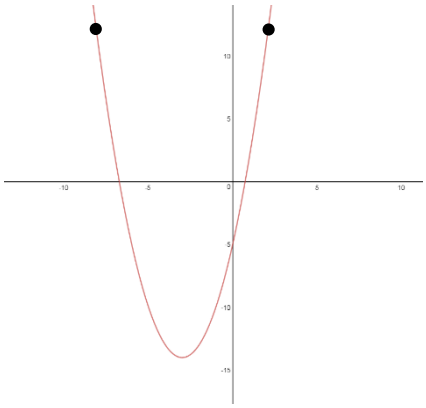
The curve $y = x^2 + 6x - 5$ is shown.

(a) Determine the coordinates of the turning point and the y-intercept

(b) Given that A is (-8, 11) write down the coordinates of B

A(-8,11)

B



Essential Skills 3

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Simplifying Surds (Non Calculator)

Simplify:

1. $\sqrt{20} + \sqrt{45} - \sqrt{5}$
2. $2\sqrt{3} - \sqrt{108} + \sqrt{75}$
3. $7\sqrt{2} + \sqrt{18} - \sqrt{128}$
4. $\sqrt{6} - \sqrt{54} - \sqrt{24}$
5. $\sqrt{160} + 2\sqrt{10} - \sqrt{90}$
6. $\sqrt{63} - \sqrt{28} - \sqrt{7}$
7. $\sqrt{44} - \sqrt{99} + 4\sqrt{11}$
8. $3\sqrt{5} + \sqrt{320} - \sqrt{180}$
9. $4\sqrt{2} + \sqrt{8} - \sqrt{98}$
10. $\sqrt{27} - 2\sqrt{12} + \sqrt{3}$

APPLYING QUESTION

The Rectangle shown has a perimeter of $\sqrt{72}$ and breadth of $\sqrt{2}$.

Calculate its length.

Essential Skills 4

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Algebraic Fractions (Non Calculator)

Write as a fraction in its simplest form:

1. $\frac{3}{x+4}+\frac{2}{x+1}$

2. $\frac{4}{x-5}+\frac{3}{x+2}$

3. $\frac{1}{x+2}-\frac{3}{x+7}$

4. $\frac{6}{2x-1}-\frac{2}{x-1}$

5. $\frac{2}{x+3}-\frac{2}{3x+1}$

6. $\frac{x-3}{5}+\frac{x+2}{2}$

7. $\frac{2b+3}{3}-\frac{b}{5}$

8. $\frac{1}{p-1}+\frac{3}{3p+5}$

9. $\frac{3x-1}{3}-\frac{2x-3}{2}$

10. $\frac{1}{x}+\frac{3}{x^2}$

APPLYING QUESTION

A cyclist cycling on difficult terrain was able to cover x km at 4 km/h

(a) Write an expression in terms of x for time of his journey.

On the return leg they took a more favourable route with 3 additional km. They were able to cycle at a speed of 6km/h

(b) Work out, as a single fraction in terms of x, the **total** time for the whole journey.

Essential Skills 5

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Changing the subject of a Formula (Non Calculator)

Change the subject to the indicated letter:

1. $A = bc^2 + d$ (c)

2. $V = \pi r^2 h$ (r)

3. $H = \sqrt{ft}$ (t)

4. $W = \frac{d^2}{p}$ (p)

5. $g = (vip)^2$ (v)

6. $A = \frac{1}{2}absinC$ (a)

7. $gh^3 - d = w$ (h)

8. $P = \frac{5hs}{t}$ (h)

9. $D = \frac{3(a+b)}{f}$ (a)

10. $T = \sqrt[3]{6t - 3}$ (t)

APPLYING QUESTION (Calculator)

A cosmetics company aim to reduce the volume of a spherical bath bomb by 20%

(a) If it originally had a volume of $480cm^3$, what will its new volume be?

(b) Calculate the radius of the resized bath bomb.

Essential Skills 6

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Fractional Indices (Non Calculator)

Write the following in root form and then evaluate:

1. $8^{\frac{2}{3}}$
2. $9^{\frac{3}{2}}$
3. $16^{\frac{5}{4}}$
4. $1000^{\frac{2}{3}}$
5. $81^{\frac{3}{4}}$
6. $32^{\frac{2}{5}}$
7. $4^{\frac{5}{2}}$
8. $64^{\frac{1}{3}}$
9. $25^{\frac{1}{2}}$
10. $125^{\frac{4}{3}}$

APPLYING QUESTIONS

1. Write $2x^{-\frac{2}{3}}$ with positive powers and evaluate when $x = 8$
2. If $f(x) = x^{\frac{5}{2}}$ evaluate $f(9)$

Essential Skills 7

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Factorising Difference of Two Squares (Non Calculator)

Factorise the following:

1. $a^2 - b^2$
2. $x^2 - 9$
3. $4a^2 - d^2$
4. $9f^2 - 64$
5. $p^2 - 25$
6. $4p^2 - 81$
7. $g^2 - 100h^2$
8. $9c^2 - 49d^2$
9. $x^2 - 121$
10. $8a^2 - 18t^2$ *careful

APPLYING QUESTION

(a) Factorise $3j^2 - 3k^2$

(b) Hence, evaluate when $j = 2.3$ & $k = 0.7$

Essential Skills 8

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Multiplying out Brackets (Non Calculator)

Multiply out and simplify:

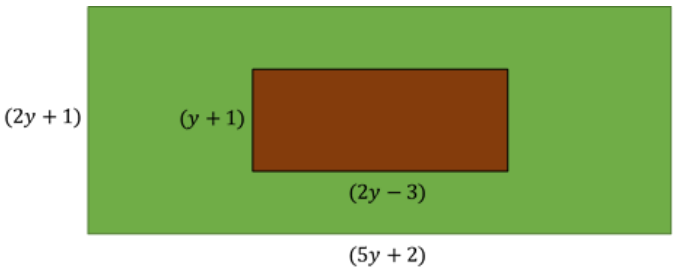
1. $3(x-3)+2(x-5)$
2. $-7(2t-3w)-11(t-1)$
3. $(x+4)(x+6)$
4. $(x-8)(x-7)$
5. $(3x+4)(2x-1)$
6. $(5x-3)(x-2)$
7. $(4x+1)(3x-2)$
8. $(x+4)^2$
9. $(2x-1)^2$
10. $(3s-4t)^2$

APPLYING QUESTION

A garden has length 5y+2 and breadth 2y + 1.

A rectangular flower bed of length 2y – 3 and breadth y + 1 is cut out the grass.

Find an expression in terms of y for the area of grass remaining



Essential Skills 9

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Multiplying out Brackets (Non Calculator)

Multiply out and simplify:

1. $(x+3)(x^2+2x+1)$
2. $(x+2)(3x^2+5x-1)$
3. $(2x+1)(x^2-3x+4)$
4. $(x-2)(x^2+5x+2)$
5. $(x-5)(x^2-3x-10)$
6. $(2x+3)(x^2-4x+3)$
7. $(3x-1)(2x^2+4x-1)$
8. $(x-1)(x^2-7x+6)$
9. $(x+8)(3x^2+x-4)$
10. $(x-4)(2x^2-2x+1)$

APPLYING QUESTION

Multiply out and simplify:

(a) $(x+2)(x-3)(x+1)$

(b) $(x+2)(x-1)^2$

(c) $(2x-3)^3$

Essential Skills 10

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Factorising Trinomials when a=1 (Non Calculator)

Factorise the following:

1. $a^2 + 6a + 8$
2. $b^2 + 11b + 30$
3. $c^2 - 8c + 12$
4. $d^2 - 13d + 40$
5. $e^2 + e - 56$
6. $f^2 - 3f - 54$
7. $g^2 + 15g + 54$
8. $h^2 + 13h - 30$
9. $j^2 - 6j - 55$
10. $3k^2 + 6k - 189$ *careful

APPLYING QUESTION

(a) Factorise $x^2 - 16$

(b) Hence, simplify $\frac{x^2-3x-28}{x^2-16}$

Essential Skills 11

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Calculations involving Fractions (Non Calculator)

Calculate the following:

1. $2\frac{1}{3} - 1\frac{2}{5}$
2. $6\frac{2}{5} \div 3\frac{1}{2}$
3. $3\frac{2}{3} \times 1\frac{10}{11}$
4. $1\frac{1}{6} + 2\frac{3}{8}$
5. $2\frac{3}{4} \div 1\frac{1}{5}$
6. $2\frac{2}{9} \times 4\frac{3}{5}$
7. $5\frac{1}{2} - 2\frac{5}{6}$
8. $\frac{1}{7}(2\frac{1}{4} + 1\frac{3}{5})$
9. $\frac{5}{6} of \frac{2}{3} + 1\frac{1}{6}$
10. $2\frac{1}{2} \times (2\frac{1}{8} - 1\frac{2}{5})$

APPLYING QUESTION

A recipe requires $1\frac{3}{4}$ cups of flour.

If the intention is to make $1\frac{1}{3}$ times the quantity on the recipe, how much flour will be required?

Essential Skills 12

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Standard Deviation (Calculator)

Calculate the mean and the standard deviation of each:

1. 14, 17, 15, 23, 20, 19
2. 8, 13, 7, 6, 8, 9, 5
3. 1.8, 3.7, 4, 2.6, 5.9
4. 102, 108, 112, 109, 110, 107
5. 47, 56, 61, 52, 59
6. 1, 2, 4, 1, 3, 2, 1
7. 9, 14, 11, 13, 8, 11
8. 33, 39, 40, 38, 35
9. 1305, 1301, 1298, 1300, 1295, 1307
10. 41, 35, 33, 46, 38

APPLYING QUESTION

The prices, in pence, at five petrol stations around Airdrie for a litre of unleaded are:

121 119 120 117 118

(a) Calculate the mean and standard deviation.

(b) Why do you think the standard deviation must be so low?

(c) If each petrol station had to put their price up by 4 pence what effect would it have on the mean and standard deviation?

Essential Skills 13

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

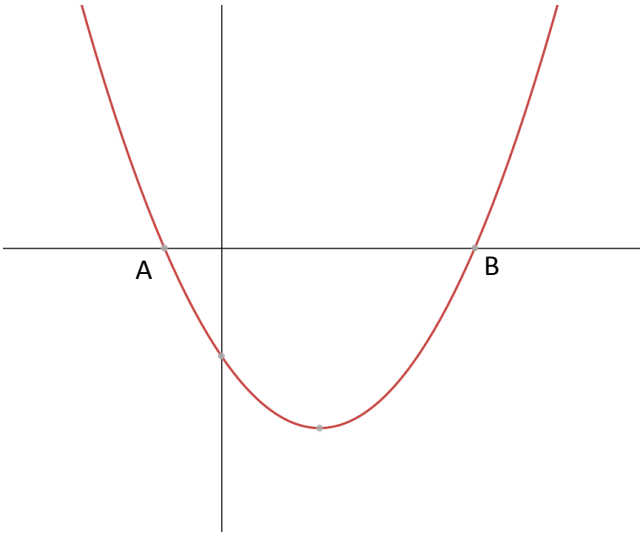
Quadratic Formula (Calculator)

Solve the following to 1 decimal place:

1. $x^2 + 6x + 2 = 0$
2. $3x^2 + 4x - 1 = 0$
3. $5x^2 - x - 3 = 0$
4. $4x^2 - 7x + 1 = 0$
5. $x^2 + 4x - 2 = 0$
6. $4 - 4x - x^2 = 0$ *Careful
7. $9x^2 - 8x + 1 = 0$
8. $2x^2 + 3x - 5 = 0$
9. $5x^2 - 9x + 2 = 0$
10. $2x^2 = 3x + 3$ *Careful

APPLYING QUESTION

The curve $f(x) = 2x^2 - 4x - 3$ is shown.



Determine the coordinates of A and B

Give your answers to 3 significant figures

Essential Skills 14

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Rationalising the Denominator (Non Calculator)

Write the following with a rational denominator in its simplest form:

1. $\frac{3}{\sqrt{5}}$ 2. $\frac{4}{\sqrt{6}}$

3. $\frac{8}{\sqrt{2}}$ 4. $\frac{1}{2\sqrt{7}}$

5. $\frac{5}{3\sqrt{3}}$ 6. $\frac{4}{3\sqrt{6}}$

7. $\frac{\sqrt{2}}{3\sqrt{5}}$ 8. $\frac{\sqrt{3}}{\sqrt{15}}$

9. $\frac{5}{\sqrt{8}}$ *Careful 10. $\frac{2}{\sqrt{48}}$ *Careful

APPLYING QUESTIONS

(1) Given that $f(x) = \frac{5}{4\sqrt{x}}$

Evaluate $f(15)$, writing your answer with a rational denominator in its simplest form.

(2) ***Non Calculator***

Calculate the mean and standard deviation of:

11 14 10 13 13 10 13

Write your answer with a rational denominator in its simplest form

Essential Skills 15

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Trigonometric Equations (Calculator)

Solve the following: $(0 \leq x \leq 360)$

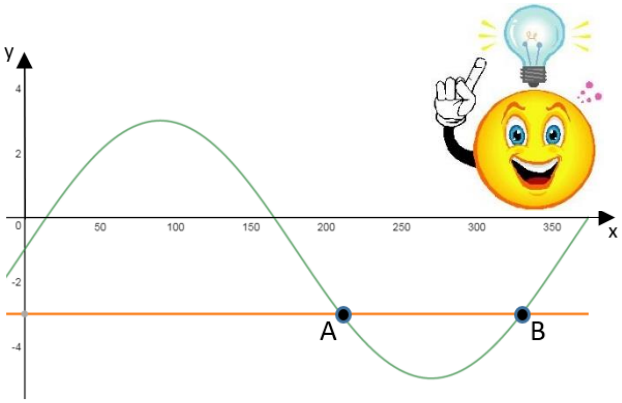
1. $2sinx - 1 = 0$
2. $2cosx - \sqrt{3} = 0$
3. $5tanx - 1 = 2$
4. $6sinx + 2 = 3$
5. $3cosx + 1 = 3$
6. $2tanx + 11 = 20$
7. $5sinx - 1 = -3$
8. $4cosx + 7 = 5$
9. $2tanx + 3 = 1$
10. $20sinx + 17 = 25$

APPLYING QUESTION

The curve $y = 4sinx - 1$ is shown.

The line $y = -3$ intersects at A and B

Determine the coordinates of A and B



Essential Skills 16

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

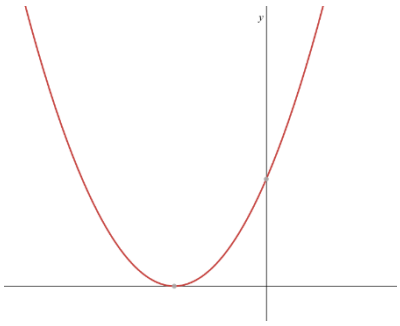
Nature of Roots: Discriminant (Non Calculator)

Determine the nature of roots on the following:

1. $x^2 + 3x + 4 = 0$
2. $4x^2 + 7x + 3 = 0$
3. $x^2 + 6x + 9 = 0$
4. $2x^2 - 5x + 2 = 0$
5. $3x^2 + 3x - 1 = 0$
6. $5x^2 - 10x + 5 = 0$
7. $3x^2 - 3x - 6 = 0$
8. $4x^2 - 1x + 2 = 0$
9. $5 - x - 2x^2 = 0$
10. $x^2 + 4 = 0$

APPLYING QUESTIONS

1. What value would the discriminant be in the parabola shown?



2. Find the value of k given that $kx^2 + 5x + 10 = 0$ has equal roots?

Essential Skills 17

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Functional Notation (Non Calculator)

1.	If	$f(x) = 3x - 4$	Evaluate:	(a)	$f(2)$	(b)	$f(-1)$
2.	If	$f(x) = x^2 - 1$	Evaluate:	(a)	$f(4)$	(b)	$f(-2)$
3.	If	$f(x) = 2x^3 + 3$	Evaluate:	(a)	$f(3)$	(b)	$f(-1)$
4.	If	$f(x) = 3x^2$	Evaluate:	(a)	$f(5)$	(b)	$f(-4)$
5.	If	$f(x) = 3x^2 - 1$	Evaluate:	(a)	$f(4)$	(b)	$f(-2)$
6.	If	$f(x) = 7 - x$	Evaluate:	(a)	$f(3)$	(b)	$f(-7)$
7.	If	$f(x) = 5 - x^2$	Evaluate:	(a)	$f(2)$	(b)	$f(-3)$
8.	If	$f(x) = -x^3$	Evaluate:	(a)	$f(1)$	(b)	$f(-4)$
9.	If	$f(x) = 4 + x^2$	Evaluate:	(a)	$f(5)$	(b)	$f(-3)$
10.	If	$f(x) = 3 + 2x - x^3$	Evaluate:	(a)	$f(2)$	(b)	$f(-1)$

APPLYING QUESTION

A function is defined as $h(x) = 24 - 5x$

(a) Evaluate $h(-3)$

(b) Express $h(p - 4)$ in its simplest form.

(c) Given that $h(t) = 59$, find the value of t.

(d) Solve $3x + 9 = 2h(x)$

Essential Skills 18

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Straight Lines (Non Calculator)

Find the equation of the line connecting the points:

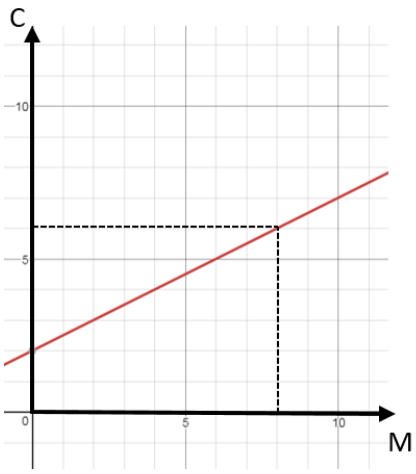
1. $A\,(2,5)$ & $B\,(8,23)$
2. $C\,(0,7)$ & $D\,(5,17)$
3. $E\,(-3,2)$ & $F\,(2,7)$
4. $G\,(-1,-4)$ & $H\,(3,4)$
5. $J\,(-4,7)$ & $K\,(1,2)$
6. $L\,(-5,0)$ & $M\,(0,-10)$
7. $P\,(-4,0)$ & $Q\,(0,5)$
8. $R\,(0,-3)$ & $S\,(4,7)$
9. $T\,(3,1)$ & $U\,(7,7)$
10. $V\,(-2,5)$ & $W\,(4,-3)$

APPLYING QUESTION

A local delivery firm charges a basic £2 for deliveries.

An extra charge is dependent on distance, as shown in the diagram.

M is the distance (miles) and C is cost (£)



(a) If an 8 mile delivery costs £6 find an equation, in terms of M and C, for the line.

(b) What would the cost be for a 25 mile delivery?

Essential Skills 19

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Simultaneous Equations (Calculator may be used on Applying Q2)

Solve the system of equations:

1. $3x + 2y = 12$
   $2x + y = 7$

2. $4x + 3y = 19$
   $5x - y = 0$

3. $2x + 7y = 18$
   $3x + 5y = 16$

4. $5x + 2y = 3$
   $4x + 3y = 1$

5. $7x - 3y = 6$
   $4x - 2y = 2$

6. $2x - 5y = 18$
   $3x + 3y = 6$

7. $x - 3y = 1$
   $2x + y = -12$

8. $5x - 3y = -12$
   $4x + y = 4$

9. $7x - 3y = -19$
   $6x - 2y = -14$

10. $12x + y = 31$
    $4x - 2y = -6$

APPLYING QUESTIONS

1. Find the point of intersection of lines $3x + 2y = 33$ and $4x - y = 22$

2. An Excelsior stadium concert has room for x standing spectators and y seated spectators.

(a) If the capacity is 12000 tickets, make an equation in terms of x and y.

(b) A standing ticket costs £28·50 and a seated ticket is £41.

Make an equation in terms of x and y given that the takings for a sold-out concert were £472, 500.

(c) How many of each ticket were sold?

Essential Skills 20

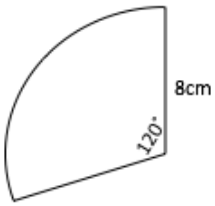
The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

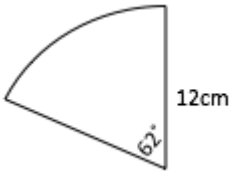
Arcs and Sectors (Calculator)

Calculate the length of arc and sector area in each:

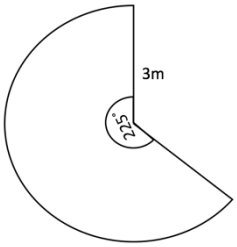
1.



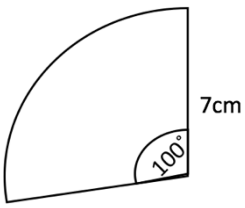
2.



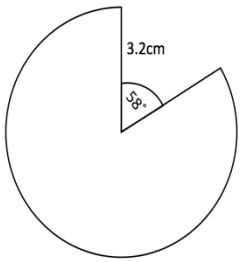
3.



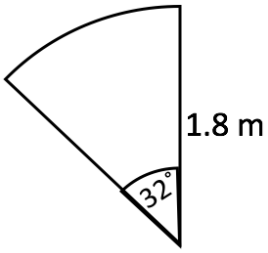
4.



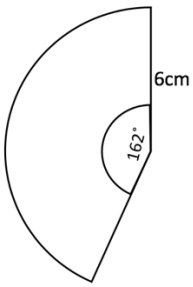
5.



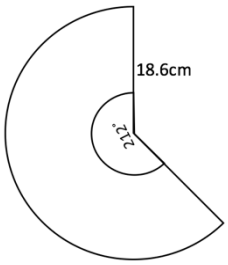
6.



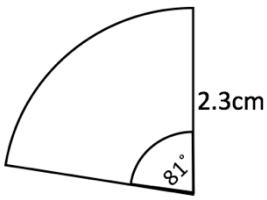
7.



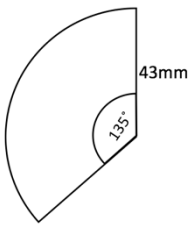
8.



9.

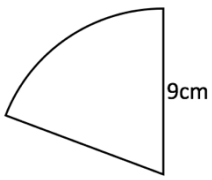


10.

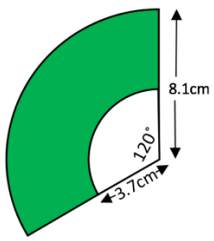


APPLYING QUESTIONS

1. The arc length of the sector shown is 10.52cm.
   What is its **area**?



2. Calculate the perimeter of the shaded section:



Essential Skills 21

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Algebraic Fractions 2 (Non Calculator)

By factorising numerators and denominators, simplify:

1. $\frac{x^2+5x+6}{3x+6}$

2. $\frac{x^2+4x-21}{2x+14}$

3. $\frac{x^2+8x+12}{x^2+6x}$

4. $\frac{x^2-9x+14}{x^2+3x-10}$

5. $\frac{2x^2-5x-3}{x^2-9}$

6. $\frac{3x-2}{3x^2+13x-10}$

7. $\frac{2x^2-50}{4x^2-19x-5}$

8. $\frac{4x^2-4x-3}{2x^2-5x+3}$

9. $\frac{2x^2+7x+3}{3x^2+8x-3}$

10. $\frac{x^2+x-56}{2x^2+11x-40}$

APPLYING QUESTION

(a) $x^2 - 3x - 54$

(b) Hence, simplify $\frac{x^2-3x-54}{3x^2+17x-6}$

Essential Skills 22

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Vectors (Non Calculator)

Find the components of the resultant vector:

1. $\begin{pmatrix}5\\2\end{pmatrix}-\begin{pmatrix}3\\4\end{pmatrix}$

2. $\begin{pmatrix}3\\2\\1\end{pmatrix}-2\begin{pmatrix}1\\0\\-3\end{pmatrix}$

3. $\begin{pmatrix}2\\-1\\7\end{pmatrix}+\begin{pmatrix}4\\-3\\5\end{pmatrix}$

4. $2\begin{pmatrix}0\\1\\8\end{pmatrix}-3\begin{pmatrix}-1\\2\\-5\end{pmatrix}$

5. $\begin{pmatrix}4\\0\\-1\end{pmatrix}+\begin{pmatrix}2\\9\\5\end{pmatrix}-\begin{pmatrix}3\\8\\-3\end{pmatrix}$

6. $2\begin{pmatrix}1\\-5\\7\end{pmatrix}+\begin{pmatrix}1\\-2\\-2\end{pmatrix}-4\begin{pmatrix}0\\-4\\2\end{pmatrix}$

Given that $\boldsymbol{a}=\begin{pmatrix}1\\3\\-2\end{pmatrix}$, $\boldsymbol{b}=\begin{pmatrix}-1\\1\\5\end{pmatrix}$ and $\boldsymbol{c}=\begin{pmatrix}2\\-5\\3\end{pmatrix}$, calculate:

7. $|\boldsymbol{a}+\boldsymbol{b}|$ 8. $|\boldsymbol{b}+\boldsymbol{c}|$ 9. $|\boldsymbol{a}+\boldsymbol{c}|$ 10. $|\boldsymbol{a}-\boldsymbol{b}|$

APPLYING QUESTIONS

1. The magnitude of vector $\begin{pmatrix}4\\k\\20\end{pmatrix}$ is 21. Find the possible values of k.

2. **a** and **b** are vectors with components $\begin{pmatrix}3\\5\\-2\end{pmatrix}$ & $\begin{pmatrix}3\\5\\1\end{pmatrix}$ respectively.

   Find the magnitude of 3**a** – 2**b**, leaving your answer as a surd in its simplest form.

Essential Skills 23

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Percentages: Reversing the Change (Calculator)

Find the original value:

1. £212 after having been increased by 6%
2. 105g after having been decreased by 30%
3. £12750 after a 2% rise
4. €6750 in a 10% off sale
5. 448ml after an increase of 12%
6. £96 after an increase of 20%
7. $79.20 after having been decreased by 20%
8. £36750 after a 5% wage rise
9. £48 after a 40% discount
10. €7.82 after a 15% increase in price

APPLYING QUESTION

On the 30th of June 2016 the exchange rate for the Euro was:

$$£1 = €1.13$$

This was a drop of 14.4% from the week earlier-before the Brexit vote.

What was the exchange rate before the referendum?

Essential Skills 24

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Indices (Non Calculator)

Simplify, leaving your answers with positive indices:

1. $\frac{x^5 \times x^6}{x^3}$

2. $\frac{x^7 \times x^{-4}}{x^2}$

3. $\frac{3x^2 \times x^4}{x^{-5}}$

4. $\frac{5x^3 \times 4x^2}{2x^3}$

5. $\frac{8x^5 \times 3x}{12x^2}$

6. $\frac{3x^2 \times 2x^{-1}}{7x}$

7. $\frac{2x^3 \times 5x}{15x^{-6}}$

8. $\frac{x^8 \times 3x^{-6}}{x^5}$

9. $\frac{2x^2y^3 \times 6x^2y}{4xy^2}$

10. $\frac{3x^2y^{\frac{1}{3}} \times 6x^{-1}y^{\frac{8}{3}}}{9x^3y^2}$

APPLYING QUESTION

(a) Simplify, $\frac{x^5y^3 \times 2x^{-1}y}{3x^2y^5}$,

leaving your answer with positive indices.

(b) Hence, evaluate $\frac{x^5y^3 \times 2x^{-1}y}{3x^2y^5}$, when $x = -3$ and $y = 2$.

Essential Skills 25

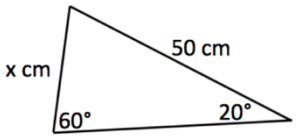
The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

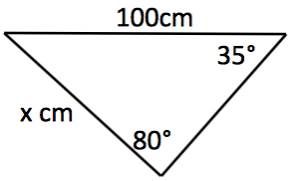
The Sine Rule (Calculator)

Calculate side, x in each:

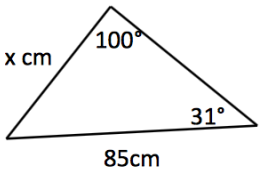
1.



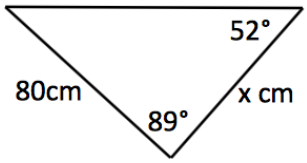
2.



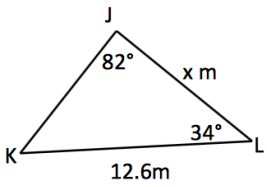
3.



4.

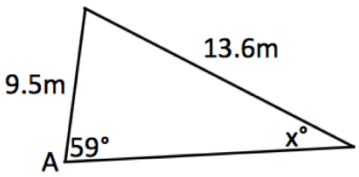


5.

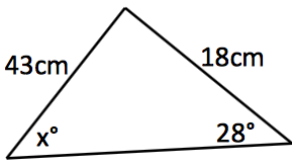


Calculate angle, x in each:

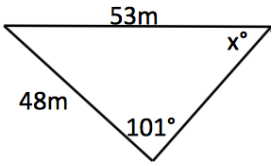
6.



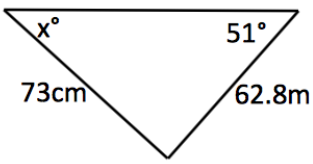
7.



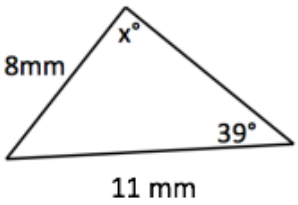
8.



9.



10.



APPLYING QUESTION

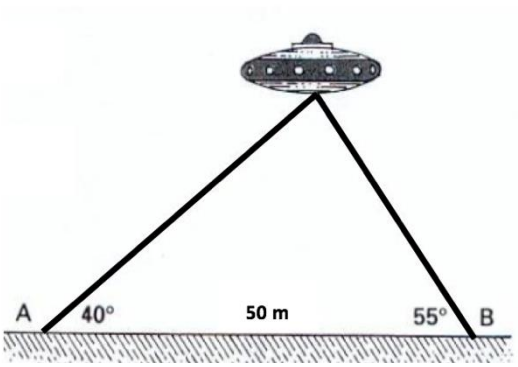
Aaron and Brandon spot a UFO above Brannock High.

Aaron measures elevation at 40˚ from his viewpoint

Brandon measures 55˚ from his.

They are standing 50 metres apart.

What height is the UFO above the ground?



Essential Skills 26

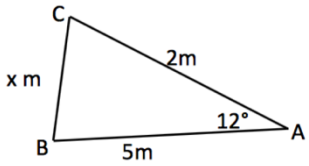
The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

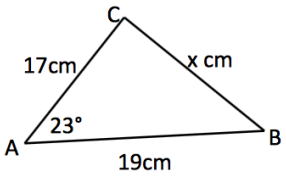
The Cosine Rule (Calculator)

Calculate side, x in each:

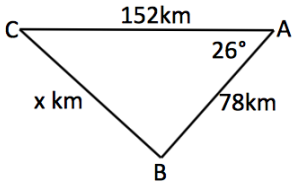
1.



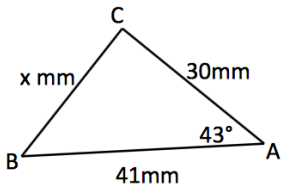
2.



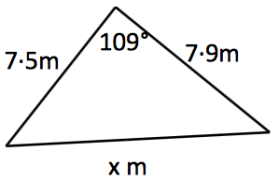
3.



4.

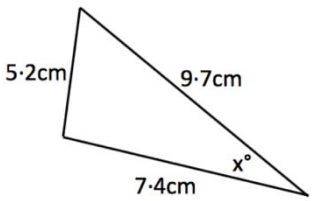


5.

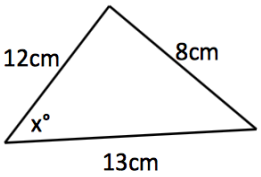


Calculate angle, x in each:

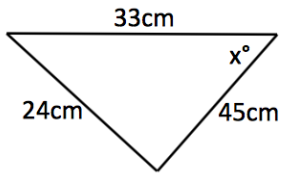
6.



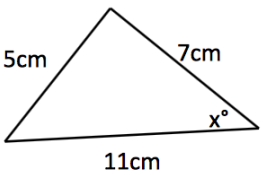
7.



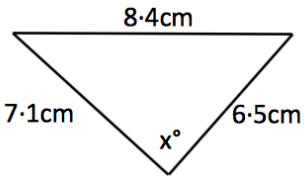
8.



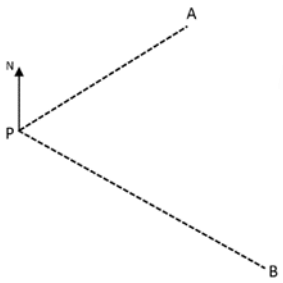
9.



10.



APPLYING QUESTION



Two drones leave from the same position, P.

Drone A flies 350 metres on a bearing of 063°.

Drone B flies 470 metres on a bearing of 134°.

Calculate the distance between the two drones.

Essential Skills 27

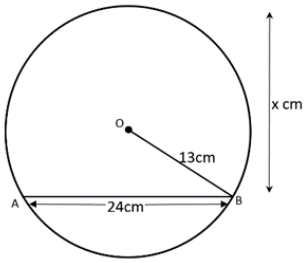
The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

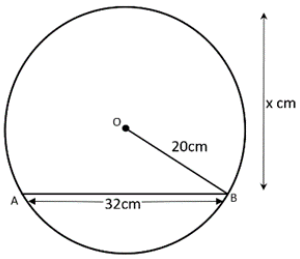
Pythagoras in Circles (Calculator)

Calculate x:

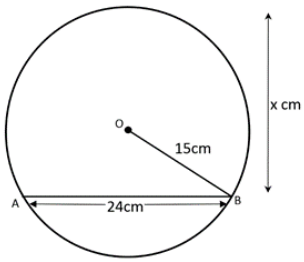
1.



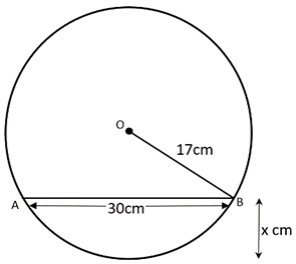
2.



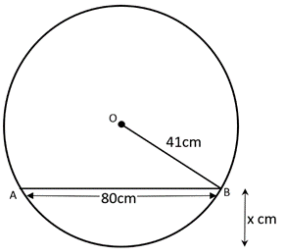
3.



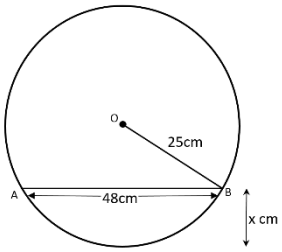
4.



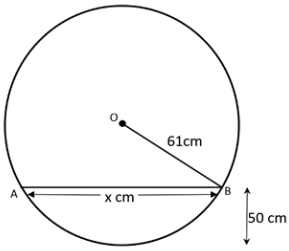
5.



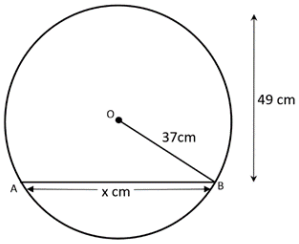
6.



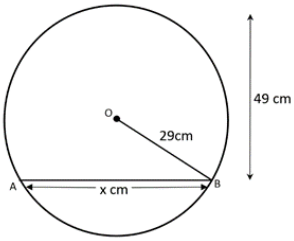
7.



8.



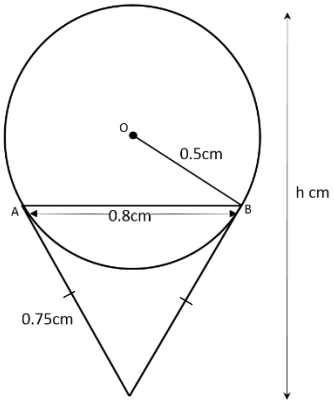
9.



APPLYING QUESTION

A pendant is designed as shown in the diagram.

Calculate its total height.



Essential Skills 28

The questions in this series of worksheets appear frequently.

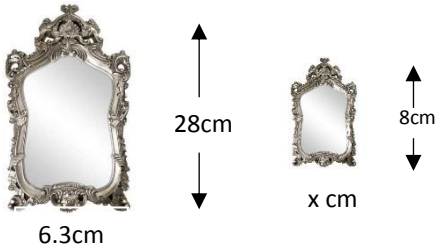
These are the GIFTS you must take to succeed

Similar Shapes (Calculator)

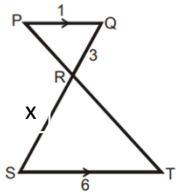
Calculate x in the following:

1.

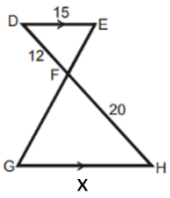
2.



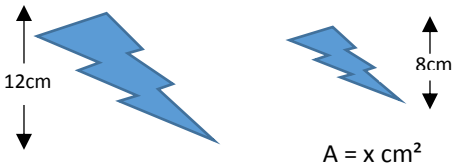
3.



4.



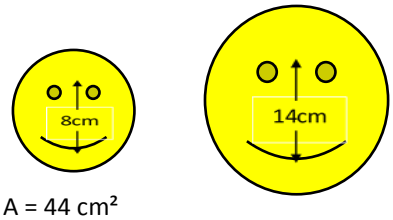
5.



A = 27 cm²

A = x cm²

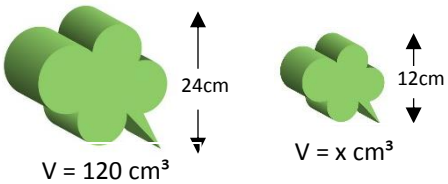
6.



A = 44 cm²

A = x cm²

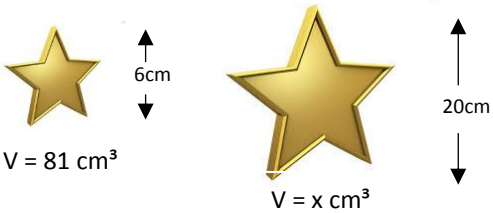
7.



V = 120 cm³

V = x cm³

8.



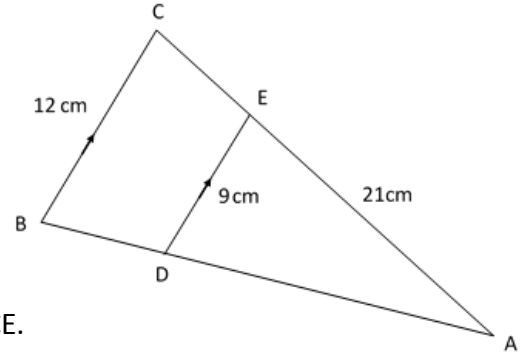
V = 81 cm³

V = x cm³

APPLYING QUESTION

In the diagram shown, triangles ABC and ADE are **mathematically similar**.

BC = 12 cm, DE = 9 cm and AE = 21 cm.



Find the length of CE.

Essential Skills 29

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Percentages: Appreciation and Depreciation (Calculator)

Calculate:

1. The interest earned on £3800 at 4% p.a after 3 years.
2. The population of a village after 4 years if it started with 1500 and decreases by 6% yearly
3. The number of bacteria after 3 hours if 30 are present initially and are increasing by 42% per hour.
4. The value of a ring, initially costing £799, after 3 year depreciation at 8.2% per year.
5. The volume of a 750ml jelly mould after 2 hours if decreasing by 5.6% per hour.
6. The school roll after 5 years if increasing by 8% per year from 630 initially.
7. The balance after 3 years when £240 is deposited with a 2.8% interest rate.
8. The sewage in a canal after 4 months if clearing removes 23% of the initial 238mg/litre per month.
9. The trade-in price of a car after 3 years. Bought for £13500, depreciating by 9% per year.
10. The value of a work of art, valued at £23000, after 9 years increasing by 12.5% per year.

APPLYING QUESTION

The population of Airdrie is 39200

Motherwell has a population of 32500

If the population of Airdrie drops by 4% yearly whilst the Motherwell population rises by 5%, how long will it take before Motherwell has a greater population than Airdrie?

Essential Skills 30

The questions in this series of worksheets appear frequently.

These are the GIFTS you must take to succeed

Quadratic Equations (Factorising) (Non Calculator)

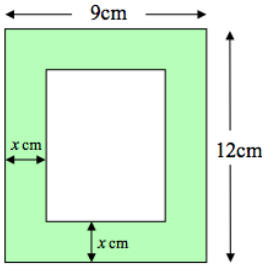
Solve:

1. $x^2 + 8x = 0$
2. $2x^2 - x = 0$
3. $x^2 - 25 = 0$
4. $4x^2 - 1 = 0$
5. $x^2 + 4x + 3 = 0$
6. $3x^2 - x - 4 = 0$
7. $5x^2 + 8x - 4 = 0$
8. $3x^2 - 8x - 3 = 0$
9. $10x^2 - 17x + 3 = 0$
10. $20 + 7x - 6x^2 = 0$

APPLYING QUESTIONS

1. Solve $4x(x + 1) = 15$

2. The dimensions of a picture frame are shown:

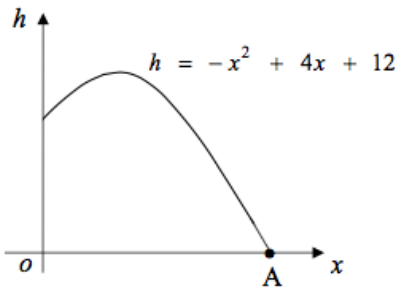


(a) Show that the area of glass at the centre is

$$A = 4x^2 - 42x + 108$$

(b) If the area of glass is $54cm^2$, find the value of x.

3.



A rock is thrown form a cliff and makes the shape of a parabola.

(a) How far will it travel before landing in the water?
(Each unit on the x axis represents 2 metres.)

(b) What was the maximum height it reached?
(Each unit on the h axis represents 2 metres.)

Answers

Essential Skills 1	
1	$x+1$
2	$x^{\frac{3}{7}}-1$
3	$6x^{\frac{4}{5}}-3$
4	$12x^2+8$
5	$5x^3+5$
6	$x^{\frac{7}{6}}-1$
7	$a^{\frac{7}{4}}-1$
8	$3b^{\frac{11}{12}}+1$
9	$6c^{\frac{7}{8}}+12$
10	$x^{-3}-1$
AQ	(a) $x^{\frac{3}{4}}+1$ (b) 9

Essential Skills 3	
1	$4\sqrt{5}$
2	$\sqrt{3}$
3	$2\sqrt{2}$
4	$-4\sqrt{6}$
5	$3\sqrt{10}$
6	0
7	$3\sqrt{11}$
8	$5\sqrt{5}$
9	$-5\sqrt{2}$
10	0
AQ	$2\sqrt{2}$

Essential Skills 2	
1	$(x+4)^2-19$
2	$(x-3)^2-10$
3	$(x+6)^2-16$
4	$(x-9)^2-81$
5	$(x-1)^2+6$
6	$(x+5)^2-12$
7	$(x+2)^2-13$
8	$(x-3)^2-3$
9	$(x+7)^2-74$
10	$(x-2)^2-15$
AQ	(a) $(x+3)^2-14$ (b) TP (-3,-14) Y intercept (0,-5) (c) B(2, 11)

Essential Skills 4	
1	$\frac{5x+11}{(x+4)(x+1)}$
2	$\frac{7x-7}{(x-5)(x+2)}$
3	$\frac{-2x+1}{(x+2)(x+7)}$
4	$\frac{2x-4}{(2x-1)(x-1)}$
5	$\frac{4x-4}{(x+3)(3x+1)}$
6	$\frac{7x+4}{10}$
7	$\frac{7b+15}{15}$
8	$\frac{6p+2}{(p-1)(3p+5)}$
9	$\frac{7}{6}$
10	$\frac{3+x}{x^2}$
AQ	(a) $\frac{x}{4}$ (b) $\frac{5x+6}{12}$

Answers

Essential Skills 5	
1	$c = \sqrt{\frac{A-d}{b}}$
2	$r = \sqrt{\frac{V}{\pi h}}$
3	$t = \frac{H^2}{f}$
4	$p = \frac{d^2}{W}$
5	$v = \frac{\sqrt{g}}{ip}$
6	$a = \frac{2A}{bsinC}$
7	$h = \sqrt[3]{\frac{w+d}{g}}$
8	$h = \frac{tP}{5s}$
9	$a = \frac{Df-3b}{3}$
10	$t = \frac{T^3+3}{6}$
AQ	(a) $384cm^3$ (b) 4.5cm

Essential Skills 7	
1	$(a+b)(a-b)$
2	$(x+3)(x-3)$
3	$(2a+d)(2a-d)$
4	$(3f+8)(3f-8)$
5	$(p+5)(p-5)$
6	$(2p+9)(2p-9)$
7	$(g+10h)(g-10h)$
8	$(3c+7d)(3c-7d)$
9	$(x+11)(x-11)$
10	$2(2a+3t)(2a-3t)$
AQ	(a) $3(j+k)(j-k)$ (b) 14.4

Essential Skills 6	
1	4
2	27
3	32
4	100
5	27
6	4
7	32
8	4
9	5
10	625
AQ	(1) $\frac{2}{x^{2/3}}, \frac{1}{2}$ (b) 243

Essential Skills 8	
1	$5x-19$
2	$21w-2st+11$
3	$x^2+10x+24$
4	$x^2-15x+56$
5	$6x^2+5x-4$
6	$5x^2-13x+6$
7	$12x^2-5x-2$
8	$x^2+8x+16$
9	$4x^2-4x+1$
10	$9s^2-24st+16t^2$
AQ	$8y^2+10y+5$

Answers

Essential Skills 9	
1	$x^3 + 5x^2 + 7x + 3$
2	$3x^3 + 11x^2 + 9x - 2$
3	$2x^3 - 5x^2 + 5x + 4$
4	$x^3 + 3x^2 - 8x - 4$
5	$x^3 - 8x^2 - 25x + 50$
6	$2x^3 - 5x^2 - 6x + 9$
7	$6x^3 + 10x^2 - 7x + 1$
8	$x^3 - 8x^2 + 13x - 6$
9	$3x^3 + 25x^2 + 4x - 32$
10	$2x^3 - 10x^2 - 7x - 4$
AQ	(a)$x^3 - 7x - 6$ (b) $x^3 - 3x + 2$ (c) $8x^3 - 36x^2 - 18x - 27$

Essential Skills 11	
1	$\frac{14}{15}$
2	$1\frac{29}{35}$
3	7
4	$3\frac{13}{14}$
5	$2\frac{7}{24}$
6	$10\frac{2}{9}$
7	$2\frac{2}{3}$
8	$\frac{11}{20}$
9	$1\frac{13}{18}$
10	$1\frac{13}{16}$
AQ	$2\frac{1}{3} cups$

Essential Skills 10	
1	$(a + 2)(a + 4)$
2	$(b + 5)(b + 6)$
3	$(c - 2)(c - 6)$
4	$(d - 5)(d - 8)$
5	$(e - 7)(e + 8)$
6	$(f + 6)(f - 9)$
7	$(g + 6)(g + 9)$
8	$(h - 2)(h + 15)$
9	$(j + 5)(j - 11)$
10	$3(k + 9)(k - 7)$
AQ	(a) $(x + 4)(x - 4)$ (b)$\frac{x-7}{x-4}$

Essential Skills 12	
1	$\overline{x} = 18, s = 3{\cdot}35$
2	$\overline{x} = 8, s = 2{\cdot}58$
3	$\overline{x} = 3 \cdot 6, s = 1{\cdot}56$
4	$\overline{x} = 108, s = 3{\cdot}4$
5	$\overline{x} = 55, s = 5{\cdot}6$
6	$\overline{x} = 18, s = 3{\cdot}35$
7	$\overline{x} = 2, s = 1{\cdot}15$
8	$\overline{x} = 37, s = 2{\cdot}92$
9	$\overline{x} = 1301, s = 4{\cdot}43$
10	$\overline{x} = 38 \cdot 6, s = 5{\cdot}13$
AQ	$\overline{x} = 119, s = 1{\cdot}58$ (b) Competition (c) Mean up by 4, Standard deviation the same.

Answers

Essential Skills 13	
1	$x = -0 \cdot 4 \ \& \ -5 \cdot 6$
2	$x = 0 \cdot 2 \ \& \ -1 \cdot 5$
3	$x = 1 \cdot 0 \ \& \ -0 \cdot 8$
4	$x = 1 \cdot 6 \ \& 1 \cdot 3$
5	$x = 0 \cdot 4 \ \& \ -4 \cdot 4$
6	$x = -4 \cdot 8 \ \& \ 0 \cdot 8$
7	$x = 0 \cdot 7 \ \& 0 \cdot 2$
8	$x = 1 \ \& \ -2 \cdot 5$
9	$x = 1 \cdot 5 \ \& \ 0 \cdot 3$
10	$x = -0 \cdot 7 \ \& \ 2 \cdot 2$
AQ	$A(-0 \cdot 581, 0) \ \& \ B(2 \cdot 58, 0)$

Essential Skills 15	
1	$30° \ \& \ 150°$
2	$30° \ \& \ 330°$
3	$31° \ \& \ 211°$
4	$9.6° \ \& \ 170.4$
5	$48.2° \ \& \ 311.8°$
6	$77.5° \ \& \ 257.5°$
7	$203.6° \ \& \ 336.4°$
8	$120° \ \& \ 240°$
9	$135° \ \& \ 315°$
10	$23.6° \ \& \ 156.4°$
AQ	$A(210°, -1) \ \& \ B(330°, -1)$

Essential Skills 14	
1	$\frac{3\sqrt{5}}{5}$
2	$\frac{2\sqrt{6}}{3}$
3	$4\sqrt{2}$
4	$\frac{\sqrt{7}}{14}$
5	$\frac{5\sqrt{3}}{9}$
6	$\frac{2\sqrt{6}}{9}$
7	$\frac{\sqrt{10}}{15}$
8	$\frac{\sqrt{5}}{5}$
9	$\frac{5\sqrt{2}}{4}$
10	$\frac{\sqrt{3}}{6}$
AQ	(1) $\frac{\sqrt{15}}{12}$ (2) $\frac{2\sqrt{6}}{3}$

Essential Skills 16	
1	$b^2 - 4ac = -7;$ *no real roots*
2	$b^2 - 4ac = 1;$ 2 *real & distinct roots*
3	$b^2 - 4ac = 0;$ 2 *real & equal roots*
4	$b^2 - 4ac = 9;$ 2 *real & distinct roots*
5	$b^2 - 4ac = 21;$ 2 *real & distinct roots*
6	$b^2 - 4ac = 0;$ 2 *real & equal roots*
7	$b^2 - 4ac = 81;$ 2 *real & distinct roots*
8	$b^2 - 4ac = -31;$ *no real roots*
9	$b^2 - 4ac = 41;$ 2 *real & distinct roots*
10	$b^2 - 4ac = -16;$ *no real roots*
AQ	(1) $b^2 - 4ac = 0$ (2) $k = \frac{5}{8}$

Answers

Essential Skills 17	
1	$(a)\ 2\ \ (b)\ -7$
2	$(a)\ 15\ \ (b)\ 3$
3	$(a)\ 57\ \ (b)\ 1$
4	$(a)\ 75\ \ (b)\ 48$
5	$(a)\ 47\ \ (b)\ 11$
6	$(a)\ 4\ \ (b)\ 14$
7	$(a)\ 1\ \ (b)\ -4$
8	$(a)\ -1\ \ (b)\ 64$
9	$(a)\ 29\ (b)\ 13$
10	$(a)\ -1\ \ (b)\ 2$
AQ	(a) 39 (b) 44 – 5p (c) t= -7 (d) x = 3

Essential Skills 18	
1	$y = 3x - 1$
2	$y = 2x + 7$
3	$y = x + 5$
4	$y = 2x - 2$
5	$y = -x + 3$
6	$y = -2x - 10$
7	$4y = -5x + 20$
8	$2y = -5x - 6$
9	$2y - 3x\ + 7 = 0$
10	$3y + 4x = 7$
AQ	(a) C= $0 \cdot 5M + 2$ (b) £14·50

Essential Skills 19	
1	$x = 2, y = 3$
2	$x = 1, y = 5$
3	$x = 2, y = 2$
4	$x = 1, y = -1$
5	$x = 3, y = 5$
6	$x = 4, y = -2$
7	$x = -5, y = -2$
8	$x = 0, y = 4$
9	$x = -1, y = 4$
10	$x = 2, y = 7$
AQ	(1) (7, 6) (2) (a) $x + y = 12000$ (b) $28 \cdot 5x + 41y = 472500$ (c) 1560 standing, 10440 seated

Essential Skills 20	
1	$16 \cdot 8cm, 67 \cdot 0cm^2$
2	$12 \cdot 99cm, 77 \cdot 91cm^2$
3	$11 \cdot 8m, 17 \cdot 7m^2$
4	$12 \cdot 2cm, 42 \cdot 8cm^2$
5	$16 \cdot 9cm, 27 \cdot 0cm^2$
6	$1 \cdot 0m, 0 \cdot 9m^2$
7	$17 \cdot 0cm, 50 \cdot 9cm^2$
8	$68 \cdot 8cm, 640 \cdot 0cm^2$
9	$3 \cdot 3cm, 3 \cdot 7cm^2$
10	$101 \cdot 3mm, 2178 \cdot 3mm^2$
AQ	(1) $47 \cdot 4cm^2$ (2) $33 \cdot 5cm$

Answers

Essential Skills 21	
1	$\frac{x+3}{3}$
2	$\frac{x-3}{2}$
3	$\frac{x+2}{x}$
4	$\frac{x-7}{x+5}$
5	$\frac{2x+1}{x+3}$
6	$\frac{1}{x+5}$
7	$\frac{2(x+5)}{4x+1}$
8	$\frac{2x+1}{x-1}$
9	$\frac{2x+1}{3x-1}$
10	$\frac{x-7}{2x-5}$
AQ	(a) $(x+6)(x-9)$ (b) $\frac{x-9}{3x-1}$

Essential Skills 23	
1	£200
2	150g
3	£12500
4	€7500
5	400ml
6	£80
7	$99
8	£35000
9	£80
10	€6·80
AQ	€1·32

Essential Skills 22	
1	$\begin{pmatrix}2\\-2\end{pmatrix}$
2	$\begin{pmatrix}1\\2\\7\end{pmatrix}$
3	$\begin{pmatrix}6\\-4\\12\end{pmatrix}$
4	$\begin{pmatrix}3\\-4\\31\end{pmatrix}$
5	$\begin{pmatrix}3\\1\\7\end{pmatrix}$
6	$\begin{pmatrix}3\\4\\4\end{pmatrix}$
7	5
8	9
9	$\sqrt{14}$
10	$\sqrt{57}$
AQ	(1) ± 5 (2) $7\sqrt{2}$

Essential Skills 24	
1	x^8
2	x
3	$3x^{11}$
4	$10x^2$
5	$2x^4$
6	$\frac{6}{7}$
7	$\frac{2x^{10}}{3}$
8	$\frac{3}{x^3}$
9	$3x^3y^2$
10	$\frac{2y}{x^2}$
AQ	(a) $\frac{2x^2}{3y}$ (b)3

Answers

Essential Skills 25	
1	19·75cm
2	58·2cm
3	44·5cm
4	63·9cm
5	11·4cm
6	36·8°
7	11°
8	62·8°
9	42·0°
10	120·1°
AQ	26·5 metres

Essential Skills 27	
1	18cm
2	32cm
3	24cm
4	9cm
5	32cm
6	18cm
7	120cm
8	70cm
9	42cm
10	
AQ	1.4cm

Essential Skills 26	
1	3·1m
2	7·4cm
3	88·8km
4	28·0mm
5	12·5m
6	32·0°
7	37·1°
8	31·2°
9	19·7°
10	76·1°
AQ	486·1m

Essential Skills 28	
1	20cm
2	2·1cm
3	18
4	25
5	$12cm^2$
6	$134.75cm^2$
7	$15cm^3$
8	$3000cm^3$
9	
10	
AQ	7cm

Answers

Essential Skills 29	
1	£474·48 interest
2	1171 people
3	85 bacteria
4	£618·12
5	668·4ml
6	925 pupils
7	£260·73
8	83·7 mg/litre
9	£10173·21
10	£66389·67
AQ	3 years (Motherwell 37622, Airdrie 34681)

Essential Skills 30	
1	$x = 0, x = -8$
2	$x = 0, x = \frac{1}{2}$
3	$x = -5, x = 5$
4	$x = -\frac{1}{2}, x = \frac{1}{2}$
5	$x = -3, x = -1$
6	$x = \frac{4}{3}, x = -1$
7	$x = \frac{2}{5}, x = -2$
8	$x = -\frac{1}{3}, x = 3$
9	$x = \frac{1}{5}, x = \frac{3}{2}$
10	$x = -\frac{4}{3}, x = \frac{5}{2}$
AQ	(1) $x = -\frac{5}{2}, x = \frac{3}{2}$ (2) (a) $l = 12 - 2x, b = 9 - 2x$, proof (b) $x = \frac{3}{2}$ (3) (a) 12m (b) 32m