Big Ideas and Concepts within the Mathematics Curriculum

This is an illustration of what Layer 1 of the Mathematics technical framework could look like.

Mathematics Purpose Statement

The work to evolve the mathematics curriculum has been taken forward by the National Core and Collaboration groups. An initial priority for this work was to clarify the purpose of mathematics within Scotland's Curriculum. This led to the development of a short purpose statement which outlines the vision for the position of mathematics, which is shown below.

In Scotland we strive for all children and young people to develop as fluent mathematical thinkers and learners who can work both independently and collaboratively to reason, investigate, and build connections. This will enable children and young people to make informed decisions, solve problems, and appreciate the mathematical relationships that shape our world.

Mathematics Big Ideas

A fundamental principle of the evolving technical framework is the development of conceptual understanding. The use of Big Ideas provides an opportunity to prioritise understanding in a curriculum area and to promote coherence. Developing Big Ideas was a starting point for the work undertaken by the Mathematics Core and Collaboration Groups.

The Big Ideas for mathematics outlined below reflect the current thinking of these groups.

Quantity, Number, and the Algebraic Properties of Number *Quantity is described through number, and the algebraic properties of number allow us to generalise relationships between these quantities.*	**Shape and Space** *Ideas of shape and space allow us to describe, visualise, and navigate the physical structure of our world.*	**Information, Data and Uncertainty** *Mathematics equips us to sort and interpret information, manage uncertainty, and make informed decisions.*
Quantity is about understanding 'how much' and 'how many'. In everyday life, we rely on our understanding of numbers and how they work. Numbers help us count, order, measure, compare, and express values. Important rules like the commutative, associative, and distributive laws of addition and multiplication are critical in understanding generalisations and making connections. We have developed counting systems that use place value to represent large and small numbers. From this, we can solve problems accurately, fluently and efficiently in different ways by using mental strategies, visual ideas and algorithms. To deepen our understanding of numbers and how they behave, we use algebra to explore patterns and relationships. This helps us generalise how numbers and operations on numbers interact. Algebra extends our understanding as we explore how quantities change, using functions to describe relationships and rates of change, and prepares us for calculus and the solving of increasingly sophisticated problems.	Ideas of shape and space support the development of our visualisation skills, help us understand the effects of transformations, and allow us to make sense of mathematical structures and symmetries. As we navigate the world around us exploring position, movement and direction we develop our spatial reasoning. We can describe location through coordinate geometry which fixes position in two- and three-dimensional space. The properties of two-dimensional shapes and three-dimensional objects play a significant role in their selection and use in a range of contexts and real-life applications. Ideas of shape and space have applications for us both within and outside of the field of mathematics.	Data is everywhere, influencing our decisions and impacting our daily lives. An awareness of different types of data helps us analyse and interpret the information in the world around us. Mathematics supports us in making informed, evidence-based decisions and predictions by critically evaluating information. Probability helps us navigate uncertainty and risk. By providing structured methods, mathematics allows us to collect, organise and represent information accurately and objectively. Statistics help us identify and make sense of patterns and trends, measure variations, and draw reliable conclusions. We can make sense of information by constructing mathematical models to tackle problems and explore solutions. In areas such as finance, making sense of information is essential for interpreting data, managing budgets, assessing risks, and making informed choices.

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Mathematics is a Language *Mathematics provides us with a shared, precise language to communicate about the world around us.*	**Mathematics is Interconnected** *Mathematics is a network of interconnected ideas that help us deepen our understanding, solve problems, and make links within and beyond the subject.*	**Mathematics is Meaning** *Mathematics encourages us to be curious, think critically, and reason logically.*
The language of mathematics encompasses all mathematical concepts and supports the deepening of conceptual understanding. It allows us to model and communicate solutions to real-world problems. By using mathematical symbols, diagrams and accurate notation we can communicate effectively across disciplines and cultures. The use of precise notation and mathematical language is essential when conveying abstract concepts. As a collaborative practice, mathematics is about discourse, communicating ideas, and posing purposeful questions that promote curiosity and creativity. Precise mathematical language underpins clear and visible thinking, informed decision-making and justification.	Mathematics is a web of connected ideas, relationships and concepts. Among these concepts is 'Patterns and Sequences', which helps us to notice and understand mathematical relationships, whilst 'Proportional Reasoning' involves understanding multiplicative relationships in a variety of contexts. We use different concepts and connections to solve problems in new and unfamiliar situations. Exploring the links between concepts can deepen understanding, develop reasoning, and support decision-making. Connections also extend beyond mathematics itself, linking to a variety of areas of learning including science, technologies and expressive arts. Noticing and communicating the connections is fundamental to sense-making and enhancing an appreciation of mathematics.	Mathematics is more than a collection of facts, rules and procedures. It is a powerful tool for understanding our world, equipping us to think critically and make evidence-based decisions such as assessing risk and managing finances. Mathematics is dynamic and evolving, with new discoveries emerging in the field, often building on its rich history. We use mathematics to make sense of uncertainty and create beauty from logic. Proof establishes the truth of mathematical results, which allows us to approach real-world problems with confidence. While finding solutions is important, value also lies in developing our skills of reasoning, problem solving, and investigating. Being mathematical requires curiosity and empowers us all to ask questions. It invites us to wonder, investigate and challenge assumptions. Mathematics is a deeply human endeavour. It is a story of intrigue and collaboration across centuries and cultures, where each generation builds on the discoveries of the past, for the benefit of humanity.

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Mathematics Concepts

The mathematics groups then worked to develop the key concepts that would be the foundations of the mathematics curriculum. Progression in mathematics would be based on deepening understanding of these concepts. A description of each of the proposed concepts has been provided below. These concepts would be the foundation for what learners need to know and be able to do at each level as they progress through the mathematics curriculum. Together the Big Ideas and Concepts will form the first layer of the evolving technical framework.

Concept	Description
Counting and Unitising	Counting and unitising form part of our earliest engagement with mathematics. Counting generally begins orally then moves on to establishing the total number of individual objects in a collection. This develops into more sophisticated counting strategies, such as counting on and unitising, where items are grouped and counted in quantities that differ from one.
Types of Number	As mathematical development progresses, new types of numbers are needed to solve increasingly sophisticated problems. Whole numbers, integers and rational numbers are commonly used within everyday contexts. Further mathematical study includes working with irrational and complex numbers.
Structure of Number	Numbers can be represented and partitioned in various ways, such as through the development of early number bonds, place-value, multiples, factors, powers, and roots. Subitising (instantly recognising the number of items in a small group without counting) is a key foundation in understanding how numbers are structured. Understanding the structure of number can enable flexible and efficient manipulation to support accurate calculations.
Operations on Number	Additive and multiplicative strategies (linking to subtraction and division) and the understanding of their associated symbols, are developed as they are applied to a greater range of numbers. These operations range in complexity from finding the total of two collections and developing early number bonds, to creating multi-step solutions to problems in everyday and mathematical contexts.
Estimation, Error and Accuracy	Estimation is used when it is not essential or possible to find an exact number or measurement. Numerical calculations and measurements can be approximated by rounding to numbers that are easier to compute mentally, helping to develop a sense of reasonableness. Approximations introduce errors, which can be measured and expressed with a degree of tolerance, ensuring appropriate levels of accuracy for the context.
Comparison	Comparison allows us to recognise that quantities and measurements are either equal or unequal. If quantities are unequal, then one quantity or measurement is greater than the other. The language of comparison appears in a range of contexts, and decisions are often made based on comparisons.
Proportional Reasoning	Proportional reasoning involves recognising and applying the relationship between two or more quantities that change in proportion to each other. This is often represented using ratios, fractions, percentages, or equations, and enables predictions to be made, problems to be solved, and logical conclusions to be drawn.
Money and Finance	Finance involves recognising the value of coins and notes, different payment methods and sources of income. Understanding key principles such as borrowing, saving, budgeting, interest calculations, pay deductions, and investments help us to assess risk and make informed financial plans and decisions. Finance links closely with other concepts such as Operation on Number, Comparison and Probability.
Patterns and Sequences	Patterns, which occur in both nature and everyday life, are situations where actions, objects, colours, shapes, or numbers repeat in a certain order. A sequence is a list or arrangement of elements that follow a particular pattern or rule. Known elements can be used to establish the rule which describes the sequence, to copy and continue the sequence, and to predict other elements within it.
Position and Movement	Movements and directions can be described using appropriate language and measurements. The location of an object or point can be communicated using appropriate language or notation. A locus of points on a Cartesian plane can be used to define algebraically a geometric entity.
Measures	Length, area, volume, mass, angle, time, and temperature can be described and measured using appropriate language, scales and units of measurement. These measurements help us to describe and understand the world around us and can be used to perform associated calculations.
Properties of Shapes and Solids	Two-dimensional shapes and three-dimensional objects can be described, classified and analysed by their geometrical properties. These properties have relevance to their use in everyday and mathematical contexts. Some shapes, or combinations of shapes, can be placed without overlaps or spaces to cover a plane (tessellation).
Symmetry	Symmetry refers to ways in which an object, often a shape, is unchanged by transformations such as translation, reflection, rotation, and scaling. Examples of symmetry can be found in the world around us in plants, flowers, animals, and buildings. A wide range of mathematical structures, relationships, and representations, for example, in number, algebra and statistics, have their own symmetries (or asymmetries).
Expressions and Equations	Expressions are mathematical phrases which include variables (quantities that can take on a range of values) which are often represented by a symbol or letter. They can be written in equivalent forms, for example by simplifying through collecting like terms, or through factorising. The use of variables allows generalisations to be made about numbers, patterns, and mathematical relationships. An equation is a mathematical sentence that includes the equals sign to show both sides have the same value/balance. Each side of the equation can include numbers and/or symbols. If an equation includes an unknown number (represented by a symbol or letter), it can be solved to find the value(s) for which the equation is true. Inequations are mathematical statements indicating that one expression is not strictly equal to another.
Justification and Proof	Mathematical reasoning can be used to justify the solutions to problems. Mathematical statements may be true in all cases, true in some cases, or never true. It can be demonstrated that such a statement is true in all cases through a logical, mathematical argument. To show that a statement is not always true, just one counterexample is needed. A proof can be expressed by using physical objects, creating appropriate diagrams, or algebraically using mathematical notation.

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Types of Data	Data is information such as words, pictures, numbers, facts, figures, or objects. Data can be classified as qualitative (categorical) or quantitative (numerical). Qualitative data can be nominal (no sense of order, for example, breeds of dogs), or ordinal (can be ordered, for example, standard clothing sizes – small, medium, large). Quantitative data can be discrete (separate values that can be counted as whole numbers) or continuous (can take any value within a given range).
Data Collection and Organisation	Data can be collected in different ways such as observations, surveys, interviews, and experiments. Technology plays an ever-increasing role in the collection of data. Care should be taken to avoid bias and sample size should be considered. Data can be classified, grouped, ordered, and counted in different ways to prepare it for analysis. Physical objects can be organised by similar properties, for example, by shape, colour, or size. Quantitative data may be grouped into intervals, sorted in ascending or descending order, or categorised based on shared characteristics. Qualitative data may be organised by key themes or by using a coding system. The purpose of collection will impact on/determine how the data is organised.
Data Representation	Data can be represented using a range of tables, graphs, and charts, the choice of which depends on the type of data collected and the purpose for which it has been collected. Technology can play an important role in the creation of these tables, graphs and charts. Visual representations can become misleading, for example by truncating scales in graphs or exaggerating differences and trends.
Data Analysis and Interpretation	Sets of quantitative data can be analysed by comparing totals and identifying the most and least, calculating averages and measures of spread, and by identifying trends and outliers. Patterns and trends can be analysed in qualitative data, highlighting key points and repeating narratives. Conclusions can then be drawn about what the data shows or does not show. This may include comparing data sets with one another and making choices, decisions, and predictions. Being aware of the different types of bias and the role this can play is key when analysing data.
Probability	Probability helps us predict the behaviour of events where the outcome is uncertain. Appropriate language and notation can be used to describe or calculate the likelihood of an event happening, enabling informed decisions and accurate predictions.
Mathematical Modelling	Mathematical modelling is the process of describing, in mathematical language, how real-world systems behave. This staged process is cyclical in nature, where a problem is analysed, a model is formulated, evaluated, and modified if necessary. An effective mathematical model can support us in making predictions and decisions, and in solving problems.
Functions and Relationships	Operations on numbers can be described by giving the relationship between a number (input) and the corresponding number after the operations have been carried out (output). A function is a relationship where there is exactly one output for every input. Not all relationships are functions. Functions and relationships can be described in different ways, for example, by using arrows mapping inputs to outputs, algebraically using function notation, or graphically using Cartesian axes.

Development of a Mathematical Concept

An illustration of the work to evolve the technical framework for mathematics is provided. The illustration focuses on the concept of Patterns and Sequences and is related to the Big Idea of Quantity, Number, and the Algebraic Properties of Number.

In this illustration the second layer gives an overview of what learners need to know about this concept at each level. The third layer clarifies the specific knowledge, skills, procedures and strategies required to develop and deepen understanding of this concept as learners progress within and between levels.

Other concepts will also relate to this Big Idea, such as Structure of Number and Operations on Number. Further content related to Patterns and Sequences will link to other Big Ideas. A glossary will also be produced to support the understanding of key terminology and language.

The evolution of the technical framework (the know-do-understand model) seeks to ensure that learning is not a disconnected collection of facts or processes. Deep learning occurs when children and young people make sense of their learning, can establish connections and apply this learning in meaningful contexts. This connects knowledge and skills to conceptual understanding.

Development of the Patterns and Sequences Concept

Concept description:

Patterns, which occur in both nature and everyday life, are situations where actions, objects, colours, shapes or numbers repeat in a certain order. A sequence is a list or arrangement of elements that follow a particular pattern or rule. Known elements can be used to establish the rule which describes the sequence, to copy and continue the sequence and to predict other elements within it.

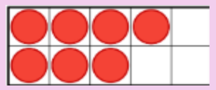
Layer 2 – Overview of the development of Patterns and Sequences	
Early	**Patterns and Sequences:** The sequence of whole numbers can be seen, copied and continued. Numbers can be arranged in order of size. Whole numbers are either even, where they can be shared into two equal groups, or odd, where they cannot be shared into two equal groups.
First	**Patterns and Sequences:** A number sequence is an ordered list that follows a pattern or rule. Numbers in a sequence can be in ascending or descending order. The patterns used to generate number sequences can be based on any type of operation on number.
Second	**Patterns and Sequences:** A square number of items can be arranged in the shape of a square. The sequence of square numbers can be found by multiplying whole numbers by themselves. A triangular number of items can be arranged in the shape of a triangle. The sequence of triangular numbers (1, 3, 6, …) increases by one more each time. The next number in a Fibonacci sequence is found by adding together the two previous numbers in that sequence.
Third	**Patterns and Sequences:** An arithmetic sequence is one where the difference between two consecutive terms is always the same. An n^{th} term formula describes a sequence and is used to find any number in that sequence using its position.
Fourth	**Patterns and Sequences:** A geometric sequence is one where the ratio of two consecutive terms is always the same. Arithmetic sequences can be used to model real-life situations.

Summary of essential vocabulary for learners				
Early	First	Second	Third	Fourth
Sequence Ascend Descend Odd numbers Even numbers	Pattern Ordered list Multiplicative number patterns Multiples	Decimal fractions Integers Square numbers Arrays Triangular numbers Cube numbers	Term Arithmetic sequence n^{th} term formula Consecutive terms Common difference	Geometric sequence Common ratio Gradient

Patterns and Sequences Concept

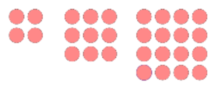
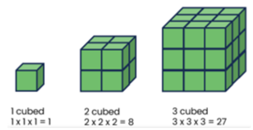
This is an illustration of part of the Mathematics technical framework, specifically the concept of Patterns and Sequences. A detailed glossary will also be produced to support the understanding of key words and phrases.

Layer 3 – Expected Knowledge and Skills/Procedures/Strategies which show progression within and across levels for the concept of Patterns and Sequences (Early to Fourth)

			End of Early Level	**Beginning of First Level**			
Know	**Do**	**Know**	**Do**	**Know**	**Do**	**Know**	**Do**
Sequences of numbers can be seen in the world around us.	Explore and identify number sequence in everyday situations and real-life experiences. For example, house numbers, page numbers, clocks and calendars.						
Number sequences can be copied and continued.	**Within the number range 1 to 5 and then 0 to 10:** Copy and continue number sequences through a range of songs, stories, rhymes and counting games. Notice and identify missing numerals when shown sequences which ascend or descend in ones. For example, 5, 6, ?, 8. Identify an error made in an ascending or descending number sequence in ones. For example, 4, 5, 6, '8', 9...	Number sequences can be copied, continued and extended.	**Within the number range 0 to 20:** When counting in ones, identify which number(s) comes next in ascending and descending oral and written sequences. For example, 13, 14, 15,? When counting in ones, identify missing numbers from ascending and descending oral and written number sequences. For example, 14, 15,?, 17 Recognise and explain an error made in an oral or written ascending or descending number sequence in ones. For example, 14, 15, 16, '18', 19...	A sequence is an ordered list of numbers that follow a specific rule. A number pattern is a rule that numbers follow which can be used to create a number sequence. For example, increasing by 5 each time.	**Within the range 0 to 100:** Identify next number(s), missing numbers and errors in number sequences that ascend and descend in ones. When choral counting, using concrete materials or looking at ordered lists of numerals, notice, discuss and identify the pattern and explain the rule. For example, the sequence of 8, 6, 4, 2 has the rule of decreasing by 2 each time. Recognise and explain an error made in an oral or written number sequence. For example, 10, 20, 30, '14', 50.	Skip counting is based on a number pattern. Number sequences can be generated by skip counting.	**Gradually increase the range to 1000.** Identify next number(s), missing numbers and errors in number sequences that ascend and descend in ones. Recognise and extend a sequence of numbers, based on skip counting, by applying the rule. For example, 3, 6, 9, ?, ?. Notice, discuss and identify the pattern within an ordered list and explain the rule. For example, the sequence of 100, 150, 200, 250 (has the rule of skip counting in 50s).
		If a number is even, the total can be shared into two equal groups. If a number is odd number, it cannot be shared equally into two groups.	Identify and demonstrate if a number to 10 is odd or even using concrete materials and visual approaches. For example, pair-wise ten frames:  Identify the next odd/even number to 10 using mathematical tools to keep track. For example, a number track or number line. Sort numerals from 0 to 10 into odd or even categories.	An even number has a ones digit of 0, 2, 4, 6 or 8. An odd number has a ones digit 1, 3, 5, 7 or 9.	Identify odd and even numbers based on the ones digit.		Continue to identify odd and even numbers based on the ones digit.

Patterns and Sequences Concept

This is an illustration of part of the Mathematics technical framework, specifically the concept of Patterns and Sequences. A detailed glossary will also be produced to support the understanding of key words and phrases.

End of First Level		Beginning of Second Level				End of Second Level	
Know	**Do**	**Know**	**Do**				
	Within the range 0 to 1000: Identify next number(s), missing numbers and errors in ascending and descending number sequences based on skip counting. For example, 7,14, ?, ?, 35, ...		**Within the range to 0 to 10 000:**		**Within the range 0 to 100 000:**		**Within the range -20 to 1 000 000:**
Number sequences can be generated based on addition or subtraction number patterns. Number sequences can be generated based on multiplicative number patterns.	Notice, discuss and identify the pattern within an ordered list and explain the rule. For example, the sequence of 72, 63, 54, 45 has the rule of subtracting 9 each time. When choral counting in multiples, record the count and explore the links and patterns. For example, notice the links between the multiples of 3 and 6. Recognise and extend a sequence of numbers by applying a multiplicative rule.		Continue to notice, discuss and identify the pattern within an ordered list and explain the rule. For example, the sequence of 800, 1700, 2600, 3500 has the rule of adding 900 each time. Identify the next number(s), missing numbers and errors in ascending and descending number sequences based on additive and multiplicative rules		Continue to notice, discuss and identify the pattern within an ordered list and explain the rule. For example, the sequence of 525, 2100, 8400 has the rule of multiplying by 4 each time. Continue to identify the next number(s), missing numbers and errors in ascending and descending number sequences based on additive and multiplicative rules		Notice, discuss and identify the pattern within an ordered list of integers and explain the rule. For example, the sequence of 23, 11, -1, -13 has the rule of subtracting 12. Identify the next number(s), missing numbers and errors in ascending and descending integer sequences based on additive rules
		A square number of items can be arranged in an array with an equal number of rows and columns:  A square number is found by multiplying a number by itself. This can be represented using a superscript 2 and is read as “squared”. For example, $4 \times 4 = 4^2$ is read as “four squared”.	Explain and demonstrate visually or with concrete materials what a square number is. Determine square numbers using arrays and known addition/multiplication facts. Explain and demonstrate visually or with concrete materials if a number is square or not. Calculate and record a range of square numbers. For example, 6 squared $6^2 = 36$	Triangular numbers describe the number of items that can be arranged in a triangle. The first row contains 1 item, the second row 2 items, the third row 3 items and so on: 	Explore and spot where triangular number patterns appear in real contexts. For example, snooker ball and bowling pin formations. Form triangular number patterns using physical objects. Explain and demonstrate visually or with concrete materials if a number is triangular or not. Explore strategies for calculating triangular numbers. Investigate classic mathematical problems involving triangular numbers. For example, Pascal’s Triangle, the handshake problem.	A cube number of items can be arranged as a cube.  A cubic number is found by multiplying an integer by itself twice. This can be represented using a superscript 3 and is read as ‘cubed’ For example, $4 \times 4 \times 4 = 4^3$ is read as ‘four cubed’. The next number in a Fibonacci sequence is found by adding together the previous two numbers in that sequence.	Determine cubic numbers using physical objects and addition/multiplication facts. Explain and demonstrate visually or with concrete materials if a number is cubic or not. Calculate and record a range of cubic numbers. Explore the history and application of Fibonacci sequences in the context of the natural world.

Patterns and Sequences Concept

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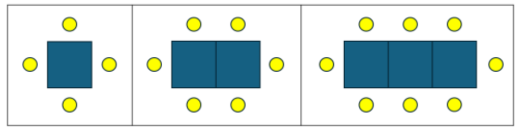
		End of Third Level		End of Fourth Level	
Know	**Do**	**Know**	**Do**	**Know**	**Do**
The numbers in a sequence are called terms. A sequence can be described by its first term and the rule for finding the next term.	Find the first few terms in a number sequence given its first term and the rule for finding the next term. For example, starting with a first term 10, use the rule add 6 to find the next 4 terms in this sequence.			A geometric sequence is one where the ratio of two consecutive terms is always the same. The next term of a geometric sequence is found by multiplying the previous term by this common ratio.	Identify whether (or not) a sequence is geometric by looking for a common ratio. Explore contexts where geometric sequences occur. For example, the rice and chessboard legend, compound interest, inflation. Continue these sequences to solve related problems.
An arithmetic sequence is one where the difference between two consecutive terms is always the same.	Identify whether or not a sequence is arithmetic by looking for a common difference. In context, identify patterns that follow an arithmetic sequence. For example, the number of people that can be seated around an increasing number of tables:  Continue these sequences to solve related problems.	A rule can describe how to find a number in a sequence using its position. This is called an n^{th} term formula.	Generate parts of a sequence using its n^{th} term formula, arithmetic sequences only. For example, $T = 4n + 6$. Explore the relationship between the n^{th} term formula of an arithmetic sequence and its common difference. For example, for $T = 4n + 6$, the sequence has a common difference of 4. Explore the relationship between the n^{th} term formula of an arithmetic sequence and the sequence of multiples of its common difference. For example, for $T = 4n + 6$, all the terms in the sequence are 6 greater than a multiple of 4.	The common difference in an arithmetic sequence appears in its n^{th} term formula. When terms in an arithmetic sequence are plotted against their position in that sequence, a straight line is formed with a gradient equal to the common difference. (Link with Patterns and Sequences in Geometry and Measure).	Establish an arithmetic sequence that a physical or pictorial pattern, or real-life situation follows. Determine its n^{th} term formula and use it algebraically or graphically to solve related problems.

Next steps:

The release of these documents will be supported by a national webinar hosted by the National Mathematics Specialist Advisor and Education Scotland.

A podcast and a LinkedIn article featuring members of the Core and Collaboration groups will also be released to support understanding of the documents as well as a link for providing feedback on the initial thinking from the mathematics groups.

Further general CIC updates can be found by following this link: - [CIC News Bulletin – Available Now – Curriculum Improvement Cycle]