

# **X100/302**

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NATIONAL  
QUALIFICATIONS  
2011

WEDNESDAY, 18 MAY  
10.50 AM – 12.00 NOON

MATHEMATICS  
HIGHER  
Paper 2

**Read Carefully**

- 1 **Calculators may be used in this paper.**
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**Table of standard derivatives:**

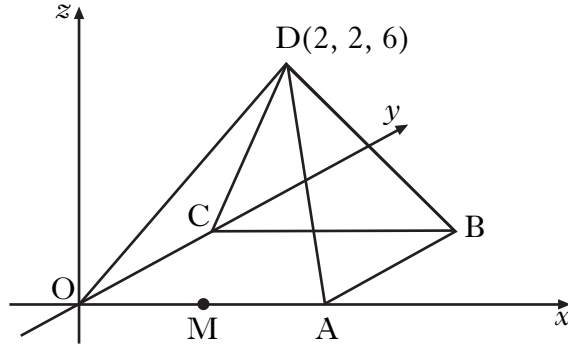
| $f(x)$    | $f'(x)$      |
|-----------|--------------|
| $\sin ax$ | $a \cos ax$  |
| $\cos ax$ | $-a \sin ax$ |

**Table of standard integrals:**

| $f(x)$    | $\int f(x) dx$             |
|-----------|----------------------------|
| $\sin ax$ | $-\frac{1}{a} \cos ax + C$ |
| $\cos ax$ | $\frac{1}{a} \sin ax + C$  |

**ALL questions should be attempted.**

1. D,OABC is a square based pyramid as shown in the diagram below.



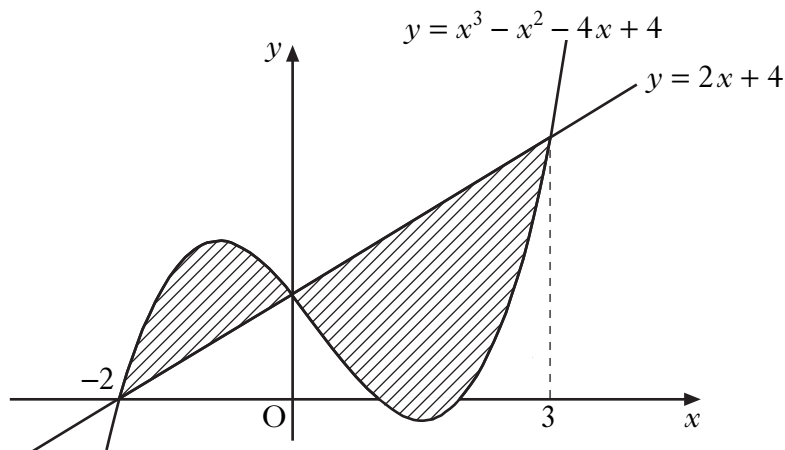
O is the origin, D is the point (2, 2, 6) and OA = 4 units.  
M is the mid-point of OA.

- (a) State the coordinates of B. 1
- (b) Express  $\vec{DB}$  and  $\vec{DM}$  in component form. 3
- (c) Find the size of angle BDM. 5
2. Functions  $f$ ,  $g$  and  $h$  are defined on the set of real numbers by
- $f(x) = x^3 - 1$
  - $g(x) = 3x + 1$
  - $h(x) = 4x - 5$ .
- (a) Find  $g(f(x))$ . 2
- (b) Show that  $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$ . 1
- (c) (i) Show that  $(x - 1)$  is a factor of  $3x^3 + 4x^2 - 5x - 2$ .
- (ii) Factorise  $3x^3 + 4x^2 - 5x - 2$  fully. 5
- (d) Hence solve  $g(f(x)) + xh(x) = 0$ . 1

**[Turn over**

3. (a) A sequence is defined by  $u_{n+1} = -\frac{1}{2}u_n$  with  $u_0 = -16$ .  
Write down the values of  $u_1$  and  $u_2$ . 1
- (b) A second sequence is given by 4, 5, 7, 11, . . . .  
It is generated by the recurrence relation  $v_{n+1} = pv_n + q$  with  $v_1 = 4$ .  
Find the values of  $p$  and  $q$ . 3
- (c) Either the sequence in (a) or the sequence in (b) has a limit.  
(i) Calculate this limit.  
(ii) Why does the other sequence not have a limit? 3

4. The diagram shows the curve with equation  $y = x^3 - x^2 - 4x + 4$  and the line with equation  $y = 2x + 4$ .  
The curve and the line intersect at the points  $(-2, 0)$ ,  $(0, 4)$  and  $(3, 10)$ .

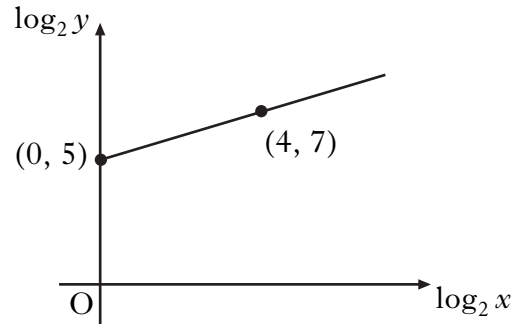


Calculate the total shaded area. 10

5. Variables  $x$  and  $y$  are related by the equation  $y = kx^n$ .

The graph of  $\log_2 y$  against  $\log_2 x$  is a straight line through the points  $(0, 5)$  and  $(4, 7)$ , as shown in the diagram.

Find the values of  $k$  and  $n$ .



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6. (a) The expression  $3 \sin x - 5 \cos x$  can be written in the form  $R \sin(x+a)$  where  $R > 0$  and  $0 \leq a < 2\pi$ .

Calculate the values of  $R$  and  $a$ .

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- (b) Hence find the value of  $t$ , where  $0 \leq t \leq 2$ , for which

$$\int_0^t (3 \cos x + 5 \sin x) dx = 3.$$

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7. Circle  $C_1$  has equation  $(x + 1)^2 + (y - 1)^2 = 121$ .

A circle  $C_2$  with equation  $x^2 + y^2 - 4x + 6y + p = 0$  is drawn inside  $C_1$ .

The circles have no points of contact.

What is the range of values of  $p$ ?

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[END OF QUESTION PAPER]

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