GLOW *'Look, Capture and Create'*

Session 4 *Biodiversity and Beauty*

Further teachers' notes on Fibonacci numbers in daisy-like flowers

by Dr Neil Paterson Education Officer at the University of Dundee Botanic Garden

Here's a Fibonacci series again:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, …

Now let's take ratios dividing each number by its predecessor:

1/1 = 1.000

2/1 = 2.000

3/2 = 1.500

5/3 = 1.666,

8/5 = 1.600

13/8 = 1.625

21/13 = 1.615

34/21 = 1.619

55/34 = 1.617

89/55 = 1.618

The ratios are converging on to a number around 1.618. In fact, by generating bigger and bigger Fibonacci numbers and taking their ratios we can get as close as we like to what the ancient Greeks called the *golden number*:

$$\Phi = (1 + \surd 5)/2 = 1.618034$$

But what about the flowers?

The little flowers in the central disk (called "disk florets") of a sunflower head originate as lumps of cells called primordia which start at the growing tip and then move outwards as the head grows. The angle between one primordium and the next to grow is close to 137.5°. This is called the golden angle – we'll see how it connects to the golden number shortly.

Before we connect this angle with Fibonacci, consider what would happen if the primordia were inserted at an angle of 90°. We'd get this pattern:

```
                    x
                    x
                    x
                    x
x   x   x   x   x   o   x   x   x   x   x
                    x
                    x
                    x
                    x
```

The florets end up very far from close-packed in the disk.

Let's look again at the golden angle. An angle can be measured in two ways – internally, the golden angle is 137.5° but externally it is equivalent to 360° minus 137.5° which equals 222.5°. The ratio of 360/222.5 is 1.618, the golden number!

Back to Fibonacci. Take the ratio 5/8 which as a fraction of 360° gives us the angle 225°. If we insert primordia at this angle we get a head with 8 radial spokes. This is because the primordia's positions repeat after 8 rounds:

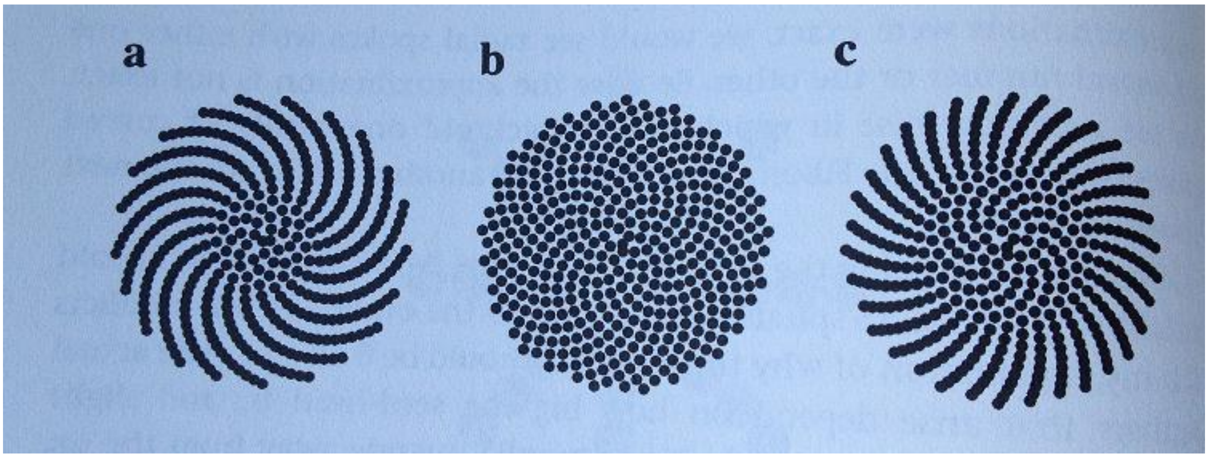
0, 5/8, 10/8 (=2/8), 15/8 (=7/8), 4/8, 1/8, 6/8, 3/8, 40/8 (=0)

If we use instead 8/13 (equivalent to 221.5°) we get a head with 13 radial spokes:

0, 8/13, 16/13 (=3/13), 11/13, 6/13, 1/13, 9/13, 4/13, 12/13, 7/13, 2/13, 10/13, 5/13, 13/13 (=0)

The spokes are again straight because eventually (after 13 rounds in this case) the primordial positions repeat exactly.

Notice that the 5/8 ratio gives an angle larger than the golden angle and 8/13 gives one smaller. If we use an angle just slightly less than the golden angle we get spirals bending to the left because the primordia never repeat *exactly* – there's drift as we keep going around (see a below). If we use an angle just slightly more than the golden angle we get spirals bending to the right (see c below). At the golden angle, what we see is the best close-packing of the little flowers and we can simultaneously see two sets of spirals, one bending to the left, the other to the right (see b below). If you count the spirals in this diagram you'll find that there are 21 in a and 34 in b, again, like 8 and 13, two consecutive Fibonacci numbers. Which numbers we actually get depend on factors such as how big the flower head is.



So we see Fibonacci numbers because ratios of Fibonacci numbers give close approximations to the golden angle which gives a close-packing solution to the problem of how to fill up the flower disk. This is compact and stronger than one made up of a few radial spokes. Natural Selection has found a good compromise which produces a near optimal solution to the problem.

[Note: a second best approximation to the golden angle, which also leads to good close-packing can be got from the "anomalous series" which is a Fibonacci starting with 4 and 7:

4, 7, 11, 18, 29, 47, …

These numbers also turn up in real flowers.]

Much of the above and the diagram come from Ian Stewart's book "The Magical Maze" (Weidenfeld and Nicholson 1997) which is a version of his Royal Institution Christmas Lectures aimed at children. The book has lots of ideas for teachers wanting to introduce connections between maths and the real world.