Education Scotland
Foghlam Alba

Scotland's Curriculum Framework
curriculum improvement cycle
Frèam-obrach Curraicealam na h-Alba
cearcall leasachadh a' churraicealaim

Curriculum Improvement Cycle

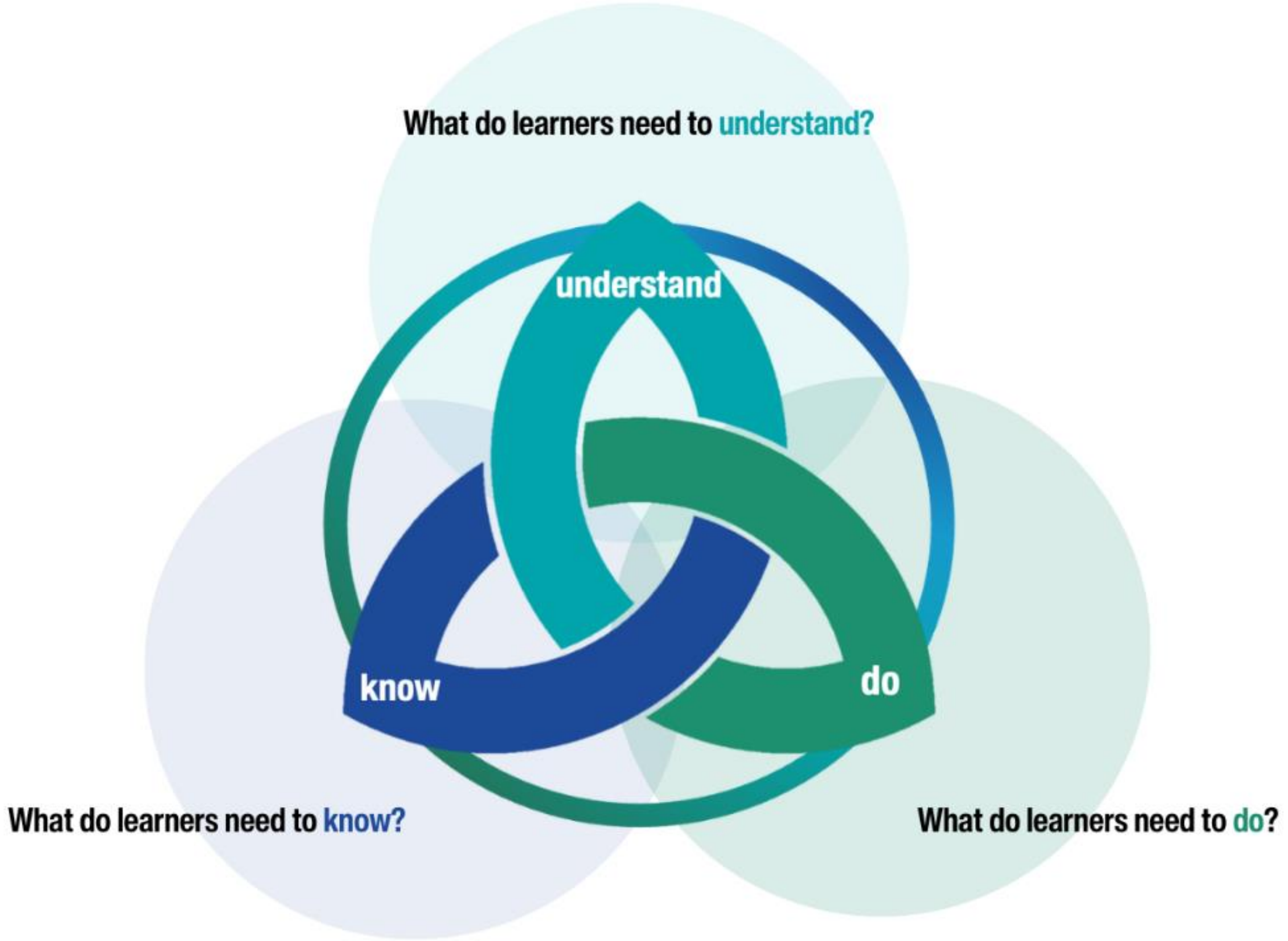
Draft Technical Framework

Numeracy and Mathematics Sample

June 2026

These are **draft materials**. The content, format and style are subject to change. It would not be appropriate to change current planning or tracking and monitoring systems in establishments at this stage.

There is **no expectation** on educators, schools or settings to do anything now with these samples. They are being shared as part of the co-design process and in advance of engagement and feedback time during the 2026/27 session.


What do learners need to understand?
understand
know
do
What do learners need to know?
What do learners need to do?

Contents

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Illustrative Big Idea

Mathematics is a Language

The language of mathematics supports the deepening of understanding. It allows us to model and communicate solutions to real-world problems. By using mathematical symbols, diagrams and accurate notation we can communicate effectively across disciplines and cultures. The use of precise notation and mathematical language is essential when conveying abstract concepts. As a collaborative practice, mathematics is about discourse, communicating ideas, and posing purposeful questions that promote curiosity and creativity. Precise mathematical language underpins clear and visible thinking, informed decision-making and justification.

Strand: Quantity, Numbers and the Algebraic Properties of Number

The focus of quantity is to understand value and size. Determining quantities in the world around us often involves solving problems that draw on a range of mathematical concepts. Numbers help us count, order, measure, compare, and express values. Number sense contributes to mathematical fluency, building accuracy, flexibility, and efficiency. The algebraic properties of number provide a lens through which to generalise how numbers and operations on numbers interact.

Please note: Work is being undertaken to explore what the Know-Do-Understand model means for practitioners who use a range of progression frameworks and guidance documents, including the ASN Milestones, to plan learning for children with complex needs.

- Early Level
- Number structures and operations

Strand: Quantity, Numbers and the Algebraic Properties of Number **Sub-strand: Number structures and operations**			
Beginning of Early Level ←		→	**End of Early Level**
Understand: Spoken words and written numerals are equivalent representations of counted quantities.			
Know	**Do** (through play, everyday routines, games, songs, stories and rhymes)	**Know**	**Do** (through play, everyday routines, games, songs, stories and rhymes)
Whole numbers can be said in words and symbolised using written numerals. Number words, numerals and quantities are connected.	Find examples of numerals in outdoor and indoor spaces. Recognise numerals which have been randomly arranged. For example, Can you point to 3? Where is 1? Identify numerals when randomly arranged. For example, What number is this? What number have you found? Explore making marks which represent numerals and quantities in different ways. Describe pictures, drawings and objects in the outdoor and indoor spaces in a numerical way. For example, I have painted three cats, I have two hands. Match numerals, spoken number words and associated quantities.	Digits are the symbols (0-9) used in the decimal number system. Digits are combined to represent numbers beyond 9.	Develop a sense of number referents. For example, 10 fingers, 4 Seasons, 6 eggs in a box, 5 people in my family. Recognise numerals which have been randomly arranged. For example, Can you point to 11? Where is 17? Identify numerals when randomly arranged. For example, What number is this? What number have you found? Identify numerals from other written symbols. For example, h, 5, c, A, ! Match numerals, spoken number words, and quantities. Make mathematical marks during role play and everyday routines. For example, making price tags, tickets or shopping lists.
Notes Whole numbers are the set of numbers 0, 1, 2, 3, 4, 5...			

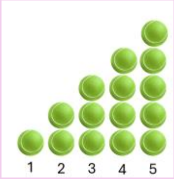
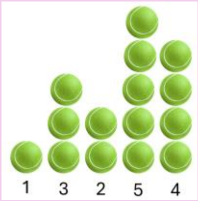
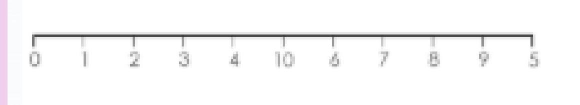
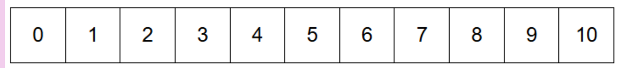
We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

A numeral is a single digit/symbol or group of digits/symbols used to represent a number.
Some learners may have experience of, or interest in, different number systems, such as Chinese, Arabic, Roman, ... and this can be explored where appropriate.
Using familiar and engaging contexts such as songs, stories, and rhymes helps learners connect number words to quantities and notice how numbers change over time.
Mathematical mark-making is how young learners use their own lines, dots, marks and symbols to represent real objects, visualise quantities and numbers and make sense of mathematics (symbolic representation).
A number referent is a known benchmark used to estimate unknown quantities, sizes, or magnitudes. For example, if you know what 10 fingers looks like, you can use that visual "referent" to estimate if a quantity shown is more or less than ten.

Includes

Working initially within the range 1 to 5, progressing to 0 to 10, and then extending to 20.

Understand: Numbers exist in a fixed order, and ordinality describes position or rank.

Know	**Do** (through play, everyday routines, games, songs, stories and rhymes)	**Know**	**Do** (through play, everyday routines, games, songs, stories and rhymes)
The order of the number words always stays the same (stable order).	Represent numbers laid out in order, using concrete materials of the same size. For example,  Recognise and correct errors made in ordered numbers. For example,  Listen to, copy and say the number words in the correct forwards and backwards order through number songs, stories and rhymes.		Listen to, copy and say the number words in the correct forwards and backwards order, through number songs, stories and rhymes. Count forwards and backwards in ones starting at any number within the range. Recognise when the numerals to 10 have been placed in the correct or incorrect order on a number track or line. For example,   Identify the numbers before, after, between, one more and one less than a given number(s). Identify which number is the largest and which is the smallest. For example, which is larger 13 or 16? Order a set of numerals from the largest or from the smallest. For example, 5, 12, 9, 2.
The whole number which comes directly after another whole number is one more. The whole number which comes directly before another whole number is one less.	Identify the numbers before and after within the range. For example, I am 3 and will be 4 on my birthday.  		
Numbers words can be used to represent position or rank, for example first, second, third... (Ordinal numbers).	Identify the position of an object or person using ordinal numbers. For example, Amina is second in the race. The red car went through the tunnel first.	Ordinal number words can be written symbolically (1st, 2nd, 3rd...)	Recognise and say ordinal numbers used in real and relevant contexts. For example, at sports events or in dates. Compare and describe the position of people or objects using ordinal numbers. For example, what colour is the 4th bead?, point to the 2nd red bead. The 3rd bead is...? 

Notes

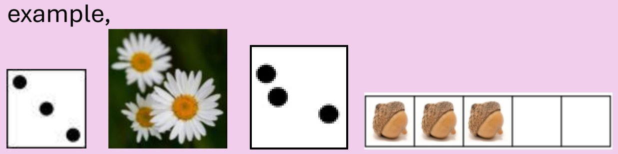
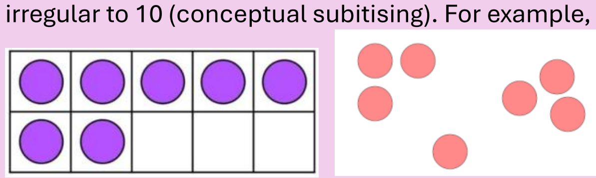
It is important to model counting backwards as part of everyday interactions as children are often less familiar with this.
Number tracks are useful when introducing the sequential order of numbers. Using a number track, where each space is discrete, prevents learners from skipping numbers or double-counting numbers.
Number lines are more useful when learners move beyond simple counting and need to understand the spaces between numbers, measure distances, or perform jumps.

Includes

Initially working within the range 1 to 5, progressing to 0 to 10, and then extending to 20.

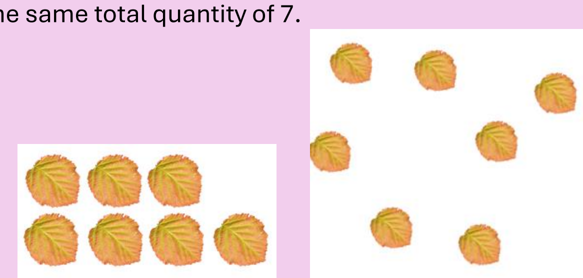
Understand: Small quantities of objects can be identified instantly without the need to count each item individually (subitising).

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

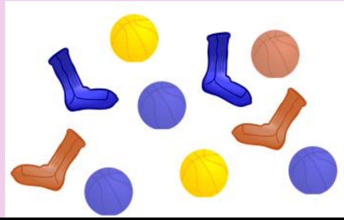
Know	**Do** (through play, everyday routines, games, songs, stories and rhymes)	**Know**	**Do** (through play, everyday routines, games, songs, stories and rhymes)
Objects up to 5 can be visually recognised without counting (perceptual subitising).	See and say the number of objects in a small group as a single unit. For example, seeing and saying 3 when shown 3 fingers or a dot pattern of 3 without the need to count 1, 2, 3. Recognise and say, without counting, 'how many' in regular and then irregular patterns to 5 (perceptual subitising). For example,  Demonstrate 'how many' using sounds and movements, when shown a visual pattern. For example, clap two times when shown two dots. Recognise and say, without counting, how many dots there are on each side of a regular 1-6 dice.	The total number of objects in a group can be broken up into smaller quantities and then combined to work out the total (conceptual subitising). Estimation is making a rough guess about a quantity. The quantity within a collection can be estimated without counting.	Recognise and say 'how many' in regular patterns and then irregular to 10 (conceptual subitising). For example,  Make numbers using finger patterns, initially sequentially and then simultaneously. Demonstrate 'how many' using sounds and movements, when shown a visual pattern. For example, jump six times when shown six dots. Estimate which collection has more than/less than, when shown two or more collections. Estimate how many there are in individual collections, then count to check.

Notes
When introducing subitising, start with noticing. Initially, learners should be encouraged to notice small groups of objects within play, stories, and everyday experiences.
A range of visuals can support the understanding of subitising. For example, using natural materials such as sticks, stones ...
Regular dot patterns are the formations on a 1 to 6 dice. Irregular dot patterns are randomly arranged.

Understand: Objects, sounds and movements can be counted to find out how many individual items there are; cardinality represents the numerosity of a set.

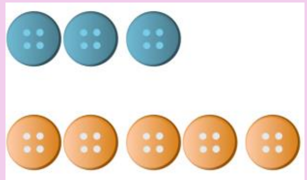
Know	**Do** (through play, everyday routines, games, songs, stories and rhymes)	**Know**	**Do** (through play, everyday routines, games, songs, stories and rhymes)
Counting is a way to keep track of 'how many' there are. An individual object (one) can be taken out of a larger collection. When counting, one number name is assigned to each object, and each object is counted only once (one-to-one correspondence). When counting, the last number word said refers to how many objects have been counted. (cardinality).	Count forwards and backwards through play, everyday routines, games, songs, stories and rhymes. Separate an individual object (one) from a larger group and refer to this object as a 'one'. For example, take one ball from the basket, one banana from the bowl. Count a range of physical objects that are the same size, colour and shape. Organise objects to support counting. For example, line them up and then move one at a time, keeping track of what has been counted. Assign and match one number word to each object when counting aloud. Identify the total number of objects in a group by counting in a variety of ways and in different contexts. Accurately count objects and answer 'how many?' with the last number counted.	Objects can be counted from any starting position in an arrangement or order (order-irrelevance). The number of objects in a set remains the same even if they are rearranged, covered, grouped or spread out, as long as none are added or taken away (conservation of number).	Match counted quantities with their corresponding numeral. Count forwards from any starting point. For example, how many balls are there? Start counting from the red one.  Recognise that the total is not affected if only the arrangement or position changes. For example, 10 balls put into a box remains 10 balls. Recognise that the total is not affected by the size of object or their layout. For example, seven leaves spread apart, seven leaves lying close together, seven small leaves, seven large leaves all have the same total quantity of 7. 

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Anything can be counted, regardless of whether it is physical or abstract. (abstraction).	Count out a given amount. For example, can you find **4** acorns, we need **5** cups for snack. Count out from a larger collection. For example, give me 5 cars from the box. Count anything, including, sounds, actions and items that cannot be touched or moved. For example, claps, shadows, bounces of a ball, number of people in your family. Keep track of 'how many' sounds and movements on fingers. For example, how many clicks, how many jumps.		Count a range of random objects explaining that the count stays the same regardless of attributes (colour, size, shape). For example, a ball, a train and a pebble is 3 and three crayons is also 3. In both examples there are 3 altogether. Keep track of 'how many' sounds and movements. For example, How many bounces of a ball? How many beats of a drum? Count out from a larger collection within the extended number range. For example, give me 12 pencils from the pot. Count a collection of objects within a larger collection. For example, How many balls? How many blue balls? How many blue things? 

Notes
Includes
Initially working within the range 1 to 5, progressing to 0 to 10, and then extending to 20.

Understand: Comparing quantities reveals whether one collection has more, less, or the same number of items.

Know	**Do** (through play, everyday routines, games, songs, stories and rhymes)	**Know**	**Do** (through play, everyday routines, games, songs, stories and rhymes)
More than, less than and the same can be used to compare and describe the number of objects in two collections.	Compare collections where the objects are the same shape, colour and size and where the amounts are obviously different. Describe these using the terms more than, less than or the same as. For example,  Compare and describe collections where the amount is closer in value. For example, there are less blue buttons/there are more orange buttons.  Compare and describe collections where the objects are different in shape, size or colour. For example, there are more sticks. 	Most and least can be used to compare and describe the number of objects in three or more collections. Equal means being exactly the same in amount, number or size. Not equal means being different in amount, number or size.	Estimate, then determine the sizes of three or more collections, and explain which one has the most, the least, or if they are the same size. Compare collections of objects indicating if these are equal or not equal in size. Compare equal collections where the objects are different in size or colour and recognise that the total is the same. For example, three small tennis balls and three large basketballs. 

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

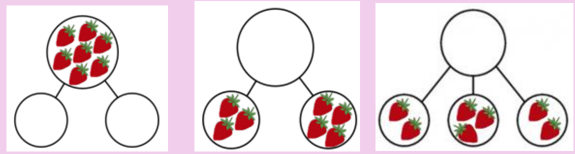
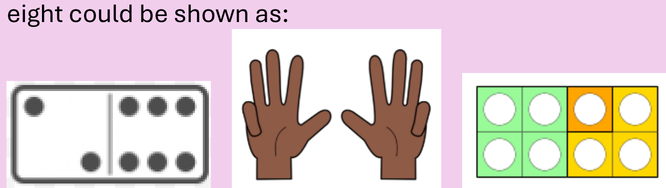
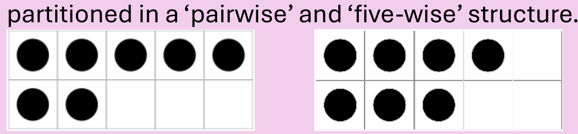
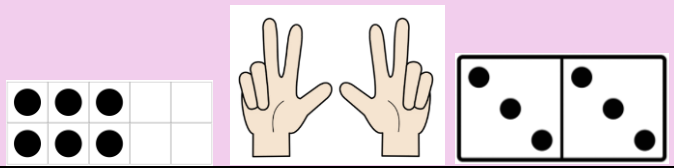
	Recognise when the amount within two collections is the same. For example, the same number of bananas. 		

Notes

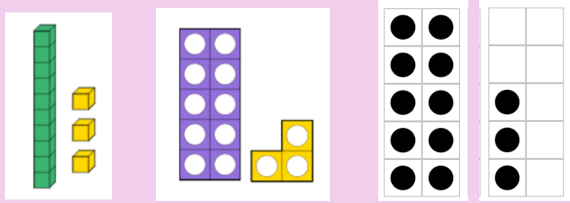
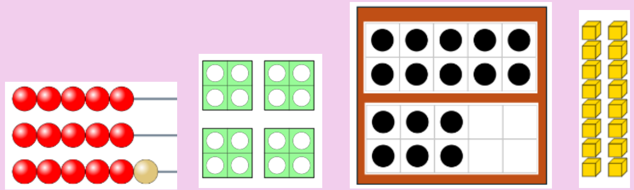
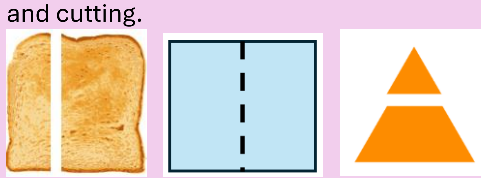
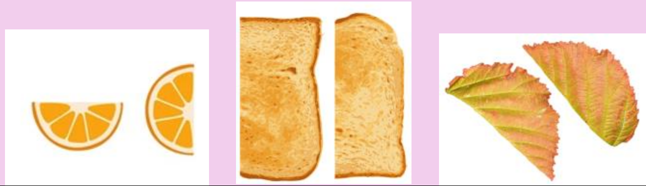
At the initial stage the emphasis is on visually recognising if the amounts are more, less or the same rather than counting to compare.

Once children are confident in visual comparison, work within the range 0 to 20 and as the collections being compared become closer in amount, encourage children to estimate which is more, most, less, least, the same and then encourage them to count to check.

Understand: Shapes, objects, sets of objects and numbers can be partitioned, and their parts can be recombined to restore the original whole.

Know	**Do** (through play, everyday routines, games, songs, stories and rhymes)	**Know**	**Do** (through play, everyday routines, games, songs, stories and rhymes)
A set of objects can be shared into smaller sized sets (not always equal sharing at this point). The same quantity (whole) can be partitioned into parts in different ways (part/part/whole, part/part/part/whole). Whole number partitions can be recombined to make the original quantity.	Share out sets of objects when playing and recognise that the total remains the same. For example, five dolls shared between two children is still five dolls. Partition sets of objects in different ways and recognise that the total remains unchanged. For example, four could be shown split into two parts (2 and 2 or 3 and 1) or three parts (2 and 1 and 1):  Partition collections of objects (up to 5) into two smaller groups without counting. Recombine partitioned amounts to 5 and recognise that this re-makes the whole.	The numbers 1 to 10 can be partitioned using a ‘pair-wise’ pattern. The numbers 6 to 10 can be partitioned using a ‘five- wise’ structure.	Recognise that when a set of objects is partitioned into smaller sized sets the total remains the same. For example,  Partition given amounts into two or more parts, in different ways, and recognise that the total remains unchanged. For example, eight could be shown as:  Recombine partitioned amounts to 10 and recognise that this re-makes the whole. Recognise and identify numbers 1 to 10 which have been partitioned in a ‘pairwise’ and ‘five-wise’ structure.  Partition sets of objects using the ‘pair-wise’ and ‘five-wise’ structures. Describe pair-wise patterns and equal partitions to 10 using the language of doubles and halves. For example, six is double three, three is half of six. 

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

		The numbers 11 – 20 can be partitioned using a ‘ten and a ones’ structure. When shapes and objects are partitioned into two equal parts each part is one half and they can be re-combined to make one whole.	Partition the numbers to 20 into tens and ones using concrete materials. For example, 13 is ten and three one and 20 is two tens and zero ones.  Show ways that numbers to 20 can be partitioned using concrete materials and visual representations. For example, sixteen could be shown as:  Recombine partitioned amounts to 20 and recognise that this re-makes the whole. Recognise when a shape or object has been partitioned into two equal parts (halves) and when it has not through folding, matching and cutting.  Recombine equal parts of shapes and objects in different orientations and layouts and recognise that this re-make a whole. 

Notes
Numbers shown in a ‘five-wise pattern’ on a ten frame build on knowledge of five, for example, seven can be seen as 5 and two more.
Numbers shown in ‘pair-wise pattern’ on a ten frame builds up the amounts in pairs.
It is important that children see ten frames with quantities show in both a pair-wise and five-wise way.
Make connections to knowledge of doubles (making equal groups).
Initially working within the range 1 to 5, progressing to 0 to 10, and then extending to 20.

Understand: Addition describes combining collections of items or increasing the size of a collection.

Know	**Do** (through play, everyday routines, games, songs, stories and rhymes)	**Know**	**Do** (through play, everyday routines, games, songs, stories and rhymes)
Addition is combining two or more collections of objects.	Count the number of objects in more than one collection, up to a total of 5. For example, combining collections of toys.	Pairs of numbers that add to a given number are called number bonds.	Identify, show and say number bonds to 10. For example, using objects to show that 7 is 7 and 0, 6 and 1, ...

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

		Addition is also increasing the number of objects in a single collection. The mathematical symbol for addition is +. The mathematical symbol used to show equality is =. The mathematical symbol used to show inequality is ≠.	Combine two or more collections to calculate the total. Count two collections where one cannot be seen (screened). For example, finding the total when 2 people are in the garden and 2 are in the house. (screened)  Calculate the total in two collections, by counting on from the larger. For example, when combining 3 and 6, counting **6**, 7, 8, 9 rather than 1, 2 ,3, 4, 5, 6, 7, 8, 9. Calculate the total by counting on when a collection has been increased by an amount. For example, adding three balls to a basket of 7 is counted **7**, 8, 9, 10 Calculate the total, by counting on, when the starting amount of objects cannot be seen. For example, when 6 counters are in the tub (unseen) and 3 are added. Calculate the total when the amount being added is unknown, using concrete materials to support. For example, calculating how many counters have been added to give a total of 9 when we started with 7. Calculate the starting amount when this is unknown using concrete materials to support. For example, calculating how many counters we started with if we add four more and now have ten. Recognise and read number sentences; where the addition and equals symbols have been used. Record a given addition number story using the correct mathematical symbols. For example, 'there are six people on the bus and two more get on, there are now eight passengers altogether' can be recorded as 6 + 2 = 8. Tell a number story based on given addition calculations. For example, 'six cats and two cats equals eight cats altogether,' when shown 6 + 2 = 8. Recognise, read and discuss number stories where the mathematical symbol for equal or not equal has been used.

Notes
Concrete materials should be used to model and carry out addition calculations, progressing to pictorial representations such as illustrations or a number line.
Mathematical symbols should be modelled alongside the use of concrete and pictorial methods before learners are expected to independently demonstrate the use of symbols.
A number sentence is a calculation represented with symbols (for example, 7 + 2 = 9).
A number story is a calculation represented in a context (for example, seven strawberries and two strawberries equal nine strawberries altogether).
When telling number stories, a range of words for addition should be modelled. For example, six **add** two, six **plus** two, six and two **more**.
It is important that learners are given opportunities to **read** number sentences aloud, **tell** number stories from given calculations and **record** number sentences when listening to a number story, and not to have an over emphasis on solving written calculations.
In an addition calculation augend + addend = sum.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

When the amount being added is unknown, this is called a missing addend (for example, calculating how many counters have been added to give a total of 9 when we started with 7).
When the starting amount is unknown, this is called a missing augend (for example, calculating how many counters we started with if we add four more and now have ten).

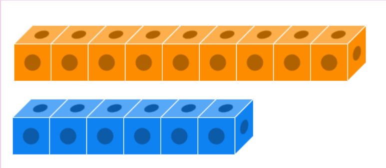
Includes

Initially work within the range 1 to 5, progressing to 0 to 10.

Excludes

Learners would not be expected to use the language of sum, addend and augend.

Understand: Subtraction describes finding the difference between quantities of items, or the quantity left when some items are removed.

Know	**Do** (through play, everyday routines, games, songs, stories and rhymes)	**Know**	**Do** (through play, everyday routines, games, songs, stories and rhymes)
Subtraction is reducing the number of objects in a collection.	Reduce the total within a group using a range of physical objects. For example, when eating snack or removing toys from a collection.		Calculate the number of objects which are left when some are removed. For example, finding how many are left when six strawberries have been eaten out of a collection of nine.
		Subtraction is also finding the difference between number of objects in two collections.	Find the difference between two collections of objects. For example, find the difference between 6 and 9.  Calculate how many objects have been taken away when given the starting number of objects and the remaining number. For example, calculate how many have been taken away from eight counters when five are left. Calculate the starting number of objects when given the number remaining and the number removed. For example, calculate how many we started with if we remove 5 and have 4 remaining.
		The mathematical symbol for subtraction is –.	Recognise and read number sentences where the subtraction symbol has been used. Record a given subtraction number story using the correct mathematical symbol. For example, I had nine stickers and gave five away, now I have four left, can be recorded as 9 – 5 = 4. Tell a number story based on given subtraction calculations. For example, nine cars take away five leaves four cars when shown 9 – 5 = 4.

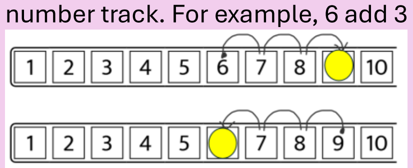
Notes

Concrete materials should be used to model and carry out the subtraction calculations, progressing to pictorial representations such as illustrations or a number line.
Mathematical symbols should be modelled alongside the use of concrete and pictorial methods before learners are expected to independently demonstrate the use of symbols.
A number sentence is a calculation represented with symbols (for example, 7 - 2 = 5).
A number story is a calculation represented in a context (for example, seven strawberries take away two strawberries leaves five strawberries).
When telling number stories, a range of words for subtraction should be modelled. For example, six **subtract** two, six **take away** two, six **remove** two, two **less than** six. Find **the difference between** six and two.
It is important that learners are given opportunities to **read** number sentences aloud, **tell** number stories from given calculations and **record** number sentences when listening to a number story, and not to have an over emphasis on solving written calculations.
In a subtraction calculation minuend – subtrahend = difference.
When the amount being subtracted is unknown, this is called a missing subtrahend (for example, calculate how many have been taken away from eight counters when five are left).
When the starting amount is unknown, this is called a missing minuend (for example, calculate how many we started with if we remove 5 and have 4 remaining).

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Includes
Initially working within the range 0 to 5, progressing to 0 to 10.
Excludes
At this stage learners would not be expected to use the language of minuend and subtrahend.

Understand: A quantity can be altered and returned to its original value by combining addition and subtraction, as these operations undo each other.

Know	**Do** (through play, everyday routines, games, songs, stories and rhymes)	**Know**	**Do** (through play, everyday routines, games, songs, stories and rhymes)
		Addition and subtraction are connected.	Identify when objects are being added or removed when listening to addition and subtraction problems in context and model using concrete materials. Recognise that when objects are removed the total of a collection becomes less but if these are added back in the collection returns to the original total. Recognise that when objects are added the total of a collection becomes more but if these are then removed the collection returns to the original total. Notice the connection between addition and subtraction using a number track. For example, 6 add 3 is 9. 9 subtract 3 is 6. 

Notes
Learners should be encouraged to notice and discuss the connection between addition and subtraction when using concrete materials and pictorial representations. At this stage they are not expected to mathematically explain inverse operations.
Includes
Working within the range 0 to 10.

Understand: Zero represents the absence of a quantity or value; adding or removing zero leaves a total unchanged.

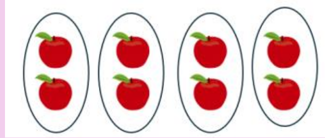
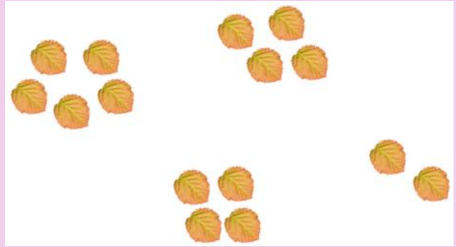
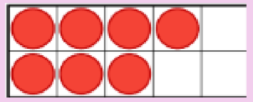
Know	**Do** (through play, everyday routines, games, songs, stories and rhymes)	**Know**	**Do** (through play, everyday routines, games, songs, stories and rhymes)
Zero means there is none of a particular quantity.	Recognise where there is zero of a particular quantity. For example, there are no oranges in a bowl, so the number of oranges is zero. Recognise and identify if there is zero, one or more than one in a small collection of everyday objects.	When adding or subtracting zero the total within the collection does not change. Zero is represented by the numeral 0.	Discuss the impact that adding/subtracting zero has on the total. Recognise and read number sentences; where zero has been used. Record a given adding / subtracting zero story using the correct mathematical symbols. For example, 'I had seven stickers, Mum gave me no more' can be recorded as 7 + 0 = 7. Tell a number story based on given adding / subtracting zero calculations.

Notes
When counting backwards some learners may say '3, 2, 1…blast off' instead of 'zero'. Whilst some number songs and rhymes might end in this way, when counting aloud, it is important that adult's model and reinforce 'zero' in the number sequence.

Understand: Equal sharing arranges a quantity into a given number of equal parts.

Know	**Do** (through play, everyday routines, games, songs, stories and rhymes)	**Know**	**Do** (through play, everyday routines, games, songs, stories and rhymes)

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

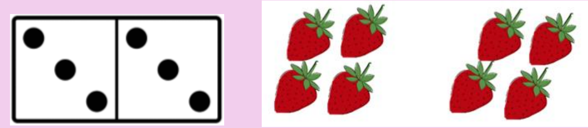
A whole amount can be split and shared. A collection of objects can be shared into smaller collections.	Split whole amounts into smaller parts. For example, everyone gets a piece of dough from the tub, we all get two scoops of cereal from the box. Share out objects one at a time and recognise if everyone has the same amount or not.	A collection of objects can be shared out equally or not equally. If a number is even, the total can be shared into two equal, whole number parts. If a number is odd, it cannot be shared into two equal, whole number parts.	Share a set of objects equally and identify how many each person gets. For example, when 8 apples are shared between 4 children each child gets two apples.  Share out objects into smaller collections and use the language of equal and not equal to describe this. For example: these leaves have not been shared equally.  Share a collection of whole objects between two and recognise if the total amount shared is odd or even. For example, 8 is even because it can be shared equally between 2, 7 is odd because there is one left over. Identify and explain if a number to 10 is odd or even using concrete materials and visual approaches. For example, pair-wise ten frames:  Identify the next odd/even number to 10 on a number track or number line. Sort odd and even numbers from 0 to 10.

Notes
When sharing out a collection of whole objects, ones which cannot be broken into smaller pieces should be used (for example, buttons, acorns, beanbags).
Excludes
Sharing into fractional parts is not required at this stage (for example, 7 in two shares of three and a half).

Understand: Equal grouping arranges a quantity into equal parts of a given size.

Know	Do (through play, everyday routines, games, songs, stories and rhymes)	Know	Do (through play, everyday routines, games, songs, stories and rhymes)
		Equal groups have the same number of objects in them. A pair is a group of two objects.	Arrange collections of objects into equal groups. For example, four sheep in each pen, three buttons for each snowman. Arrange a collection of objects into equal groups and identify how many groups there are. For example, put three pencils into each pot, how many pots will we need? Arrange themselves and objects into pairs. For example, gloves, wellies, socks.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

		When two groups have exactly the same amount, this is called double. When making equal groups of objects from a collection, there may be some left over.	Describe two equal groups using the language of doubles. For example, double three equals six, eight is double four.  Arrange themselves and objects into groups of different amounts and begin to recognise that there might be some left over. For example, put these strawberries into groups of four, two groups of four with two left over. 

Notes
At this stage, whole objects which cannot be broken into smaller pieces should be used.

- Patterns and sequences

Strand: Quantity, Numbers and the Algebraic Properties of Number
Sub-strand: Patterns and sequences

Beginning of Early Level ⟵⟶ **End of Early Level**

Understand: Mathematical sequences reveal patterns that occur in nature, daily routines and lived experiences.

Know	**Do** (through play, everyday routines, games, songs, stories and rhymes)	**Know**	**Do** (through play, everyday routines, games, songs, stories and rhymes)
In the world around us numbers often follow a pattern, this is called a number sequence.	Notice where numbers follow a pattern in everyday situations. For example, house numbers, page numbers, clocks and calendars. Copy and continue number sequences through a range of songs, stories, rhymes and counting games. Notice and identify missing numerals when shown sequences which ascend or descend in ones. For example, 5, 4, ?, 2. Identify an error made in an ascending or descending number sequence in ones. For example, 2, 3, '5', …		Identify which number(s) comes next in ascending and descending oral and written sequences, when counting in ones. For example, 13, 14, 15, ? Identify missing numbers from ascending and descending oral and written number sequences, when counting in ones. For example, 14, 15, ?, 17 Recognise and explain an error made in an oral or written ascending or descending number sequence in ones. For example, 14, 15, 16, '18', 19…

Notes
Learners should have experiences working with oral and written/visual number sequences (for example, a number track with numerals covered).
Patterns are also explored within the Shape, Space and Measurement Strand.
Includes
Initially working within the range 1 to 5, progressing to 0 to 10, then extending to 20.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

- First Level
- Number structures and operations

Strand: Quantity, Numbers and the Algebraic Properties of Number
Sub-strand: Number structures and operations

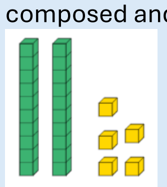
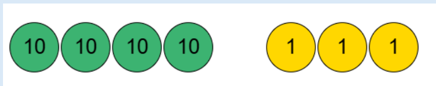
Beginning of First Level ⟵⟶ **End of First Level**

Understand: Whole numbers extend without limit, so quantities can be counted indefinitely.

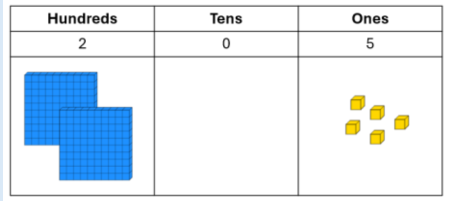
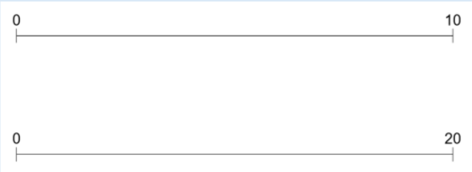
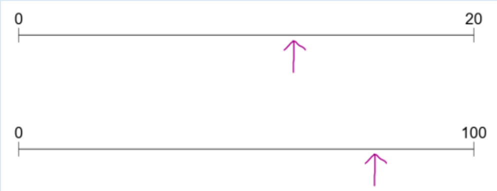
Know	Do	Know	Do	Know	Do
The language of less than, greater than, equal to can be used to compare quantities of items.	Compare visually and describe quantities when the amounts are obviously different. Compare visually and describe quantities of items when their amounts are less obviously different, counting to check description is accurate. Count on and back, in ones, from any starting number. Skip count in tens off the decade (14, 24, 34...). For example, when counting off the decade, Hamza counts 14, 24, 34, 40... Is he correct? Why/why not?		Count forwards and backwards, in ones, from any starting number. Skip count in hundreds on the hundred. For example, 400, 300, 200, ...		Skip count in hundreds off the decade. For example, 676, 576, 476, ...

Notes
Whole numbers are the set of numbers 0, 1, 2, 3, 4, 5 ...
Encourage learners to estimate before counting and check estimates for accuracy.
It is important to model skip counting backwards at the same time as skip counting forwards.
Learners could investigate the origin of numbers and different ways of counting, appreciating the influence of different cultures. They could compare different number systems and how they have evolved over time (for example, zero emerged in India around the 7th century, and the numbers used globally today are a Hindu-Arabic numeral system).
Learners could also investigate Roman numerals; connected to social subjects.
Includes
Initially working within the range 0 to 100, progressing to 200, 300 before extending to 1000.

Understand: The base ten system uses symbols (digits) to express quantities in an organised way, with each place representing a value ten times greater than the one on its right.

Know	Do	Know	Do	Know	Do
Ten ones are equal to one ten and vice versa. The base ten system (decimal number system) uses exactly ten digits (0 to 9) to build any number.	Represent whole numbers as combinations of units of different sizes (ones, or tens and ones), showing how quantities can be composed and decomposed.  Interpret the value of each digit in a number by analysing its position and the unit it represents within the place value system in 2-digit whole numbers. For example, in the	One hundred is equal to ten groups of ten or one hundred ones. Three-digit whole numbers have three place values, the ones place, the tens place and the hundreds place.	Represent whole numbers to 100 in different ways, showing how they are composed of tens and ones to form the same quantity.  Interpret the value of each digit in a number by analysing its position and the unit it represents within the place value system in 3-digit whole numbers. For	One thousand is equal to ten groups of one hundred or one hundred groups of ten or one thousand ones.	Represent whole numbers to 1000 in different ways, showing how place value units combine to form the same quantity.

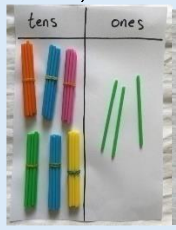
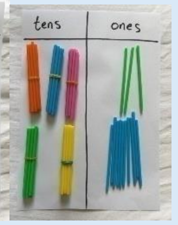
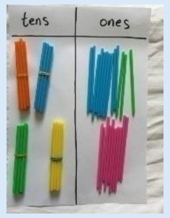
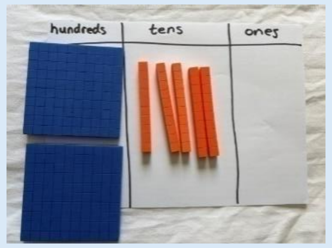
We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Two-digit whole numbers have two place values, the ones place and the tens place. Zero is used as a placeholder.	number 90, the 9 represents 9 tens and has a value of 90 and the 0 is used to show that the 9 is in the tens place. Represent whole numbers in different ways and explain how equivalent quantities are shown across representations.		example, in the number 205, the 2 represents 2 hundreds and has a value of 200, the 0 represents 0 tens (and holds the tens place) and the 5 represents 5 ones and has a value of 5.  		
Decades of whole numbers are sets of ten numbers which have the same digit in the tens place.	Identify which decade whole numbers 10 to 99 belong to. For example, 84 lies in the decade 80 to 89. 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99	Numbers can be placed on a number line between their neighbouring decades. Numbers can be rounded to the nearest 10 as an approximation.	Place two-digit numbers on marked then empty number lines.  Round two-digit numbers to the nearest ten using a number line.	Numbers can be rounded to the nearest 100 as an approximation.	Round three-digit numbers to the nearest hundred. Round three-digit numbers to the nearest ten.
The language less than, greater than, equal to can be used to compare whole numbers.	Identify whether whole numbers are greater than, less than or equal to each other based on their relative position.			Numbers can be compared using the mathematical symbols. The mathematical symbol used to show less than is <. The mathematical symbol used to show more than is >.	Represent the relationship between whole numbers using appropriate symbols. For example, 67 is less than 98 is written as 67 < 98.
Magnitude refers to the size or value of a number.	Order whole numbers in ascending or descending order by comparing their magnitude.		Order whole numbers in ascending or descending order, gradually increasing the range towards 1000.		Order whole numbers to 1000 in ascending or descending order.
The position of a number on the number line indicates its size (magnitude) in relation to the numbers to either side of it.	Estimate where the same number lies on number lines with different scales by considering how far it is between the endpoints. For example, estimating where 8 would be on each of these number lines.  Estimate the number that is being indicated on empty number lines with different scales. For example, 0 to 10, 0 to 20, 0 to 100.  				

Notes
Using a number line/hundred square provides a strong visual and spatial representation of number order, magnitude and relationships.
The use of base ten material supports the understanding of how numbers are composed.
When discussing the use of the greater than and less than symbols the meaning of the equals sign should be reinforced.
Includes
Initially working within the range 0 to 100, progressing to 200 and 300 before extending to 1000.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Understand: Numbers can be partitioned flexibly to reveal their structure and support efficient calculation.

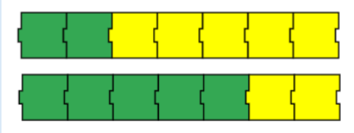
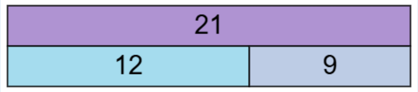
Know	Do	Know	Do	Know	Do
Two-digit whole numbers can be partitioned into tens and ones in canonical, and non-canonical form. Whole numbers can be partitioned into two or more parts. This is the part-whole relationship.	Represent whole numbers as tens and ones and partition them in different ways, explaining how each representation shows the same quantity. For example, 63 is 6 tens and 3 ones (canonical) but 63 is also 5 tens and 13 ones, 4 tens and 23 ones (non-canonical form).  Create and describe part-whole relationships by composing and decomposing numbers into parts that combine to form the same whole. For example, Maria thinks the number 64 can only be partitioned into two parts including 60 and 4, 50 and 14… Is she correct? Could the number also be partitioned in other ways?	Three-digit whole numbers can be partitioned into hundreds, tens and ones, in canonical form.	Represent three-digit whole numbers partitioned into hundreds, tens and ones. For example, the number 250 can be partitioned into 2 hundreds and 5 tens.  Represent numbers, initially within 300, in different ways by applying known number facts to generate and explain equivalent forms. For example, 220 is double 110, 100 + 120, 55 + 55 + 55 + 55 …	Three-digit whole numbers can be partitioned into hundreds, tens and ones, in canonical and non-canonical form.	Represent three-digit whole numbers partitioned into hundreds, tens and ones in canonical and non-canonical ways using appropriate materials. For example, the number 684 can be partitioned into 6 hundreds, 8 tens and 4 ones (canonical) or 5 hundreds, 18 tens and 4 ones… (non-canonical). Represent numbers in different ways by applying known number facts to generate and explain equivalent forms. For example, 484 is double 242, 400 + 80 + 4, 200 + 284…

Notes
Flexible partitioning supports calculation strategies at this level and later levels. For example, when calculating 18 + 23 it can become 23 + 7 +11, when dividing 84 by 6 it can become 60 and 24 each divided by 6.
Includes
Working within the range 0 to 100, progressing to 200, 300 before extending to 1000.

Understand: Additive relationships explain how quantities are increased, decreased, combined or compared.

Know	Do	Know	Do	Know	Do
The equals sign is used to demonstrate that both sides of a mathematical sentence have the same value, like the two sides of balanced scale. Mathematical symbols, +, -, =, ≠ can be used to record mathematical sentences horizontally. Addition is commutative. This means that changing the order of the numbers	Represent and explain the equivalence of different calculations by proving that they produce the same value. For example, 10 + 2 = 8 + 4. Construct mathematical sentences showing equalities symbols appearing in different positions. For example, 15 = 12 + 3, 10 + 6 = 16, 11 – 2 = 4 + 5, 14 ≠ 10 + 3.				Efficiently perform addition calculations of three or more numbers.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

in an addition calculation does not affect the total (sum). Subtraction is the inverse operation of addition. An additive fact family is a group of related maths facts that shows the connections between addition and subtraction and the commutative law. There are a range of strategies for carrying out addition and subtraction calculations.	Explain and represent the commutative property of addition by showing that the order of quantities does not affect the result. For example, showing that 2 + 5 = 5 + 2.  Explain how addition and subtraction are inverses by showing how a quantity can be changed and then restored. For example, start with 12 and add 8 to get 20. Start with 20 and subtract 8 to get back to 12. Represent visually and explain relationships between related number facts, including how operations can be used to generate equivalent and inverse results. For example,  21 = 12 + 9, 9 +12 = 21 21 – 9 = 12, 21 – 12 = 9. Apply known number facts when solving addition and subtraction calculations within 20, using doubles, near doubles, teen facts and number bonds. For example, 8 + 8 = 16, 10 + 6 = 16, 16 = 20 – 4. Apply a range of addition and subtraction strategies to calculations. Interpret and solve problems by identifying when to use addition or subtraction and using (appropriate) representations to support and record the calculation process.	When three or more numbers are added together, the way they are grouped has no effect on the sum/total (associative law). Numbers can be rearranged and reordered to make addition calculations easier. Efficient strategies help to solve calculations quickly and accurately without completing unnecessary steps.	Add multiple numbers (three or more) efficiently by rearranging and reordering them while maintaining the same total. Check the accuracy of addition calculations using subtraction and vice versa. Apply known number facts when solving addition and subtraction calculations within 100 using doubles and halves, number bonds to 100. For example, 30 + 30 = 60, half of 50 = 25, 100 = 90 + 10. Select and use efficient strategies based on known number relationships to solve addition and subtraction problems. For example, when calculating 54 – 52 it would be more efficient to find the difference between the numbers rather than counting back. Solve contextual and non-contextual problems which involve two addition or two subtraction calculations.	The written algorithms for addition and subtraction can provide an alternative way to record calculations as they become more complex.	Use rounding to estimate the result of addition and subtraction calculations by adjusting numbers to nearby convenient values. Apply known number facts when solving addition and subtraction calculations within 1000 using doubles, halves and different ways to make 1000. For example, 200+200 = 400, half of 900 = 450, 400 + 600 = 1000. Apply and justify the efficiency of using different addition and subtraction strategies when calculating within 100. Solve two-step contextual and non-contextual problems which involve both addition and subtraction. Connect different representations of (two-digit) addition and subtraction to formal written algorithm, explaining how each shows the same calculation. Solve addition or subtraction calculations of two-digit numbers using the written algorithm.

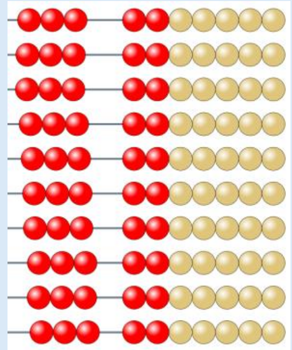
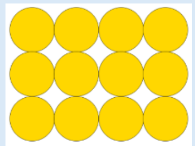
Notes
Includes
Initially working within the number range 0 to 30, progressing to 0 to 100, and then extending to 1000.
Connections should be made to place value and flexible partitioning.
Encourage learners to estimate before calculating and check reasonableness of solutions across all number ranges.
As the number range increases learners should use rounding to estimate.
Appropriate strategies include partitioning, rounding or compensating. For example, when calculating 18 + 23, this could be:

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

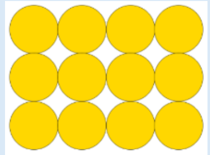
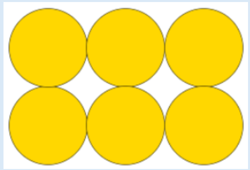
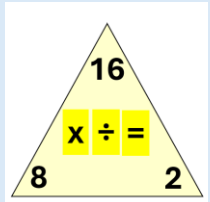
- Partitioned into tens and ones by calculating 10 + 20 + 8 + 3
- Rounded to 20 first by calculating 18 + 2 + 21
- Compensated to 20+21.

Mathematical tools such as empty number lines, place value materials, bar models or jottings can be used to keep track of the calculations.
Initially learners should fluently:
Find 1 more and 1 less to 100, 10 more and 10 less on the decade to 100, add and subtract within 20, and find doubles and halves within 10 (whole numbers only).
Then move on to:
Finding fluently 10 more and 10 less off the decade to 100 (for example, 10 more than 84), adding and subtracting within 20 mentally, and begin to extend towards 100.
Then move on to:
Adding and subtracting mentally within 100.

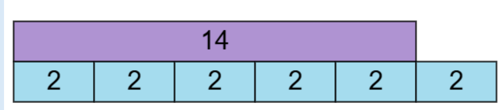
Understand: Multiplicative relationships involve organising quantities into equal groups and equal parts.

Know	Do	Know	Do	Know	Do
Objects can be equally grouped and counted. It is more efficient to skip count in equal jumps rather than count in ones.	Group items into equal sets to count them more efficiently. For example, pairs of socks counted in twos, tally marks counted in fives. Skip count forwards and backwards in equal steps from and to zero in 2s, 5s and 10s using appropriate mathematical tools to keep track. For example, 	Repeated addition is the same as skip counting forwards in equal groups.	Connect repeated addition and skip counting by showing how both represent equal steps that produce the same total. For example, 2+2+2+2 or 2, 4, 6, **8**. Skip count from zero in equal steps of 2s, 5s and 10s and use the pattern to determine totals when some groups are not visible. For example, the value of notes and coins, the number of boots in total, 		Skip count forwards and backwards in equal steps from and to zero in 3s, 4s and 6s using appropriate mathematical tools to keep track For example,  Skip count from zero in equal steps of 3s, 4s and 6s and use the pattern to determine totals when some groups are not visible. For example, eggs in boxes, wheels on cars. 
		An array shows objects grouped in an orderly way into rows and columns. For example, 3 rows of 2 or 2 columns of 3. Rows go side to side (are horizontal). Columns go up and down (are vertical). Repeated addition can be represented in terms of multiplication and by using arrays. Multiplication is finding the total number in a set of equal groups.	Make arrays which represent given contextualised number stories. For example, building an array to represent 3 boxes of 6 cupcakes. Describe physical or pictorial arrays using the language of repeated addition and multiplication. For example, this array represents 4 + 4 + 4, 3 rows of four and 3 x 4. 	Multiplication and division by 1 leaves a quantity unchanged. When any quantity is multiplied by zero, the result is zero.	Explain, using concrete materials, what it means to multiply and divide by 1, and why these leave a quantity unchanged. Explain why multiplication by zero results in zero. Model the repeated addition of 2 using arrays and visual representations, identifying at least the first ten multiples, making connections between multiplication, skip counting, and contextualised problems. Model the repeated addition of 4 using arrays and visual representations, identifying at least the first ten multiples, making

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

		The mathematical symbol for multiplication is ×.	Skip count rows or columns to find totals within given arrays and connect to the related multiplication fact. For example, 4, 8, **12**, three rows of 4 is 12**,** 3 × 4 = 12 (three multiplied by 4 equals 12). 	Multiples of 2 and 4 are connected through doubling and halving.	connections between multiplication, skip counting, and contextualised problems. Recognise connections between the 2 and 4 times tables. For example, 4 × 5 is double 2 × 5 and 2 × 3 is half of 4 × 3. Recognise and read number sentences where mathematical symbols have been used to record multiplication and division by 2 or 4.
		Multiplication is commutative. This means that changing the order of the numbers in a multiplication calculation does not affect the total (product).	Model the commutative nature of multiplication using arrays. For example, showing that 2 rows of 8 and 8 rows of 2 use the same number of counters. Make different arrays with set numbers of objects, connecting each to repeated addition and multiplication facts. For example arranging 8 counters in 2 by 4, 4 by 2, 1 by 8 and 8 by 1 arrays.	Mathematical symbols, x, ÷ and = can be used to record calculations horizontally.	Record multiplication and division calculations from given contextualised number stories using mathematical symbols. For example, 2 baskets each have 6 apples in them can be recorded as 2 x 6 = 12 or 6 x 2 = 12. Tell number stories based on given multiplication and division facts from the 2 and 4 times tables. For example, 20 ÷ 4 could be 20 apples being placed into 5 packs of 4.
	Select whether grouping or sharing is involved when listening to contextualised number stories and model this using concrete materials	Division is a way to share or group numbers equally. The mathematical symbol for division is ÷.	Describe a given array using the language of division. For example, there are 20 pine cones arranged as 4 equal rows of 5. This shows 20 divided into 5 equal groups of 4 or 4 equal groups of 5.	Multiples of 5 and 10 are connected through doubling and halving.	Apply the multiplication and division activities from the 2 and 4 times tables to the 5 and 10 times tables.
Sharing out items does not always result in equal shares.	Model and explain how equal shares are or are not possible using collections of concrete materials. For example, 8 counters can be equally shared 4 ways, but would be shared unequally 3 ways.		Describe a given array using the language of multiplication and division For example,  This represents three groups of two, two groups of three, six shared two ways, six shared three ways, 3 × 2, 2 × 3, 6 ÷ 2, and 6 ÷ 3.	Multiples of 3 and 6 are connected through doubling and halving. Division reverses the effect of multiplication, and vice versa (inverse operations).	Apply the multiplication and division activities from the 2 and 4 times tables to the 3 and 6 times tables. Explain this inverse operation using concrete materials or visual representations. For example, start with 4 and multiply it by 5 to get 20. Start with 20 and divide it by 5 to get back to 4.
Grouping problems do not always result in equal groups.	Model and explain how equal groups are or are not possible using collections of concrete materials. For example, 10 pebbles can made into equal groups of 5 but not equal groups of 4.				
			Explain and illustrate that when a quantity is arranged in equal shares, the more shares there are, the smaller each one will be. For example, 10 stickers shared between 2 children compared to 10 stickers shared between 5 children.	A multiplicative fact family is a group of related maths facts that shows the connection between division and multiplication, the commutative law and inverse operations.	Check the accuracy of multiplication calculations using division and vice versa. Explain and illustrate number facts and make connections between multiplication and division using concrete materials and visual representations. For example, 
			Explain that when a quantity is arranged in equal groups, the larger each group is, the fewer groups can be made. For example, 12 oranges arranged into	A multiple is the result of multiplying one whole number by another.	Identify division problems which do, or do not share or group equally when dividing by 2, 3, 4, 5, 6 and 10. For example, 12 toys can

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

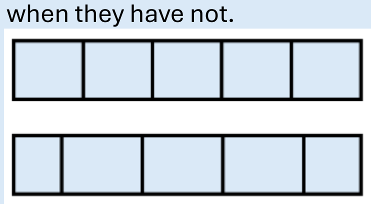
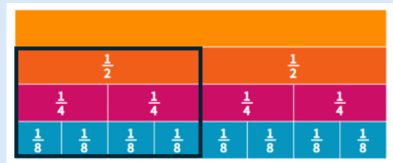
		Division problems do not always result in equal, whole number shares or groups. The remainder is the number left over when an amount cannot be grouped or shared equally in the required way.	groups of 6 compared to 12 oranges arranged into groups of 4. Solve contextual and non-contextual division problems which do not share or group equally using concrete materials. For example, 12 tennis balls cannot be shared equally five ways or grouped in fives. There are 2 tennis balls left over (there is a remainder of 2).	Division is not commutative.	be shared equally among 3 people because 12 is a multiple of 3, but 14 toys cannot be shared equally between 5 people because 14 is not a multiple of 5.  Identify multiples of 2,5,10 progressing to 3,4,6, and 100, making connections to skip counting and the commutative law. For example, the multiples of 5 are 5, 10, 15, 20, 25 …, identifying multiples of six from other known tables. Illustrate and explain using concrete materials that the commutative law does not apply to division. For example, 6 ÷ 3 ≠ 3 ÷ 6

Notes

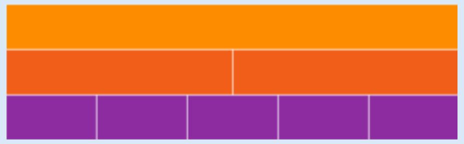
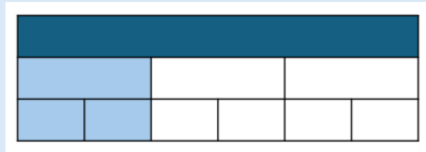
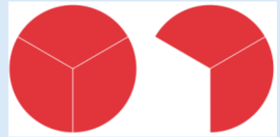
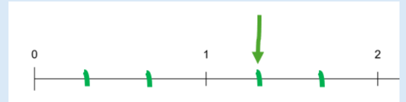
When introducing multiplication and division through arrays, use up to 30 objects as this will provide a good number of possible arrangements.
Connections should be made to place value and flexible partitioning.
Appropriate mathematical tools to keep track when skip counting include multiple sets of dice patterns, hundred square and 100 bead two colour abacus.
As learners become familiar with multiplication and division facts within the 2, 3, 4, 5, 6 and 10 times tables they should be presented with a variety of contextualised problems where they need to identify which of the four operations is required.
It is important that learners spend time working on both sharing and grouping.
It is important to remind learners that:

- sharing is knowing the total number of objects to be shared out and how many equal groups you want to make. You are working out how many objects each share will get.
- grouping is knowing the total number of objects and the size each equal group should be. You are working out how many groups you will be able to make.

Understand: Fractions represent equal parts of a whole and can be used for counting, comparing and measuring.

Know	Do	Know	Do	Know	Do
A fraction is a number which represents part of a whole or a measure.	Discuss and describe real and relevant contexts where halves and quarters are used as part of a whole and as measure. For example, I have eaten one half of my sandwich (part of a whole), the jug is one half full (measure).	A unit fraction represents one of a number of equal parts of a whole and has one as the numerator.	Identify and represent unit fractions to tenths using concrete materials and visual representations. For example, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{10}$. Read fractional notation correctly. For example, one fifth ($\frac{1}{5}$) and not one over five.	A non-unit fraction represents more than one of the same unit fractions. It has a numerator which is not equal to 1.	Identify and represent unit and non-unit fractions to tenths visually and using manipulatives. For example, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{7}{10}$. Connect concrete, visual, oral and symbolic representations of unit and non-unit fractions of shapes, objects and measures. For example, three quarters of the rectangle are shaded blue. 
Fractions can be greater than one. For example, one and a half.	Discuss and describe real and relevant contexts where fractions greater than one are used. For example, I drank one and a half glasses of water.		Connect concrete, visual, oral and symbolic representations of unit fractions of shapes, objects and measures.		
One half can be represented symbolically as $\frac{1}{2}$.	Read fractional notation of halves and quarters correctly. For example, one quarter ($\frac{1}{4}$) and not one over four.		Recognise when objects, shapes and measures have been partitioned equally and when they have not. 	Fractions are equivalent to each other if they represent the same value.	Represent equivalent fractions to tenths using concrete materials and visual representations. For example, $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$. 
One quarter is one of four equal parts of one whole object, shape or measure.	Connect concrete, visual, oral and symbolic representations of halves and quarters of shapes, objects and measures.				
One quarter can be represented symbolically as $\frac{1}{4}$.	Recognise when objects, shapes and measures have been quartered and when they have not.				

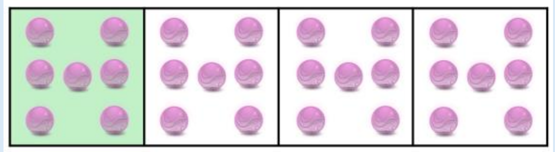
We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Fractions are written in terms of a numerator and a denominator. Four quarters of a shape, object or measure can be recombined to make the whole. One half is equal to two quarters of the same whole. Quarters are smaller than halves of the same whole	Explain the purpose of the numerator and denominator in fractions. For example, one quarter has the denominator 4 because there are 4 equal parts, and a numerator 1 because we are describing one of those four equal parts. Recombine quarters of shapes, objects and measures and recognise that this re-makes the whole. Represent halves and quarters using drawings of shapes split into two or four equal parts. Recognise and explain the connection between halves and quarters using concrete materials and visual representations. Count forwards and backwards orally in halves and whole numbers using halves of everyday items and shapes: one half, two halves/one whole, three halves/one and half, four halves/two…	For a given whole, the greater the number of equal parts, the smaller the size of each part. Unit fractions of the same whole can be compared and put into order. Fractions can be made up of smaller fractions of the same whole.	Compare and order the size of unit fractions of the same whole using concrete materials and visual representations. For example, for the same whole, fifths are smaller than halves.  Identify connections between larger and smaller fractions of the same whole using concrete materials and visual representations. For example, one third is the same as two sixths of the same whole.  Count forwards and backwards orally in quarters and whole numbers, using mathematical tools such as fraction tiles, bar models or a marked number line to keep track.	An improper fraction has a numerator which is greater than its denominator and has a value greater than one whole. A mixed number is a number greater than one, which has a whole number part and a fractional part. Improper fractions can be expressed as mixed numbers and vice versa.	Explain and model the equivalence of mixed numbers and improper fractions using concrete materials and visual representations. For example, $\frac{5}{3}$ is equivalent to 1 and two thirds.  Connect concrete, visual, oral and symbolic representations of improper fractions and mixed numbers of shapes, objects and measures. Place halves, thirds, quarters and fifths on a number line marked in whole numbers. For example, four thirds  Count forwards and backwards orally in unit fractions to tenths and whole numbers, using mathematical tools such as fraction tiles, bar models or marked number lines to keep track.

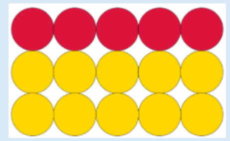
Notes

Learners should be introduced to fractions as a part-whole, where a whole is divided into equal parts and as a measure, where the fraction is relative to a unit and is often shown on a number line.
Learners should be able to calculate unit fractions of a quantity before being introduced to improper fractions and mixed numbers.
Appropriate concrete materials are folded strips of paper, fraction tiles or fraction circles.
As fractions become harder to compare, fraction strips are more appropriate than fraction circles, as circles are difficult to divide perfectly.
Rectangles and Bars (Area Models) are much easier for learners to physically fold, draw, and partition equally.
It is important when working with fractions, that learners explore ideas such as, 'Is one half always bigger than one quarter?' to understand that this comparison can only be made if we are finding those fractions of the same whole.
Make connections to time, half past, quarter past, in Space, Shape and Measurement Strand.

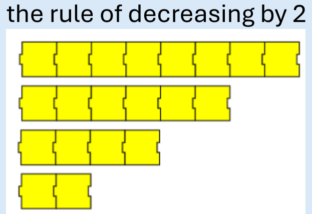
Understand: Unit fractions represent equal parts of a whole, with the denominator specifying the total number of parts and therefore the size of each part.

Know	Do	Know	Do	Know	Do
				Halves and quarters can describe equal parts of a quantity. They can be found by splitting a collection of items into two and four equal shares.	Discuss and describe real and relevant contexts where halves and quarters of quantities are found. For example, a sale where items are half price; in a class of 24 children, one quarter of the people own a dog. Find halves and quarters of quantities using concrete materials and visual representations. For example, one quarter of 28 is 7. 

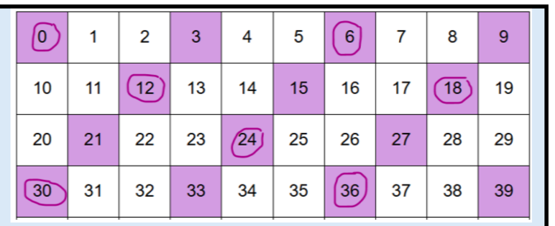
We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

				A unit fraction of a quantity can be found using division.	Describe the connection between finding halves and quarters of quantities. For example, half of 20 is 10, so a quarter of 20 is 5 (half of 10). Find the associated fractions of a quantity using the 2, 3, 4, 5, 6 and 10 times tables, (whole number answers only). Represent these calculations in an array. For example, $\frac{1}{3}$ of $15 = 15 \div 3 = 5$  Find the total in a quantity when shown one half or one quarter of that quantity. For example, show half of a collection as 6 counters with the other half unseen (screened).
Notes Connect finding a fraction of a quantity to multiplicative relationships. It is important when working with fractions, that learners explore ideas such as, ‘Is one half always more than one quarter?’ to understand that this comparison can only be made if we are finding those fractions of the same quantity. **Excludes** The use of abstract approaches when finding a whole given a part. Initially use screened concrete materials before moving on to pictorial approaches such as bar models.					

- Patterns and sequences

Strand: Quantity, Numbers and the Algebraic Properties of Number **Sub-strand: Patterns and sequences**					
Beginning of First Level ⟷ **End of First Level**					
Understand: Additive patterns involve sequences where the same number is added or subtracted each time, so the difference between terms remains constant.					
Know	**Do**	**Know**	**Do**	**Know**	**Do**
A number pattern is a rule which can be used to generate an ordered list of numbers, which is called a sequence. The numbers in a sequence are called terms.	Identify next terms, missing terms and errors in number sequences that ascend and descend in ones. Notice, discuss and identify number patterns, and describe the rule when choral counting and when making patterns using concrete materials. For example, the sequence of 8, 6, 4, 2 has the rule of decreasing by 2 each time.  Recognise and correct errors made in oral number sequences. For example, 10, 20, 30, **14**, 50.	Skip counting is based on a number pattern and generates a sequence.	Identify next terms, missing terms and errors in number sequences that ascend and descend in ones within the extending number range. Notice, discuss and identify the pattern within ordered lists and describe the rule. For example, the sequence of 100, 200, 300, 400 has the rule of skip counting forwards in 100s. Extend sequences of numbers, based on skip counting, by applying the rule. For example, 35, 30, 25, … Identify odd and even numbers based on their ones digit within the extending number range.	Number sequences can be generated based on addition or subtraction number patterns. Skip counting to and from zero generates a sequence of multiples.	Notice, discuss and identify the pattern within sequences and explain the rule. For example, the sequence of 74, 70, 66, 62 has the rule of subtracting 4 each time. Identify next terms, missing terms and errors in ascending and descending number sequences based on addition and subtraction patterns. For example, 231, 236, ?, ?, 251,256, … Record the count and describe the connections and patterns when choral counting in multiples. For example, noticing the connections between the multiples of 3 and 6. **0**, 3, **6**, 9, **12**, 15, **18**, … 0, 6, 12, 18, …

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

The even numbers form an additive sequence that begins at 0 and increases by 2 each time. The odd numbers form an additive sequence that begins at 1 and increases by 2 each time.	Identify odd and even numbers based on their ones digit.				

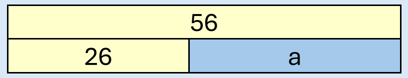
Notes
Work on patterns and sequences provides an opportunity to build fluency in additive and multiplicative facts.
Choral counting is a counting aloud routine where the count is recorded to enable pattern recognition.
Appropriate concrete materials are counters, cubes or other objects of the same size.
All even numbers have a ones digit of 0, 2, 4, 6 or 8.
All odd numbers have a ones digit 1, 3, 5, 7 or 9.
Includes
Working within the range 0 to 100, then gradually increasing the range towards 1000.

- Expressions, equations and relationships

Strand: Quantity, Numbers and the Algebraic Properties of Number
Sub-strand: Expressions, equations and relationships

Beginning of First Level ← → **End of First Level**

Understand: Symbols and visual representations model unknown quantities and make mathematical relationships explicit.

Know	Do	Know	Do	Know	Do
A picture or symbol can be used to represent an unknown number in a number statement.	Record addition and subtraction calculations with unknown numbers from given number stories. For example, giving some of my eight cookies away and being left with 3 could be recorded as 8 – ☐ = 3.		Record addition and subtraction calculations with unknown numbers from given number stories within the extending number range. For example, having some trading cards, getting 15 more and now having 24 could be recorded as ☐ + 15 = 24.	Letters of the alphabet can be used to represent an unknown number in a number statement.	Find missing values in addition and subtraction calculations, including those with letters of the alphabet representing an unknown value, using visual approaches when appropriate. For example, 26 + a = 56 
Fact families and known facts can be used to identify the missing value in addition and subtraction calculations.	Find missing values in addition and subtraction calculations, including those with a picture or symbol representing an unknown value. For example, 6 + ☐ = 8, ✓ - 4 = 3.	A range of addition and subtraction strategies (including using concrete materials and visual approaches) can be used to identify the missing value in calculations.	Find missing values in addition and subtraction calculations, including those with a picture or symbol representing an unknown value, within the extending number range. For example, 20 + ☐ = 28 or ☐ - 7 = 7.		

Notes
Appropriate concrete materials and visual approaches include counters, base ten materials, number lines and bar models.
Explicit connections should be made to the understanding of addition and subtraction.
Includes
Initially working within the number range 0 to 10, progressing to 0 to 30, and then extending to 100.

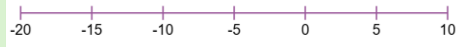
We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

- Second Level
- Number structures and operations

Strand: Quantity, Numbers and the Algebraic Properties of Number
Sub-strand: Number structures and operations

Beginning of Second Level ⟷ **End of Second Level**

Understand: Integers extend the number system to include values less than zero, enabling the representation of negative quantities.

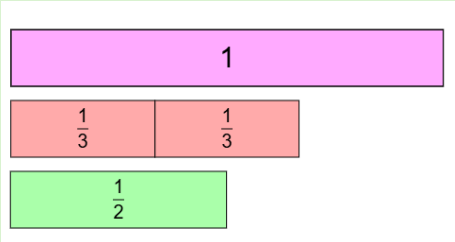
Know	Do	Know	Do	Know	Do
		The number line can be extended below zero. Numbers greater than zero are positive. Numbers less than zero are negative and a minus sign (-) is placed before the number to show this. Integers is the name given to the set of numbers … -3, -2, -1, 0, 1, 2, 3…	Recognise, say, and write numbers below zero. For example, -5 is negative five and not minus five. Place positive and negative numbers accurately on number lines. Identify missing numbers on a number line which extends below zero.  Count in integers forwards and backwards, across zero, using visual tools. For example, negative two, negative one, zero, one, two … Compare and order integer values. For example, 1 > -3, 70 707 < 77 000.		Skip count forwards and backwards across and below zero. For example, 10, 5, 0, negative 5, negative 10 …  Compare and order integer values to solve problems in context such as comparing temperatures. For example, -5°C is colder than -2°C because -5 < -2. Notice, discuss and identify the additive pattern within an ordered list of integers and describe the rule. For example, the sequence 23,11, -1, -13 has the rule of subtracting 12. Identify the next number(s), missing numbers and errors in ascending and descending integer sequences based on additive rules.

Notes
When counting in integers, it is important to model counting backwards as well as forwards.
Identify and discuss contexts where negative numbers occur. For example, temperature, finance or game scoring.
It is important for learners to work with both horizontal and vertical number lines.
Number lines could include counting sticks, drawn and digital number lines.

Understand: Rational numbers allow quantities between integers to be represented and compared.

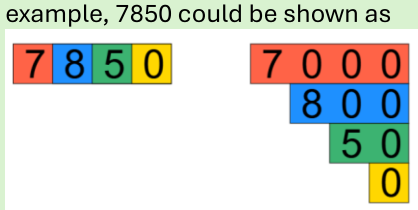
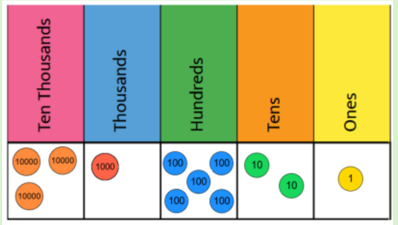
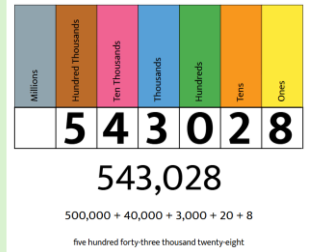
Know	Do	Know	Do	Know	Do
				The set of rational numbers includes all the integers and the fractions that lie between them.	Express integers as fractions. For example, 5 could be written as $\frac{5}{1}, \frac{15}{3}, \ldots$ Identify and place simple positive and negative rational numbers on a number line. For example, place -3, 4, 0, $\frac{1}{2}$ and $-1\frac{1}{2}$ onto a marked number line.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Fractions are numbers which can be ordered and compared. An equivalent fraction can be found by multiplying or dividing the numerator and denominator by the same number.	Place fractions to tenths in order using visual representations such as a fraction wall or a number line marked with whole numbers.  Compare the size of fractions using mathematical symbols. For example, $\frac{1}{2} < \frac{2}{3}$. Find equivalent fractions using multiplication and division, making connections to visual representations. For example, $\frac{1}{2}$ is equivalent to $\frac{2}{4}, \frac{3}{6}, \frac{5}{10}$ …	Fractions can be compared and ordered by expressing them with the same (common) denominator or numerator.	Compare the size of pairs of fractions with related denominators by expressing them with a common denominator. For example, $\frac{4}{25}$ and $\frac{1}{5}$ can be written as $\frac{4}{25}$ and $\frac{5}{25}$ and so $\frac{4}{25} < \frac{1}{5}$. Compare the size of pairs of fractions with related numerators by expressing them with a common numerator. For example, $\frac{2}{5}$ and $\frac{4}{9}$ can be written as $\frac{4}{10}$ and $\frac{4}{9}$ and so $\frac{2}{5} < \frac{4}{9}$.		Compare the size of pairs of fractions with unrelated denominators by expressing them with a common denominator or numerator. For example, $\frac{5}{6}$ and $\frac{3}{4}$ can be written as $\frac{10}{12}$ and $\frac{9}{12}$ and so $\frac{5}{6} > \frac{3}{4}$.

Notes
Related denominators are denominators that are connected because one is a multiple of the other (they can easily be changed to match).
Fraction walls are essential visual tools which allow learners to visualise, compare, and understand relationships between different fractional values.
It is important that learners continue to explain equivalency visually so that they do not become overly reliant on procedural approaches.

Understand: The base ten system represents quantities using digits, with each place ten times the value of the one to its right.

Know	Do	Know	Do	Know	Do
Four-digit whole numbers can be partitioned into thousands, hundreds, tens and ones. In a base ten number system each column, as we move left, has a place value ten times greater. These place values (10, 100, 1000…) are powers of ten.	Represent whole numbers visually, within an extending number range. For example, 7850 could be shown as  Skip count from any given whole number in thousands.	Five-digit whole numbers can be partitioned into tens of thousands, thousands, hundreds, tens and ones.	Represent whole numbers visually, within an extending number range. For example, 31 521 could be shown as  Skip count from any given whole number in tens of thousands.	Six-digit whole numbers can be partitioned into hundreds of thousands, tens of thousands, thousands, hundreds, tens and ones.	Represent whole numbers visually, within an extending number range. For example, 543 028 could be shown as  Skip count from any given whole number in hundreds of thousands. Compare and order numbers to one million where the same digits are used in different place values. For example, ordering 108 808, 180 080, 118 000, 108 880.

Notes
Skip counting in multiples of ten can strengthen learners' understanding of place value.
It is important to model skip counting backwards at the same time as skip counting forwards.

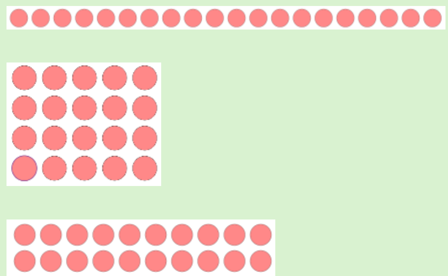
We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Appropriate resources/tools may initially include arrow cards and Gattegno charts before progressing to place value counters and place value charts.
Within the extending number range, learners should be supported to:

- identify and describe the value of a digit based on its place. For example, in the number 7850, the 8 represents eight hundreds and has a value of 800
- identify, say and write whole numbers
- compare and order numbers

Includes
Initially working within the range 0 to 10 000, progressing to 0 to 100 000, and then extending to 1 000 000.

Understand: The multiplicative structure of whole numbers greater than 1 can be described by factor pairs of smaller whole numbers.

Know	Do	Know	Do	Know	Do
		A factor is a whole number that divides into another whole number with no remainder. A number divided by one of its factors will give another factor and these two numbers are called a factor pair.	Describe numbers using the language of factors. For example, the factors of 18 are 1, 2, 3, 6, 9, 18. Represent a number in terms of its factors by identifying pairs of numbers that multiply to produce it. For example, 20 could be represented as 1 x 20, 2 x 10 or 4 x 5. 		

Notes
The use of concrete materials or visual tools could support the understanding of factor pairs. These resources include counters or multi-link cubes arranged as an array, a multiplication grid.
When working with factor pairs, certain categories of numbers have special or unusual factor pair patterns including square numbers, which have a repeated factor pair. For example, 16 = 1 x 16, 2 x 8 and **4 x 4**
When discussing square numbers, connections should be made to the Patterns and Sequences Sub-strand.

Understand: Scaling by 10, 100, and 1000 is enabled by the multiplicative relationship between places in the base-ten number system.

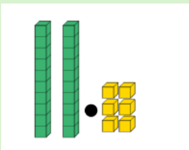
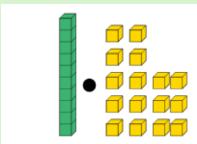
Know	Do	Know	Do	Know	Do
Scaling means changing the size, value, or amount of something by multiplying or dividing it by a constant value. When numbers are multiplied by 10, their digits are moved one place to the left making the number 10 times larger. When numbers are multiplied by 100, their digits are moved two places to the left making the number 100 times larger. When numbers are divided by 10, their digits are moved one place to the right making the number 10 times smaller.	Multiply any one or two–digit whole numbers by 10. For example, 24 × 10 = 240. Multiply any one-digit whole number by 100. For example, 5 × 100 = 500.		Multiply any one or two-digit whole number by 10 or 100. Divide up to four-digit multiples of 100 by 10 or 100 (whole number answers only). For example, 9000 ÷ 100 = 90.	When numbers are multiplied by 1000, their digits are moved three places to the left making the number 1000 times larger. When numbers are divided by 1000, their digits are moved three places to the right making the number 1000 times smaller.	Multiply any one-digit, two-digit or three- digit whole number by 10, 100 or 1000. Divide up to six-digit multiples of 10 by 10, 100 or 1000 (whole number answers only). For example, 180 000 ÷ 1000 = 180.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

When numbers are divided by 100, their digits are moved two places to the right making the number 100 times smaller. Strategies used when multiplying dividing whole numbers by 10 and 100 can be used when working with decimal fractions.	Divide three-digit multiples of 10 or 100 by 10. For example, 410 ÷ 10 = 41. Divide three-digit multiples of 100 by 100. For example, 800 ÷ 100 = 8. Multiply decimal fractions to 1 dp by 10 using concrete materials and visual tools. For example, 3·2 x 10 = 32. Divide two and three-digit whole numbers by 10, using concrete materials and visual tools to find quotients which are decimal fractions. For example, 45 ÷ 10 = 4·5, 634 ÷ 10 = 63·4		Multiply decimal fractions to 1 and 2 dp by 10 and 100 using concrete materials and visual tools. For example, 58·26 × 10 = 582·6. Identify where zero is needed as a placeholder. For example, 1798·3 x 100 = 179 830. Divide whole numbers by 10 or 100, using concrete materials and visual tools to find quotients which are decimal fractions. For example, 256 ÷ 100 = 2·56.	Strategies used when multiplying dividing whole numbers by 10, 100 and 1000 can be used when working with decimal fractions.	Multiply decimal fractions of up to 3 dp by 10, 100 or 1000, identifying where zero is needed as a placeholder. Divide whole numbers and decimal fractions by 10, 100 or 1000, to find quotients which are decimal fractions of up to 3 dp.

Notes
When multiplying a whole number by a multiple of ten, zero is used as a placeholder in empty columns.
Appropriate concrete materials or visual tools can be used and include place value counters and place value grids.
Make connections with metric conversions in the Shape, Space and Measurement Strand.

Understand: Decimal place value extends the base ten system so precision increases as place values extend to the right of the decimal point.

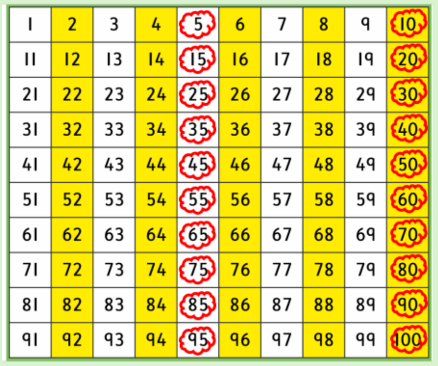
Know	Do	Know	Do	Know	Do
A decimal fraction is a fraction that is expressed as a number which has a decimal point. The decimal point (·) separates the whole number part from the fractional number part in a decimal fraction. Decimal place value refers to the value of a digit based on its position to the right of the decimal point. The first digit after the decimal point represents tenths.	In decimal fractions, identify and describe the value of a digit based on its place. For example, in the number 3·6 (which is equal to $3\frac{6}{10}$), the 3 represents 3 ones and the 6 represents 6 tenths. Visually represent decimal fractions to 1 dp partitioned in different ways. For example, 2·6 is 2 ones and 6 tenths, but it is also 1 one and 16 tenths.  	The second digit after the decimal point represent hundredths.	In decimal fractions identify and describe the value of a digit based on its place. For example, in the number 4·65 (which is equal to $4\frac{65}{100}$), the 4 represents 4 ones, the 6 represents 6 tenths and the 5 represents 5 hundredths. Visually represent decimal fractions to 2 dp partitioned in different ways. For example, 2·63 is 2 ones and 63 hundredths but is also 1 one, 16 tenths and 3 hundredths.	The third digit after the decimal point represents thousandths.	In decimal fractions identify and describe the value of a digit based on its place. For example, in the number 3·206 (which is equal to $3\frac{206}{1000}$), the 3 represents 3 ones, the 2 represents 2 tenths and the 6 represents 6 thousandths. Explain how decimal fractions can be partitioned in different ways up to 3 dp.

Notes
Learners should investigate the use of decimal fractions in real and relevant contexts or example, for 1 dp - measurements of length and body temperature, for 2 dp - money and measurements, for 3 dp - measurements of distance and mass.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Learners should be supported to recognise, say and write decimal fractions within the extending number range (1 dp, 2 dp then 3 dp). For example, $\frac{7}{10}$ can be expressed as 0·7.
Appropriate resources/tools could include fraction bars split into tenths, hundred squares, base ten materials and decimal place value charts and counters.

Understand: Operations on whole numbers follow consistent rules regardless of size, allowing the same methods to be applied across all magnitudes.

Know	Do	Know	Do	Know	Do
Addition and subtraction strategies used for whole numbers up to 1000 can also be applied to numbers greater than 1000.	Solve contextual and non-contextual addition and subtraction problems justifying choice of strategy based on efficiency.	Technology can support efficiency and accuracy of calculations.	Solve addition and subtraction problems using known strategies and verify solution with a calculator. Solve addition and subtraction problems using spreadsheet, including with formula. For example, =SUM(..)	Subtraction is not commutative.	Solve contextual and non-contextual addition and subtraction problems justifying choice of strategy from a range, including calculators or spreadsheets where appropriate. Explain and represent that changing the order of numbers in subtraction changes the result. For example, 12 – 5 ≠ 5 – 12.
A product is the result of multiplying 2 or more numbers together.	Identify products in given calculations using the 2, 3, 4, 5, 6 and 10 times tables. For example, as 3 × 2 = 6, the product of 3 and 2 is 6. Recognise connections between the 2, 4 and 8 times tables. Model the repeated addition of 8 using arrays and visual representations, identifying at least the first ten multiples, making connections between multiplication, skip counting, and contextualised problems. Recognise connections between the 3, 6 and 9 times tables. Model the repeated addition of 9 using arrays and visual representations, identifying at least the first ten multiples, making connections between multiplication, skip counting, and contextualised problems. Build the 7 times table using concrete materials, visual representations and by making connections to other tables. Recognise and read number sentences where mathematical symbols have been used to record multiplication by 8, 9 or 7.	A common multiple is one that appears in two or more times tables. Smaller multiples can be combined to find a larger product (the distributive law). The grid method is a way of representing a partitioning approach to multi-digit multiplication using the distributive law.	Carry out a range of multiplication and division calculations using known number facts and number patterns. For example, 5 × 13 could be calculated as 5 × 10 add 5 × 3. Find common multiples of two or more numbers by skip counting or listing the multiples. For example, 5, **10**, 15, **20** ... and 2, 4, 6, 8, **10**, 12, 16,18, **20**. Therefore 10 and 20 are the first two common multiples of 2 and 5.  Give examples of the distributive law within the times tables. For example, 7 × 3 = 5 × 3 + 2 × 3 Multiply two-digit multiples of 10 by a single digit using known number facts. For example, 50 × 3 = 5 × 3 × 10 = 150. Multiply two-digit whole numbers by single-digit whole numbers using the grid method. For example, to find 75 × 3 × | 70 | 5 3 | 210 | 15		Multiply three and four-digit whole numbers by single-digit whole numbers using the grid method. For example, to find 174 × 5 × | 100 | 70 | 4 5 | 500 | 350 | 20

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Record multiplication calculations from given contextualised number stories using mathematical symbols. For example, 6 packs each containing 9 individual toilet rolls can be recorded as 6 x 9 = 54.

Tell a number story based on given multiplication facts from the 8, 9 or 7 times table. For example, 3 x 8 = 24 could represent 3 packs of 8 batteries, making 24 batteries in total.

Find the product of given calculations using all times tables to 10.

Recognise and read number sentences where mathematical symbols have been used to record division by 8, 9 or 7.

Record division calculations from given contextualised number stories using mathematical symbols. For example, 72 pencils split into boxes of 8 would give 9 boxes can be written as 72 ÷ 8 = 9.

Tell a number story based on given division facts by 8, 9 or 7. For example, 20 ÷ 4 could be twenty balls shared equally between four baskets would give 5 balls in each basket.

Identify division problems which do, or do not share or group equally when dividing by 8,9 or 7. For example, 14 toys can be shared equally among 7 people because 14 is a multiple of 7, but 14 toys cannot be shared equally between 8 people because 14 is not a multiple of 8.

In division calculations the dividend ÷ divisor = quotient.

When sharing, the quotient represents the quantity of objects in each share. When grouping, the quotient represents the number equal of groups created.

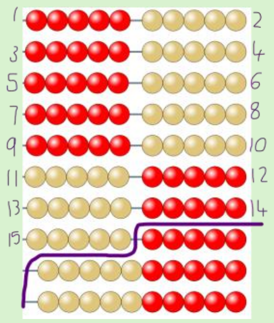
A number is divisible by another if the quotient is a whole number.

If a number is not divisible by another, the answer to a division problem could be given as a fraction, or as a whole number quotient with a remainder.

70 × 3 = 210
5 × 3 = 15
So, 75 × 3 = 210 + 15 = 225

Describe the solution to a division calculation using the term quotient. For example, when 12 toys are shared between 3 children, the quotient (number in each share) is 4 as 12 ÷ 3 = 4. When 15 toys are put into groups of 5, the quotient (number of groups) is 3 as 15 ÷ 5 = 3.

Solve division of two-digit numbers by single-digit numbers which require both grouping and sharing and use a range of representations, including concrete materials and/or visual representations to explain solutions. For example, 75 ÷ 5 could be calculated using tally marks or a Rekenrek to keep track.



Divide two-digit numbers by single-digit numbers through non-canonical partitioning (whole number quotients only). For example, 84 ÷ 6 can be partitioned as 60 ÷ 6 and 24 ÷ 6.

÷	10	4
6	**60**	**24**

60 ÷ 6 = 10
24 ÷ 6 = 4
So, 84 ÷ 6 = 10 + 4 = 14

Solve division problems which do not share or group equally and explain whether solutions should be expressed as a fraction or as a quotient and remainder. For example, sharing apples versus sharing toy cars.

The extended written algorithm and the written algorithm for multiplication can provide a more efficient way of recording calculations as they become more complex.

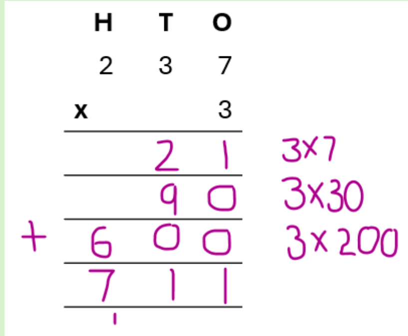
The grid method and the written algorithms can be extended to multiply any whole number by a single digit whole number.

When three or more numbers are multiplied together, the way they are grouped has no effect on their product (associative law).

Numbers can be rearranged and reordered to make multiplication calculations easier.
Round bracket symbols ( ) can be used in mathematics to indicate how numbers in a calculation are being grouped.

5 × 174 = 500 + 350 + 20
= 870

Make connections between the grid method and written representations of two-digit and three-digit multiplication by a single digit.



Use appropriate strategies to multiply larger whole numbers by single digit whole numbers.

Perform multiplication calculations of three or more numbers efficiently by rearranging and reordering. For example, when multiplying 4, 7 and 25, it is more efficient to multiply the 4 and 25 first, then multiply by 7.

Indicate the grouping of numbers in multiplication calculations using round brackets. For example, 2 x 3 x 5 = (2 x 3) x 5 = 2 x (3 x 5).

Choose from a range of approaches when multiplying whole numbers by a single digit, including a written algorithm and the use of a calculator, justifying choice based on efficiency.

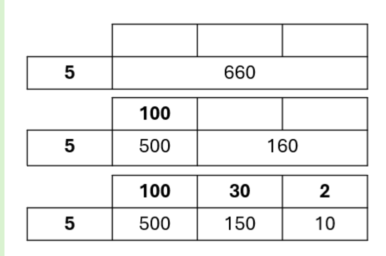
Solve division of three-digit whole numbers by single-digit whole numbers through partitioning (whole number quotients only). For example, 348 ÷ 3 could be partitioned as 300 ÷ 3, 30 ÷ 3 and 18 ÷ 3.

÷	100	10	6
3	**300**	**30**	**18**

300 ÷ 3 = 100
30 ÷ 3 = 10
18 ÷ 3 = 6
So, 348 ÷ 3 = 100 + 10 + 6 = 116

Make connections between the grid method and written representations of

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

				The written algorithm for division can provide a more efficient way of recording calculations as they become more complex.	two-digit and three-digit division by a single digit and solve using the written algorithm. For example, to calculate 660 ÷ 5  Interpret and explain solutions to contextualised problems involving division with remainders. For example, 84 cupcakes shared equally among 8 plates gives each plate ten and a half cupcakes. 84 tennis balls shared equally into 8 bags gives each bag ten balls with four remaining.
	Identify the operation(s) required when listening to or reading two-step contextualised problems.		Solve two-step problems involving two of the four operations within 100. For example, given that Alex has £13 and Nanette has £25, calculating how much each person would receive if the money was shared equally.		Solve contextual and non-contextual problems using multiplication and division with whole number answers, choosing from a range of strategies, including the use of a calculator, justifying choice based on efficiency. Solve multi-step problems involving more than two of the four operations using concrete materials, visual representations and informal jottings to keep track.

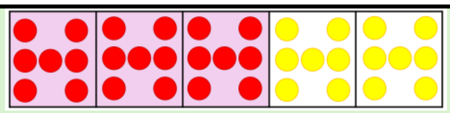
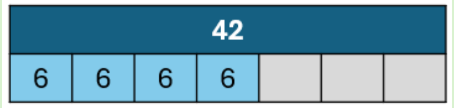
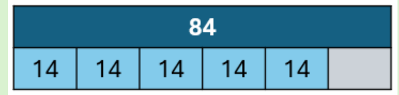
Notes
Encourage learners to estimate before calculating and check reasonableness of solutions across all number ranges.
Appropriate strategies for addition and subtraction include partitioning, rounding, compensating and the written algorithm.
Appropriate strategies for multiplication and division include use of the commutative, associative and distributive properties, the grid method, the written algorithm and use of a calculator.
Arrays constructed using concrete materials can continue to support understanding of multiplicative relationships.
When calculators are first introduced teachers should ensure that learners are supported to use them effectively.

Excludes
Calculations that require consideration of the order of operations.

Understand: Fractions represent proportional relationships between quantities that can be calculated and compared.

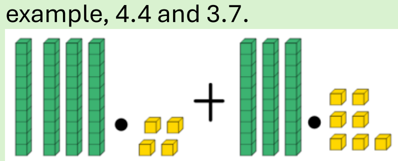
Know	Do	Know	Do	Know	Do
A non-unit fraction of a quantity can be found using division and multiplication.	Build fluency by using division strategies to find unit fractions of whole numbers. For example, $\frac{1}{5}$ of 35 is the same as 35 ÷ 5. Find non-unit fractions of whole numbers using concrete materials and visual approaches. For example, $\frac{3}{5}$ of 35.		Find unit fractions of two-digit whole numbers by making connections to division by a single digit number (whole number quotients only). For example, finding $\frac{1}{6}$ of 84.		Find unit fractions of three-digit whole numbers by making connections to division by a single digit number. For example, finding $\frac{1}{3}$ of 348. Find non-unit fractions of two-digit whole numbers by using visual representations and making

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

	 Fig. S16		Build fluency by using known facts to find non-unit fractions of whole numbers. For example, finding $\frac{4}{7}$ of 42.  $\frac{1}{7}$ of 42 = 42 ÷ 7 = 6 $\frac{4}{7}$ of 42 = 6 × 4 = 24		connections to division and multiplication. For example, finding $\frac{5}{6}$ of 84.  $\frac{1}{6}$ of 84 = 84 ÷6 = 14 $\frac{5}{6}$ of 84 = 14 × 5 = 70 Solve contextualised problems involving calculating and comparing proportions of quantities. For example, comparing $\frac{3}{4}$ of £120 and $\frac{2}{5}$ of £130.

Notes
Initially work within known times tables facts then move beyond times tables facts for unit fractions, but within times tables facts for non-unit fractions.
Appropriate concrete materials/visual tools include counters, base ten materials and bar models.
Make connections to known division and multiplication strategies.
Includes
Whole number solutions only.

Understand: Operations on decimal fractions follow the same consistent rules as whole numbers, with place value underpinning precision.

Know	Do	Know	Do	Know	Do
The number sequence can be expressed in fractions and decimal fractions.	Count in tenths using a number line or a counting stick to keep track. For example, $\frac{11}{10}, \frac{10}{10}, \frac{9}{10}$ ... Count in decimal fractions (tenths) using a number line or a counting stick to keep track. For example, 1·8, 1·9, 2, 2·1, 2·2 ... Compare decimal fractions to 1 dp using the correct mathematical symbols. For example, 1·2 > 0·9. Order decimal fractions to 1 dp in ascending and descending order. For example, putting 0·2, 2·1, 1·2 in ascending order.		Count in decimal fractions (hundredths) a number line or counting stick to keep track. For example, 0·97, 0·98, 0·99, 1, 1·01, 1·02 ... Order decimal fractions to 1 and 2 dp in ascending and descending order and compare using the correct mathematical symbols. For example, putting 2·05, 0·25, 2·5 in descending order and stating that 2·05 < 2·5.		Order whole numbers and decimal fractions of up to 3 dp in ascending and descending order and compare using the correct mathematical symbols. For example, putting 1·375, 13, 1·37, 13·75, 1·075 in descending order and stating that 1·37 < 1·375.
		Strategies used when adding and subtracting whole numbers can be used when working with decimal fractions.	Add and subtract decimal fractions to 1 dp, progressing to 2 dp using concrete materials and visual representations. For example, 4.4 and 3.7. 		Solve addition and subtraction problems involving decimal fractions of up to 3 dp in the contexts of money and measurement.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

			Make connections between the concrete, visual and written forms of addition and subtraction calculations to 2 dp. Solve simple addition and subtraction problems to 2 dp in the contexts of money and measurement.		
				Strategies used when multiplying whole numbers by a single digit can be used when working with decimal fractions. Division can result in decimal fractions when quantities do not divide into whole numbers.	Multiply decimal fractions to 1 and 2 dp by a single digit whole number using concrete materials and/or visual tools. For example, 69·35 × 6. Interpret and explain decimal fraction solutions to division problems in the context of money and measure. For example, when £47 is shared equally between two people, each person receives £23.50.

Notes
It is important to model counting backwards at the same time as counting forwards.
Appropriate concrete materials/visual tools to support addition and subtraction of decimal fractions include base 10 materials, place value counters, coins, place value charts and bar models.
Make use of real-life contexts of measure and money and connect learning across the Shape, Space and Measurement Strand, and the Finance Sub-strand.
Excludes
Decimal division at this stage should be restricted to problems in the context of money or measurement.

Understand: Rounding replaces exact values with approximations that remain close to the original.

Know	Do	Know	Do	Know	Do
Numbers can be rounded as an approximation to a required degree of precision. Different degrees of precision are required depending on the context.	Round whole numbers to the nearest 10, 100 or 1000 to estimate the solution to a given problem. Discuss and describe real and relevant contexts where estimation and rounding are used.		Round whole numbers to the nearest 10, 100, 1000 or 10 000 to estimate the solution to a given problem.		Round whole numbers to any specified power of 10 to estimate the solution to a given problem.
Decimal fractions to 1 dp can be placed on a number line between their neighbouring whole numbers.	Place decimal fractions to 1 dp on marked then empty number lines.  Round decimal fractions to 1 dp to the nearest whole number, using a number line to support.	Decimal fractions to 2 dp can be placed on a number line between their neighbouring tenths. In some contexts, estimates and approximations must be more accurate than the nearest whole number.	Place decimal fractions to 1 and 2 dp on empty number lines. Round decimal fractions to 2 dp to the nearest whole number or tenth, using a number line to support.		Round decimal fractions of up to 3 dp to the nearest whole number, tenth or hundredth.

Notes
Relevant contexts could include for example, the difference in the degree of precision required when stating the attendance at a concert versus the attendance at school.
Contexts which require more accuracy than the nearest whole number may include timings of runners in 100 metre races, ingredients in a recipe or doses of medicine.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

- Proportional reasoning

Strand: Quantity, Numbers and the Algebraic Properties of Number
Sub-strand: Proportional reasoning

Beginning of Second Level ←——————————————→ **End of Second Level**

Understand: Percentages represent proportions on a common base of 100, allowing direct comparison between quantities.

Know	Do	Know	Do	Know	Do
		Percentages are fractions with a denominator of 100. The mathematical symbol used to represent percentage is %. Fractions, decimal fractions and percentages can be used as equivalent forms of an operator.	Convert and explain the equivalence of given percentages, fractions and decimal fractions. For example, 25% is 25 per 100, 0·25, $\frac{25}{100}$, $\frac{1}{4}$. Find 10%, 20%, 25% and 50% of two-digit numbers using knowledge of equivalence with unit fractions and visual approaches such as bar models (whole number quotients only). For example, finding 20% of 85 is the same as one fifth of 85.	Fractions, decimal fractions and percentages can be expressed in equivalent forms to be compared and ordered. Known percentages can be scaled or combined to find other percentages of quantities.	Compare and order the size of fractions, decimal fractions and percentages of the same quantity by expressing them in equivalent forms. For example, by ordering $\frac{1}{4}$, 0·2 and 30%. Solve contextualised problems by finding 10%, 20%, 25% and 50% of two-digit numbers (whole number quotients only). For example, finding the cost of a £55 rucksack after a 20% discount has been applied. Solve contextualised problems by scaling or combining known percentages. For example, using 10% to find 30%, using 50% and 25% to find 75%.

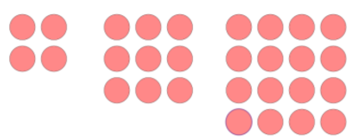
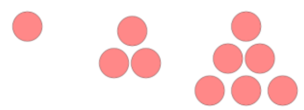
Notes
Make connections with multiplication, division and fractions.
It is useful at this stage to discuss and describe real and relevant contexts where percentages are used. For example, sale prices, finance, and advertising.
It is important when working with fractions, decimal fractions and percentages that children explore ideas such as, 'Is 50% always more than 25%?' to understand that this comparison can only be made if we are working with the same quantity.

- Patterns and sequences

Strand: Quantity, Numbers and the Algebraic Properties of Number
Sub-strand: Patterns and sequences

Beginning of Second Level ←——————————————→ **End of Second Level**

Understand: Some patterns in shapes and nature follow numerical rules that determine how they grow and change.

Know	Do	Know	Do	Know	Do
A square number of items can be arranged in an array with an equal number of rows and columns: 	Explain and model visually or with concrete materials what a square number is. Find square numbers using arrays and known multiplication facts. Justify and explain visually or with concrete materials whether a number is a square number.	A triangular number describes the number of items that can be arranged in a triangle. The first row contains 1 item, the second row 2 items, the third row 3 items and so on: 	Create triangular patterns with everyday items and recognise where they appear in real contexts. For example, stacked cans, snooker balls and bowling pin formations. Justify and explain visually or with concrete materials whether a number is triangular.	A cubic number of items can be arranged in a three-dimensional array with equal numbers of rows, columns and layers.	Explain and model visually or with concrete materials what a cubic number is. Find cubic numbers using physical objects and multiplication facts. Justify and explain using number facts and concrete materials whether a number is cubic.

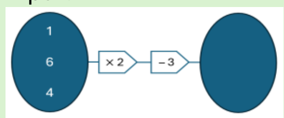
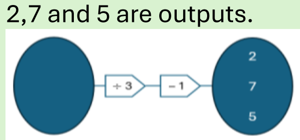
We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

A square number is found by multiplying a number by itself. The mathematical symbol for squared is ^{2} (superscript two).	Calculate and record a range of square numbers. For example, $4 \times 4 = 4^2$ is read as “four squared” and is equal to 16.		Investigate triangular numbers by exploring classic mathematical problems. For example, Pascal’s Triangle, the handshake problem.	 A cubic number is found by multiplying a number by itself and then by itself again. The mathematical symbol for cubed is ^{3} (superscript three). The next number in a Fibonacci sequence is found by adding together the previous two numbers in that sequence.	Calculate and record a range of cubic numbers. For example, $4 \times 4 \times 4 = 4^3$ is read as “four cubed” and is equal to 64. Develop an awareness of the history and application of Fibonacci sequences in the context of the natural world. 
Notes Appropriate concrete materials to show a square and triangular numbers are counters, cubes or other objects of the same size. Appropriate concrete materials to show cubic numbers are 3D stackable objects of the same size.					

- Expressions, equations and relationships

Strand: Quantity, Numbers and the Algebraic Properties of Number **Sub-strand: Expressions, equations and relationships**
Beginning of Second Level ⟷ **End of Second Level**
Understand: Expressions, equations and functions represent relationships between quantities using symbols and variables, forming part of the language of mathematics.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Know	Do	Know	Do	Know	Do
Fact families and known facts can be used to identify the missing value in multiplication and division calculations	Find missing values up to three-digit addition and subtraction calculations using visual approaches when appropriate. For example, $124 = 200 - t$ Find missing values up to two-digits in multiplication and division calculations using visual approaches when appropriate. For example, 40 =▯ × 10 or 6 ÷ ✸ = 3 or b ÷ 5 = 7	A variable is a quantity that can take on different values and is represented by a dedicated symbol or a letter. A term is a single number, a variable or a product of numbers and variables. An algebraic expression is a term or group of terms separated by operators and may contain more than one variable. An equation states that two expressions are equal in value. Equations can be solved by finding the unknown value for which the equation is true. Expressions and equations can be used to model situations and solve problems in context.	Identify variables in everyday contexts. For example, a person's height of a person, the number of green blocks in a tub, the cost of a banana. Build expressions in everyday contexts. For example, the total cost of 3 bananas could be represented by the expression $3b$, the difference between the number of green bricks and blue bricks could be represented by the expression $g - b$. Model addition and subtraction problems with one unknown and one solution making connections between visual representations and algebraic notation. For example, I got £62 in birthday money and I now have £138. How much money did I have before my birthday? This could be modelled by $a + 62 = 138$ and using a bar model: Total = £138 Amount before birthday | £62 birthday money		Model multiplication and division problems with one unknown and one solution making connections between visual representations and algebraic notation. For example, 3 bananas cost 90 pence. What is the cost of each banana? This could be modelled by $3b = 90$ and using a bar model: Total = 90 p Cost of one banana | Cost of one banana | Cost of one banana Model and solve problems with one unknown and one solution using algebraic notation (one-step and two-step examples only). For example, I double a number and add 12 and I get 732. What number did I start with? could be modelled by $732 = 2a + 12$ and by using a bar model: Total = 732 a | a | 12
				A function is a relationship between two sets of numbers, describing how each element in the first set (input) is mapped on to exactly one element in the second set (output). Functions can be represented visually, and these are often called function machines.	Calculate outputs for given inputs of single-step and two-step functions represented visually as function machines. For example, 1,6 and 4 are input.  Calculate inputs for given outputs of single-step and two-step functions represented visually as function machines by using inverse operations. For example, 2,7 and 5 are outputs. 

Notes

Explicit connections should be made to across addition, subtraction, multiplication and division.
Variables help create general rules and allow us to express relationships between quantities.
At this stage learners should be focusing on modelling and solving problems using equations rather than procedurally solving abstract equations.
Learners could investigate the origin of algebra. They could investigate how it developed across multiple cultures over time as a way of representing and solving problems involving unknown quantities and relationships. For example, Persian mathematician Al-Khwarizmi formalised systematic methods for solving equations and his work gave us the word **"algebra"**.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

- Third Level
- Number structures and operations

Strand: Quantity, Numbers and the Algebraic Properties of Number
Sub-strand: Number structures and operations

Beginning of Third Level ←→ **End of Third Level**

Understand: For natural numbers greater than 1, the number of factors a number has determines whether it is prime or composite.

Know	Do	Know	Do
Prime numbers are natural numbers which have exactly two factors, themselves and one.	Identify prime numbers using knowledge of factors. Use the definition of prime numbers to reason about the classification of the number 1.		
Composite numbers are the product of two or more prime numbers.	Express composite numbers as a product of prime factors and use these to perform calculations efficiently.		

Notes
Natural numbers are the set of numbers 1, 2, 3, 4, 5 …
Work initially with prime numbers to one hundred.
Counters and arrays may support the initial understanding of primes and composites. For example, prime numbers cannot be arranged as arrays with multiple rows and columns.
Composite numbers may or may not have composite factors but will always be able to be expressed in terms of prime factors only.
At this stage it would be useful to discuss and investigate the use of prime numbers in real-life and mathematical contexts.
Digital tools exist for prime factorisation that may be useful to experience.

Understand: Index notation provides a compact way to express repeated multiplication, enabling the efficient calculation and generalisation of powers.

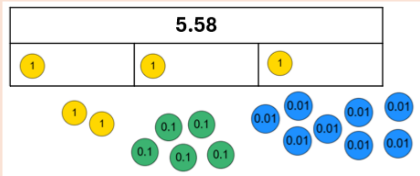
Know	Do	Know	Do
		Index notation is an efficient way of expressing repeated multiplication. There is an inverse relationship between powers and roots.	Express repeated multiplications using index notation. For example, $5 \times 5 \times 5 \times 5 = 5^4$. Explain how squared numbers, cubed numbers, and their corresponding roots, are connected through inverse operations. For example, as $4^3 = 64, \sqrt[3]{64} = 4$. Apply these connections to evaluate whole number roots.

Notes
Connect squared numbers, cubed numbers, square roots and cube roots to the area of squares, the volume of cubes and their dimensions.
Excludes
Negative roots are not required at this stage.

Understand: Operations on decimal fractions follow the same consistent rules as for whole numbers, with place value underpinning precision.

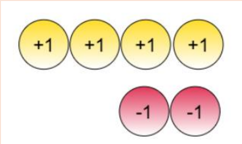
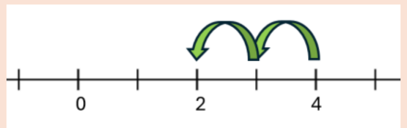
Know	Do	Know	Do
Strategies used when multiplying and dividing whole numbers can be used when working with decimal fractions.	Multiply any one, two or three digit whole number or decimal fraction by powers of 10 to 1 000 000. For example, $7.82\ x\ 10\,000 = 78200$		Calculate, interpret and explain solutions to increasingly complex multi-step contextualised problems which use the four operations.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

	Multiply decimal fractions to 1, 2 or 3 dp by a single digit whole number, justifying solutions using concrete materials and/or visual approaches. For example, $9.352 \times 6 = 56.112$. Multiply any one, two or three digit number by any number which is a product of a single digit and any whole number power of ten. For example, 3.4 × 400 = 3.4 × 4 ×100. Divide decimal fractions to 1 or 2 dp by a single digit whole number, initially using concrete materials and visual approaches. For example, 5.58 ÷ 3 = 1.86.  Fig T1 Divide 2 and 3 digit whole numbers by a single digit whole number where solutions are decimal fractions to 1 or 2 dp, initially using concrete materials and visual representations. For example, 124 ÷ 8 = 15.5 Divide any two or three digit number by any number which is a product of a single digit and 10, 100 or 1000 (whole number and exact decimal fraction answers only).	Solutions to calculations resulting in decimal fractions may need to be rounded or truncated depending on the context.	Round solutions which are decimal fractions to a given degree of precision. Justify solutions which have been rounded or truncated. For example, the number of full 230 ml cups that can be poured from a 2-litre bottle is 8, even though 2000 ÷ 230 = 8.7 (to 1 dp).

Notes
When working with whole numbers and decimal fractions, appropriate concrete materials and visual tools include place value counters and bar models.
Make use of real-life contexts of money and measure and connect learning across the Shape, Space and Measurement Strand, and the Finance Sub-strand.

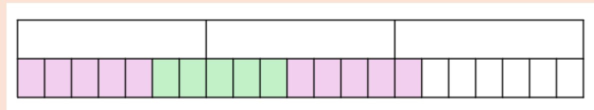
Understand: Operations on integers follow the same consistent rules as for whole numbers, extending additive and multiplicative patterns and relationships across zero.

Know	Do	Know	Do
The additive inverse of a number is the number that is added to the original to give zero. A positive unit and a negative unit that combine to make zero together is known as a zero pair. Subtraction is addition of the additive inverse. The properties of multiple and division remain when working with negative integers.	Identify the additive inverse of given integers. For example, the additive inverse of 2 is -2, the additive inverse of -5 is 5. Add and subtract integers using appropriate manipulatives, making use of zero pairs. Connect this to its representation on a number line. For example, adding 4 and negative 2.  Multiply and divide integers by applying repeated addition, continuing patterns and applying the connections between multiplication and division (inverse operations).		Add and subtract integers using a number line to keep track as necessary. Build fluency by applying multiplication and division strategies when working with integers.

Notes
When working with integers appropriate concrete materials and visual tools include double sided counters and number lines.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Understand: Operations on fractions follow the same consistent rules as for whole numbers, extending additive and multiplicative patterns and relationships to parts of a whole.

Know	Do	Know	Do
	Add and subtract fractions, including improper fractions, with common denominators, initially using a number line to keep track. Add and subtract mixed numbers with common denominators, initially using pictorial approaches or a number line to keep track.	The highest number that is a factor of two or more numbers is called their highest common factor (HCF).	Identify the highest common factor of two or more numbers by listing their factors. For example, Factors of 32: 1, 2, 4, 8, **16**, 32 Factors of 48: 1, 2, 3, 4, 6, 8, 12, **16**, 24, 48. The HCF of 32 and 48 is 16. Express fractions in their simplest form efficiently by first finding the highest common factor of their numerator and denominator.
The smallest multiple that is shared by two or more numbers is called their lowest common multiple (LCM).	Identify the lowest common multiple of two or more numbers by listing their first few multiples. For example, Multiples of 6: 6, **12**, 18, 24 … Multiples of 4: 4, 8, **12**, 16 … Therefore, the LCM of 6 and 4 is 12.		Add and subtract mixed numbers with related and unrelated denominators, initially using concrete materials or pictorial approaches, expressing solutions in their simplest form.
Expressing fractions with common denominators allows for efficient addition and subtraction.	Add and subtract fractions with related denominators by finding equivalent fractions with common denominators (including improper fractions), initially justifying choice of denominator and solutions using pictorial approaches. For example, $\frac{2}{3}+\frac{1}{6}=\frac{4}{6}+\frac{1}{6}=\frac{5}{6}$. 		Multiply a whole number by a fraction, using pictorial approaches, and the commutative law. For example, $\frac{5}{7}\times 3=\frac{15}{7}=2\frac{1}{7}$ 
		Finding a fraction of a quantity is the same as multiplying the quantity by that fraction.	Solve contextualised problems by multiplying a whole number by a fraction.
	Add and subtract fractions with unrelated denominators by finding equivalent fractions with any common denominators (including improper fractions), initially justifying choice of denominator and solutions using pictorial approaches. For example, $\frac{2}{3}+\frac{1}{4}=\frac{8}{12}+\frac{3}{12}=\frac{11}{12}$. Express fractions in their simplest form.		
Proper and improper fractions can be simplified by dividing the numerator and denominator by a common factor. A fraction is in its simplest form if the only common factor of its numerator and denominator is 1.	Solve contextualised problems by adding or subtracting fractions, expressing solutions in their simplest form.		

Notes
Related denominators are denominators that are connected because one is a multiple of the other (they can easily be changed to match).
When simplifying fractions, connections can be made to the prime factors of the numerator and denominator.

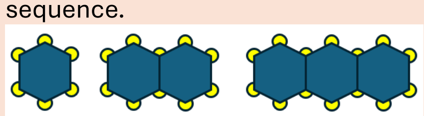
We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

- Proportional reasoning

Strand: Quantity, Numbers and the Algebraic Properties of Number Sub-strand: Proportional reasoning			
Beginning of Third Level ⟷ End of Third Level			
Understand: When variables are proportional, an increase or decrease in one causes a proportional change in the other.			
Know	**Do**	**Know**	**Do**
	Find non-unit fractions of three and four-digit whole numbers (whole number quotients only).	A ratio can be used to compare two or more quantities expressed in terms of their relative sizes.	Express the relationship connecting sets of two or three quantities as a ratio. For example, if there are 3 blue counters and 5 red counters, the ratio of blue to red is 3:5 (“three to five”).
Percentages of a quantity can be found by multiplying that quantity by the equivalent decimal fraction.	Find any percentage of any quantity, using a calculator or spreadsheet where appropriate. Solve problems in everyday contexts by increasing or decreasing a quantity by a given percentage. For example, pay rises, interest or depreciation.	A ratio is in its simplest form if the only common factor of its terms is 1.	Express ratios in their simplest (whole number) form. For example, in a lucky dip there are 20 losing tickets and 5 winning tickets, the ratio of losing to winning simplifies from 20:5 to 4:1.
Two variables are directly proportional if they have a constant quotient. This algebraic relationship can be used to model everyday situations.	Recognise variables which are directly proportional to one another.		Find related quantities by using equivalent ratios and given quantities. For example, if the safety ratio of children to adults is 12:1 and there are 3 adults in the group, they can safely look after up to 36 children.
	Model the multiplicative relationship between two variables which are directly proportional to one another to find the value of one variable given the other. For example, Scones Grams of flour 12 450 6 225 18 675 Constant quotient (rate) is 37.5 g per scone.	Two variables are inversely proportional if they have a constant product. This algebraic relationship can be used to model everyday situations.	Recognise variables which are inversely proportional to one another. Model the multiplicative relationship between two variables which are inversely proportional to one another to find the value of one variable given the other. For example, Number of workers Hours taken 4 12 8 6 1 48 Constant product is 48 hours of work.
Notes Everyday situations which demonstrate a directly proportional relationship include buying groceries, purchasing fuel, driving at constant speed, scaling up a recipe, and hourly wages. Proportion tables can be used to organise calculations. Everyday situations which demonstrate an inversely proportional relationship include the length of time a fixed quantity of resource will last when used at a particular rate and how long a task will take given the number of workers assigned to it.			

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

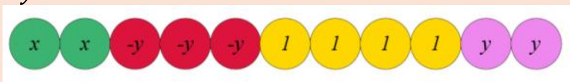
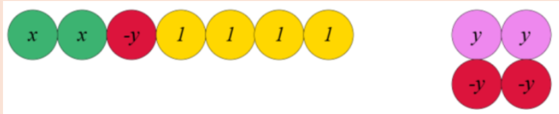
- Patterns and sequences

Strand: Quantity, Numbers and the Algebraic Properties of Number Sub-strand: Patterns and sequences			
Beginning of Third Level ⟵			⟶ **End of Third Level**
Understand: Sequences of numbers that follow multiplicative rules can lead to patterns of rapid growth or decay.			
Know	**Do**	**Know**	**Do**
	Identify and describe the multiplicative pattern within ordered lists. Continue sequences which have a multiplicative pattern.		Identify missing numbers and errors in number sequences based on multiplicative rules. Describe how the terms in multiplicative sequences change compared to additive ones.
Notes Examples of sequences which have multiplicative patterns include a sequence that starts with 2 and terms are multiplied by 3 each time or a sequence that starts with 13 and terms are divided by 10 each time. Compare additive and multiplicative sequences, such as one which adds 2 each time to one which multiplies by 2 each time. Explore the long-term behaviour of such sequences.			
Understand: Rules describing sequences enable the efficient identification of the next term or any particular term based on its position.			
Know	**Do**	**Know**	**Do**
A sequence can be described by its first term and the rule for finding the next term. An arithmetic sequence is one where the difference between two consecutive terms is always the same.	Find the first few terms in number sequences given their first term and the rule for finding the next term. For example, starting with a first term 10, use the rule add 6 to find the next 4 terms in this sequence. Identify whether sequences are arithmetic by looking for a common difference. In context, identify patterns that follow an arithmetic sequence. Describe these patterns in words and continue these sequences to solve related problems. For example, finding the number of yellow dots in the 7th picture in this sequence.  Fig T5	A rule can be described to find a particular term in a sequence by using its position in the sequence. This is called an n^{th} term formula.	Generate parts of sequences using their nth term formula (arithmetic sequences only). For example, T = 4n + 6 generates the sequence 10, 14, 18, ... Describe the relationship between the n^{th} term formula of arithmetic sequences and their common difference. For example, T = 4n + 6, the sequence has a common difference of 4. Describe the relationship between the n^{th} term formula of arithmetic sequences and the sequence of multiples of their common difference. For example, for T = 4n + 6, all the terms in the sequence are 6 greater than a multiple of 4.
Notes Geometric patterns such as mosaics from around the world could provide examples of geometric representations of numerical patterns.			

Expressions, equations and relationships

Strand: Quantity, Numbers and the Algebraic Properties of Number Sub-strand: Expressions, equations and relationships			
Beginning of Third Level ⟵			⟶ **End of Third Level**
Understand: Like terms combine to simplify expressions because each variable represents the same quantity.			
Know	**Do**	**Know**	**Do**

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Terms in an expression which contain exactly the same variable(s) and exponents are called like terms. Like terms can be added and/or subtracted. This is known as collecting like terms. Expressions can be simplified by collecting like terms into a single term.	Identify like terms within expressions. For example, in the expression $2x - 3y + 4a + 2y$ there are two like terms, $-3y$ and $2y$. Simplify expressions by collecting like terms, initially using concrete or digital manipulatives. For example, $\ \ 2x - 3y + 4 + 2y$ $= 2x - y + 4$  		Simplify expressions algebraically by collecting like terms. Simplify multi-variable expressions and justify why terms can or cannot be combined. For example, $\ \ 2x - 3xy + 4 + 6x - 7$ $= 8x - 3xy - 7$

Notes
Constant terms are like terms that can be collected.
Appropriate concrete materials include algebra discs.
Online resources exist which provide digital manipulatives.

Understand: Techniques for solving equations can be applied to inequations, which describe the way in which one expression is not equal to another.

Know	Do	Know	Do
An equivalent equation can be created by applying the same operation to each of its sides.	Solve one and two-step equations of the form $ax \pm b = c$ where a, b and c are whole numbers. Record working steps and solutions algebraically.	An inequation states that one expression is unequal in value to another. An inequation has a set of solutions for which the inequation is true. The subject of a formula is the variable that is expressed in terms of the others.	Solve, by applying inverse operations, one and two-step equations of the form $ax \pm b = c$ where a, b and c are integers. Model contextualised problems with one and two-step equations and solve them algebraically. Solve algebraically a variety of one and two-step inequations and explain what the solutions mean. For example, $3x - 5 < 16$ $3x < 21$ $x < 7$ so, the inequation is true for any value of x less than 7. Change the subject of linear formulae using the same algebraic strategies as for solving equations algebraically (single step examples only).

Notes
Appropriate algebraic strategies include balancing approaches and using inverse operations; however, manipulatives and/or pictorial approaches should still be used.
Relevant contexts which could be modelled include for example, using the information that 5 tickets and a booking fee of £2 comes to a cost of £127 to find the cost of each ticket.
Includes
Examples where the position of the variable is varied.
Excludes
Equations with negative solutions initially.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Understand: The construction of expressions and formulae describe unambiguously the operations involved and what order they are carried out.

Do	Do	Know	Do
Substitution is the process of replacing variables with specific values and can be used to evaluate an algebraic expression. A formula is an equation that describes a relationship between two or more variables.	Substitute positive values to evaluate expressions and make use of known or given formulae.		Substitute positive and negative values to evaluate expressions, including those with brackets. Check solutions to equations using substitution.
Functions are often written as $f(x)$ where x is the input and $f(x)$ is the output.	Describe, using function notation, single-step and two-step functions represented visually as function machines. For example, $f(x) = 2x - 3$. Calculate outputs for given inputs of single-step and two-step functions represented in function notation. For example, if $f(x) = 3x + 5$, $f(4) = 3 \times 4 + 5$ $= 17$		Calculate inputs for given outputs of single-step and two-step functions represented in function notation by establishing and solving an appropriate equation. For example, if $f(x) = 3x + 5$ and $f(x) = 23$ $3x + 5 = 23$ $3x = 18$ $x = 6$

Notes
Examples of appropriate known or given formulae could include evaluating $a - 4b$ given positive values for a and b, finding the area of a circle given its radius.
Learners should be encouraged to estimate to check the reasonableness of solutions.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

- Fourth Level
- Number structures and operations

Strand: Quantity, Numbers and the Algebraic Properties of Number
Sub-strand: Number structures and operations

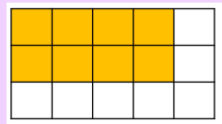
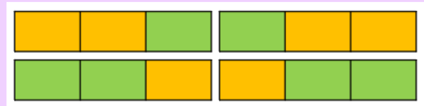
Beginning of Fourth Level ⟷ **End of Fourth Level**

Understand: Scientific notation uses powers of ten to represent numbers, enabling very large and very small numbers to be written and compared efficiently.

Know	Do
Any number can be written in scientific notation, $a \times 10^n$, where $1 \leq \lvert a \rvert < 10$ and n is an integer. The arithmetic operations on number can be applied to numbers expressed in scientific notation.	Convert numbers to and from scientific notation. Solve contextualised problems which involve numbers expressed in scientific notation, using a calculator as appropriate.

Elaboration
Appropriate contexts could be taken from other curriculum areas including science and technologies.

Understand: Operations on mixed numbers follow the same consistent rules as for fractions, scaling quantities to find parts of parts, and determining the number of fractional groups in a quantity.

Know	Do
Finding a fraction of a fraction is the same as multiplying these fractions together. In grouping problems, the size of each group can be a fraction.	Solve contextualised problems by adding and subtracting mixed numbers, expressing solutions in their simplest form. Multiply fractions by fractions, including improper fractions, initially using a pictorial approach. For example, $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$  Solve contextualised problems by multiplying fractions, expressing solutions in their simplest form. Solve contextualised problems that involve division by a fraction (whole number answers only), using visual approaches. For example, working out how long four tins of cat food would last if a cat eats two thirds of a tin each day. 

Notes
Using a pictorial approach to develop an understanding of multiplication and division of fractions avoids an over reliance on memorised procedures. The area model provides a useful pictorial approach multiplication of fractions.
Make connections to increasing a quantity by a fraction of that quantity and its equivalence to percentage increase. Possible contexts could be an overtime rate of time and a half, adding VAT of 20% or one fifth to a price.

Understand: Approximation introduces error and controlling precision, throughout calculations, minimises this error.

Know	Do
Significant figures are digits in a number that carry meaningful information about its precision and accuracy. Numbers can be rounded to a specified number of significant figures.	Express solutions to contextualised problems to a specified or appropriate number of significant figures. Identity the margin of error associated with values rounded to given numbers of significant figure.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Premature rounding impacts on the accuracy of a solution.	Solve contextualised, multi-step problems, using a calculator as appropriate, maintaining accuracy until the final solution is found. Notice and discuss the errors that could be introduced by premature rounding.

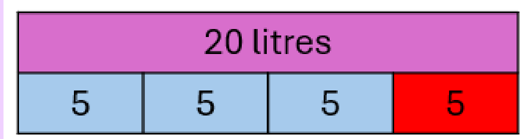
Notes
Make connections to area and volume work in the Shape, Space and Measurement Strand. It would be useful to make connections with the sciences on precision in calculations, making clear the reasons for any differences in approach.

- Proportional reasoning

Strand: Quantity, Numbers and the Algebraic Properties of Number
Sub-strand: Proportional reasoning

Beginning of Fourth Level ⟷ **End of Fourth Level**

Understand: The proportional relationship between the constituent parts of a whole can be described using a ratio.

Know	Do
Ratios can be used to describe and find proportions of a whole and to compare the balance of constituent parts within different mixtures.	Split quantities into a given ratio and solve associated contextualised problems. For example, 20 litres of purple paint is made by mixing blue and red paints in the ratio of 3:1.  This means that there are 4 shares of 5 litres, with 15 litres of blue paint and 5 litres of red paint.

Notes
Make connections to scientific contexts.

- Patterns and sequences

Strand: Quantity, Numbers and the Algebraic Properties of Number
Sub-strand: Patterns and sequences

Beginning of Fourth Level ⟷ **End of Fourth Level**

Understand: Sequences that follow additive and multiplicative patterns can model real-life situations to make predictions and solve problems.

Know	Do
A geometric sequence is one where the ratio of two consecutive terms is always the same. The next term of a geometric sequence is found by multiplying the previous term by this common ratio. The common difference in an arithmetic sequence appears in its n^{th} term formula. When terms in an arithmetic sequence are plotted against their position in that sequence, a straight line is formed with a gradient equal to the common difference.	Identify whether sequences are geometric by looking for a common ratio. Investigate contexts from everyday life and history where geometric sequences occur. For example, the rice and chessboard legend, compound interest, and inflation. Continue these sequences to solve related problems. Establish an arithmetic sequence that models a physical or pictorial pattern, or real-life situation. Determine its n^{th} term formula and use it algebraically or graphically to solve related problems.

Notes
Connect the terms in an arithmetic sequence with linear functions in Patterns and Sequences in the Shape, Space and Measurement Strand.
Arithmetic and geometric sequences have a rich and ancient history across the world, such as in the construction of the pyramids in Egypt.
Excludes
Finding the nth term of Geometric sequences and finding the sum of terms in any sequence.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

- Expressions, equations and relationships

Strand: Quantity, Numbers and the Algebraic Properties of Number
Sub-strand: Expressions, equations and relationships

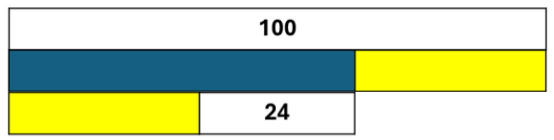
Beginning of Fourth Level ⟷ **End of Fourth Level**

Understand: The use of brackets enables expressions to be written in equivalent forms according to the distributive law.

Know	Do
The distributive law can be applied to find equivalent forms of expressions. This is known as expanding brackets and its inverse operation is known as factorising.	Find equivalent expressions by expanding brackets using the distributing law and simplifying where appropriate by using manipulatives and algebraic approaches. Factorise expressions by identifying the highest numerical common factor using manipulatives and algebraic approaches. Model everyday situations using expressions with and without brackets. For example, adding 2 to a number then multiplying by 6 has the same result as multiplying by 6 then adding 12.

Notes
Use of manipulatives/visual approaches including algebra tiles and the grid method support connections to multiplication and the distributive law for numerical calculations.
Includes
Multiplication across brackets by a positive or negative number only.

Understand: Algebraic skills form an expanding toolkit that enables mathematicians to model and solve problems of increasing complexity.

Know	Do
Quadratic equations contain a term with a squared variable.	Solve, by applying inverse operations, linear equations of the form $ax \pm b = cx \pm d$ where a, b, c and d are integers. Solve linear equations by expanding brackets. Solve quadratic equations of the form $x^2 = a$ where $a > 0$ and explain why there are two solutions. For example, if $x^2 = 9$, then $x = \pm 3$ as $3^2 = 9$ and $(-3)^2 = 9$. Using the same strategies as for solving equations algebraically, change the subject of a formula, (single or two-step examples only). Solve problems with two unknowns and one solution using manipulatives or visual representations. For example, finding the number of blue and yellow counters given that the total number of counters is 100 and there are 24 more blue counters than yellow counters. This could be solved using bar models: 

Notes
Algebra, although abstract at this stage of learning, is the language that forms the basis for further mathematical study.
Excludes
Equations with solutions that are not integers. However, it should be made clear that this is for simplicity at this stage, and solutions to equations and inequations can be fractions.
At this stage learners are not expected to use an algebraic approach to find the solution to problems with two unknowns and one solution.

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.

Version History

Version	Date	Detail
Version 1	23 June 2026	First released

We gratefully acknowledge the use of images created using MathsBot, Toy Theater and Pixabay, used with permission.