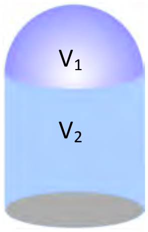
National 5 Learning Checklist – Expressions & Formulae

Topic	**Skills**	**Extra Study / Notes**	🟥	🟨	🟩
Rounding					
Round to decimal places	**e.g.** $25.1241 \rightarrow 25.1$ ***to 1 d.p.*** $34.676 \rightarrow 34.68$ ***to 2 d.p.***				
Round to Significant Figures	**e.g.** $1276 \rightarrow 1300$ ***to 2 sig. figs.*** $0.06356 \rightarrow 0.064$ ***to 2 sig. figs.*** $37{,}684 \rightarrow 37{,}700$ ***to 3 sig. figs.*** $0.005832 \rightarrow 0.00583$ ***to 3 sig. figs.***				
Surds					
Simplifying	Learn Square Numbers: 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169. Use square numbers as factors: **e.g.** $\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$				
Add/Subtract	**e.g.** $\sqrt{50} + \sqrt{8} = \sqrt{25} \times \sqrt{2} + \sqrt{4} \times \sqrt{2} = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$				
Multiply/Divide	**e.g.** $\sqrt{5} \times \sqrt{15} = \sqrt{5 \times 15} = \sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$ $\frac{\sqrt{48}}{\sqrt{3}} = \sqrt{\frac{48}{3}} = \sqrt{16} = 4$				
Rationalise Denominator	Remove surd from denominator. **e.g.** $\frac{1}{\sqrt{3}} = \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \frac{\sqrt{3}}{3}$				
Indices					
Use Laws of Indices	**1.** $a^x \times a^y = a^{x+y}$ **e.g.** $a^2 \times a^3 = a^{2+3} = a^5$ **2.** $a^x \div a^y = a^{x-y}$ $a^7 \div a^4 = a^{7-4} = a^3$ **3.** $(a^x)^y = a^{xy}$ $(a^4)^5 = a^{4\times5} = a^{20}$ **4.** $\frac{1}{a^x} = a^{-x}$ $\frac{1}{a^3} = a^{-3}$ **5.** $a^0 = 1$ $a^0 = 1$				
Scientific Notation / Standard Form	The first number is always between 1 and 10. **e.g.** $54{,}600 = 5.46 \times 10^4$ $0.000978 = 9.78 \times 10^{-4}$ $(1.3 \times 10^5) \times (8 \times 10^3) = 10.4 \times 10^8 = 1.04 \times 10^9$				
Evaluate using indices	**e.g.** $27^{\frac{2}{3}} = \sqrt[3]{27}^2 = 3^2 = 9$				
Algebra					
Expand Single Bracket	$3(x+4) = 3x + 12$				
Expand Two Brackets	Use **FOIL** (**F**irsts **O**utsides **I**nsides **L**asts) or another suitable method $(x+3)(x-2) = x^2 + 3x - 2x - 6 = x^2 + x - 6$				
	Know that every term in the first bracket must multiply every term in the second. **e.g.** $(x+2)(x^2 - 3x - 4) = x^3 - 3x^2 - 4x + 2x^2 - 6x - 8$ $= x^3 - x^2 - 10x - 8$				
Simplify Expression	Put together the terms that are the same: **e.g.** $x^2 + 4x + 3 - 2x + 8 = x^2 + 2x + 11$ $a \times a \times a = a^3$				
Factorise – Common Factor	Take the factors each term has in common outside the bracket: **e.g.** $4x^2 + 8x = 4x(x+2)$ **NB:** Always look for a common factor first.				

Algebra Contd.					
Factorise – Difference of Two Squares	Always takes the same form, one square number take away another. Easy to factorise: **e.g.** $x^2 - 9 = (x + 3)(x - 3)$ $5x^2 - 125 = 5(x^2 - 25)$ *(Common factor first)* $= 5(x + 5)(x - 5)$				
Factorise – Trinomial (simple)	Use any appropriate method to factorise: **e.g.** Opposite of FOIL: • Factors of first term are Firsts in brackets. • Lasts multiply to give last term and add to give middle term. $x^2 - x - 6 = (x - 3)(x + 2)$				
Factorise – Trinomial (hard)	This is more difficult. Use suitable method. Using opposite of FOIL above with trial and error. **NB:** The Outsides add Insides give a check of the correct answer: **e.g.** $3x^2 - 13x - 10$ = ~~$(3x - 5)(x + 2)$~~ *Check*: $3x \times 2 + (-5) \times x = 6x - 5x = -x$ ✗ $= (3x + 2)(x - 5)$ *Check*: $3x \times (-5) + 2 \times x = -15x + 2x = -13x$ ✓ If the answer is wrong, score out and try alterative factors or positions. Keep a note of the factors you have tried.				
Complete the Square	**e.g.** $x^2 + 8x - 13 = (x + 4)^2 - 13 - 16 = (x + 4)^2 - 29$				
Algebraic Fractions					
Simplifying Algebraic Fractions	**Step 1:** Factorise expression **Step 2:** Look for common factors. **Step 3:** Cancel and simplify $\frac{6x^2 - 12x}{x^2 + x - 6} = \frac{6x\cancel{(x-2)}}{(x+3)\cancel{(x-2)}} = \frac{6x}{x+3}$				
Add and Subract Fractions	Find a common denominator. This can be done either by working out the lowest common denominator, or by using Smile and Kiss $\frac{5a}{b} + \frac{3d}{2c} = \frac{10ac}{2bc} + \frac{12bd}{2bc} = \frac{10ac + 12bd}{2bc} = \frac{5ac + 6bd}{bc}$				
Multiply Fractions	Multiply top with top, bottom with bottom: $\frac{3a}{7c} \times \frac{4b}{5d} = \frac{12ab}{35cd}$				
Divide Fractions	Invert second fraction and multiply: $\frac{6x^2}{7y} \div \frac{4x}{3z} = \frac{6x^2}{7y} \times \frac{3z}{4x} = \frac{18x^2z}{28xy} = \frac{2xz}{14y}$				
Volumes					
Volume of a prism	V = Area of base x height				
Volume of a cylinder	$V = \pi r^2 h$				
Volume of a cone	$V = \frac{1}{3}\pi r^2 h$				
Volume of a sphere	$V = \frac{4}{3}\pi r^3$				
Rearrange each of the formulae to find an unknown	**e.g.** Cylinder has volume 400cm^3 and radius 6cm, find the height $V = \pi r^2 h$ $h = \frac{400}{\pi \times 6^2}$ $\frac{V}{\pi r^2} = h$ $h = \ldots$				

Volumes Contd.					
Volume of composite shapes	These are two of the above combined: Label them V_1 and V_2 **e.g.**  $V_1 = \frac{4}{3}\pi r^3 \div 2$ $V_1 = \dots$ $V_2 = \pi r^2 h$ $V_2 = \dots$				
Gradient					
Find the gradient of a line joining two points	Know that gradient is represented by the letter ***m*** **Step 1:** Select two coordinates **Step 2:** Label them (x_1, y_1) (x_2, y_2) **Step 3:** Substitute them into gradient formula **e.g.** $(\overset{x_1}{-4}, \overset{y_1}{4}), (\overset{x_2}{12}, \overset{y_2}{-28})$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-28) - 4}{12 - (-4)} = -\frac{-32}{16} = -2$				
Circles					
Length of Arc	This finds the length of the arc of a sector of a circle: $LOA = \frac{angle}{360} \times \pi d$ or $\frac{LOA}{\pi d} = \frac{angle}{360}$ For harder questions rearrange formula to find angle				
Area of Sector	$AOS = \frac{angle}{360} \times \pi r^2$ or $\frac{AOS}{\pi r^2} = \frac{angle}{360}$ For harder questions rearrange formula to find angle				

National 5 Learning Checklist - Relationships

Topic	**Skills**	**Extra Study / Notes**	🟥	🟨	🟩
Straight Line					
Gradient	• Represented by *m* • Measure of steepness of slope • Positive gradient – the line is increasing • Negative gradient – the line is decreasing				
Y-intercept	• Represented by *c* • Shows where the line cuts the y-axis • Find by making x = 0				
Find the gradient of a line joining two points	Know that gradient is represented by the letter ***m*** **Step 1:** Select two coordinates **Step 2:** Label them (x_1, y_1) (x_2, y_2) **Step 3:** Substitute them into gradient formula **e.g.** $(\overset{x_1}{-4}, \overset{y_1}{4}), (\overset{x_2}{12}, \overset{y_2}{-28})$ $m=\frac{y_2-y_1}{x_2-x_1}=\frac{(-28)-4}{12-(-4)}=\frac{-32}{16}=-2$				
Find equation of a line (from gradient and y-intercept)	**Step 1:** Find gradient *m* **Step 2:** Find y-intercept *c* **Step 3:** Substitute into $y = mx + c$ (see above for definitions)				
Find equation of a line (from two points)	Use this when there are only two points (i.e. no y-intercept) **Step 1:** Find gradient **Step 2:** Substitue into $y - b = m(x - a)$ where (a, b) are taken from either one of the points				
Rearrange equation to find gradient and y-intercept	**e.g.** $3y + 6x = 12$ $3y = -6x + 12$ $y = -2x + 4$ $m = -2$, $c = 4$				
Sketch lines from their equations	**Step 1:** Rearrange equation to the form $y = mx + c$ (see note above) **Step 2:** Draw a table of points **Step 3:** Plot points on coordinate axes				
Solving Equations / Inequations					
Solving Equations	Use suitable method: **e.g.** $5(x + 4) = 2(x – 5)$ $5x + 20 = 2x – 10$ $5x = 2x – 30$ $3x = 30$ $x = 10$				
Solving inequations	Solve the same way as equations. **NB:** When dividing by a negative change the sign: **e.g.** $-3x \leq 15$ $x \geq -5$				
Simultaneous Equations					
Solve by sketching lines	**Step 1:** Rearrange lines to form $y = mx + c$ **Step 2:** Sketch lines using table of points (as above) **Step 3:** Find coordinate of point of intersection				
Solve by substitution	This works when one or both equations are of the form $y = ax + b$ **e.g.** Solve $3x + 2y = 17$ ① $y = x + 1$ ② Sub equation 2 into 1: $3x + 2(x + 1) = 17$ $5x + 2 = 17$ $x = 3$ so $y = 3 + 1 = 4$				

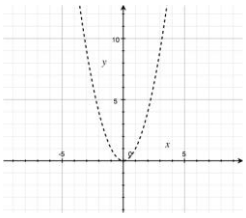
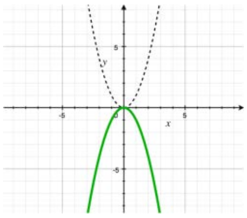
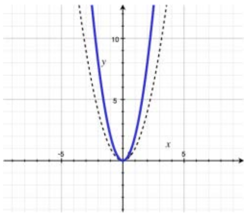
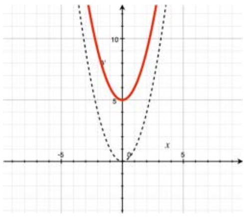
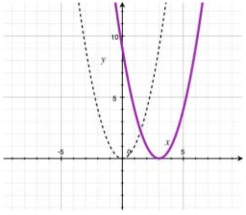
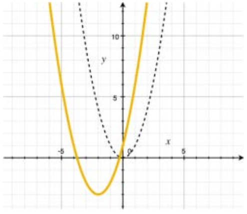
Simultaneous Equations Contd.

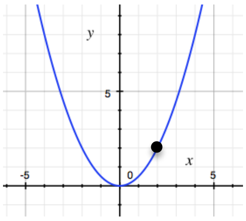
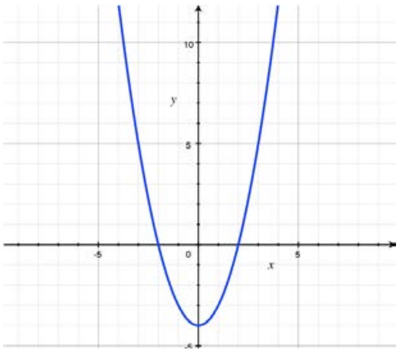
Solve by Elimination	**Step 1:** Scale equations to make one unknown equal with opposite sign. **Step 2:** Add Equations to eliminate equal term and solve. **Step 3:** Substitute number to find second term. **e.g.** $4a + 3b = 7$ ① $2a - 2b = -14$ ② ① x 2 $8a + 6b = 14$ ③ ② x 3 $6a - 6b = -42$ ④ ③ + ④ $14a = -28$ $a = -2$ substitute $a = -2$ into ① $4(-2) + 3b = 7$ $3b = 15$ $b = 5$ Ans. $a = -2, b = 5$				
Form Equations	Form equations from a variety of contexts to solve for unknowns				

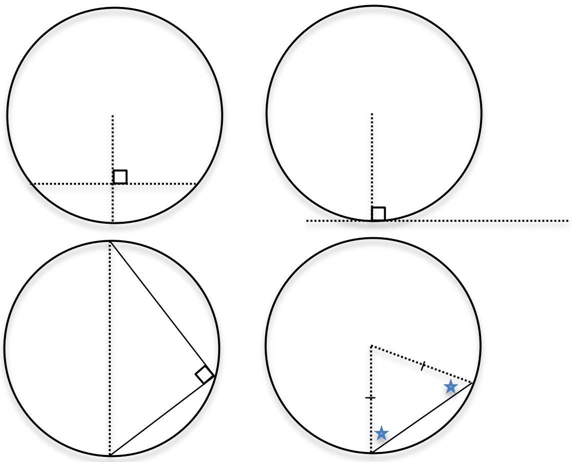
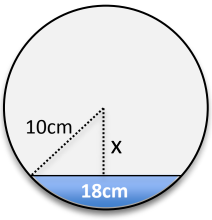
Change the Subject

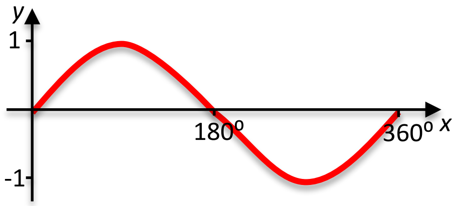
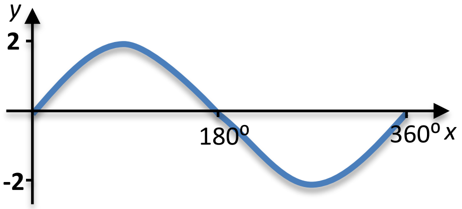
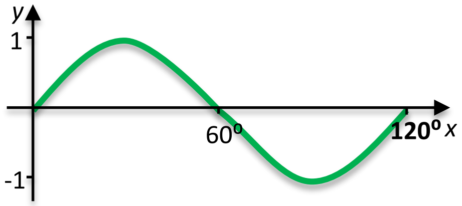
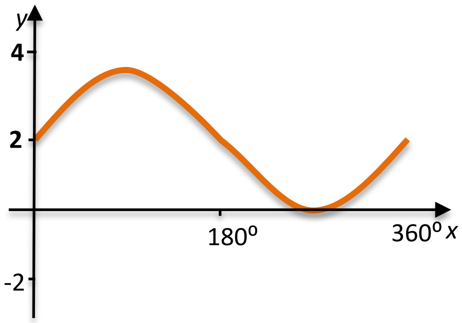
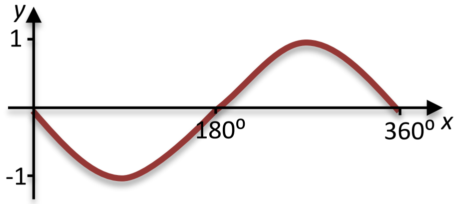
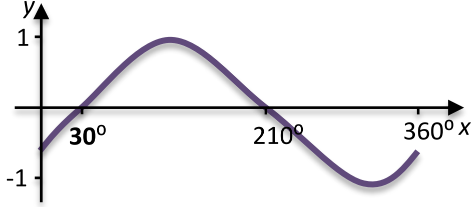
Linear Equations	Rearrange equations change the subject: **e.g.** $D = 4C - 3$ [**C**] $D + 3 = 4C$ $\frac{D+3}{4} = C$ $C = \frac{D+3}{4}$ $y = 5(z + 6)$ [**z**] $\frac{y}{5} = z + 6$ $\frac{y}{5} - 6 = z$ $z = \frac{y}{5} - 6$				
Equations with powers or roots	**e.g.** $V = \pi r^2 h$ [**r**] $\frac{V}{\pi h} = r^2$ $r = \sqrt{\frac{V}{\pi h}}$				

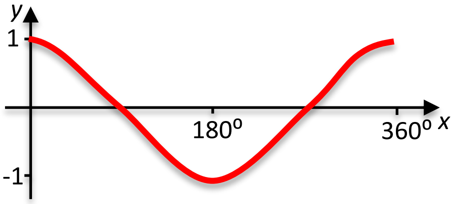
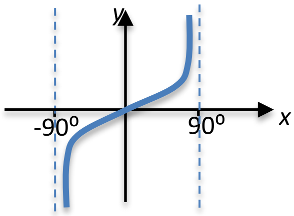
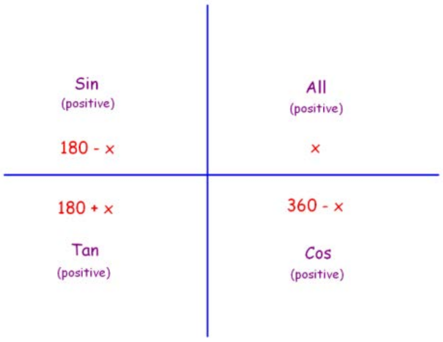
Quadratic Functions

Quadratics and their equations	$y = x^2$  $y = -x^2$  $y = 2x^2$  $y = x^2 + 5$  $y = (x - 3)^2$  $y = (x + 2)^2 - 3$ 				

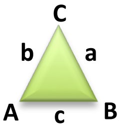
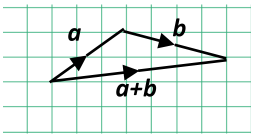
Equations of quadratics $y = kx^2$	**Step 1:** Identify coordinate from graph **Step 2:** Substitute into $y = kx^2$ **Step 3:** Solve to find k **e.g.** **Coordinate:** (2, 2) **Substitution:** $2 = k(2)^2$ $2 = 4k$ $k = 0.5$ **Quadratic:** $y = 0.5x^2$ 				
Sketching Quadratics $y = k(x + a)^2 + b$	**Step 1:** Identify shape, if k = 1 then graph is +ve or if k = -1 then the graph is -ve **Step 2:** Identify turning point (-a, b) **Step 3:** Sketch axis of symmetry x = -a **Step 5:** Find y-intercept (make x = 0) **Step 4:** Sketch information				
Sketching Quadratics (Harder) $y = (x + a)(x – b)$	**Step 1:** Identify shape (+ve or -ve) **Step 2:** Identify roots (x-intercepts) x = -a, x = b **Step 3:** Find y-intercept (make x = 0) **Step 4:** Identify turning point **e.g.** $y = (x + 4)(x – 2)$ +ve graph ∴ Minimum turning point Roots: x = 2, x = -4 y-intercept: $y = (0 + 4)(0 – 2) = -8$ Turning Point (-1, -9) (see below) **NB:** Turning point is halfway between roots. x-coord = $(2 + (-4)) \div 2 = -1$ y-coord = $(-1 + 4)(-1 – 2) = -9$				
Solving Quadratics (finding roots) – Algebraically	**Step 1:** Factorise quadratic **Step 2:** Set each factor equal to zero **Step 3:** Solve each factor to find roots **e.g.** $y = x^2 + 4x$ $x(x + 4) = 0$ $x = 0$ or $x + 4 = 0$ $x = 0$ or $x = -4$ $y = x^2 – 5x – 6$ $(x – 6)(x + 1) = 0$ $x – 6 = 0$ or $x + 1 = 0$ $x = 6, x = -1$				
Solving Quadratics (finding roots) – Graphically	Read roots from graph  x = 2, x = -2				
Solving Quadratics – Quadratic Formula	When asked to solve a quadratic to a number of decimal places use the formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $y = ax^2 + bx + c$				

	e.g. Solve y = $x^2 – 6x + 2$ to 1 d.p. a = 1 b = -6 c = 2 $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4\times1\times2}}{2\times1}$ $x = \frac{6\pm\sqrt{28}}{2}$ $x = \frac{6+\sqrt{28}}{2}$ $x = \frac{6-\sqrt{28}}{2}$ $x = 5.6$ $x = 0.4$				
Discriminant	**$b^2 – 4ac$** where $y = ax^2 + bx + c$ The discriminant describes the nature of the roots $b^2 – 4ac > 0$ two real roots $b^2 – 4ac = 0$ equal roots (tangent to axis) $b^2 – 4ac < 0$				
Using the Discriminant	**Example 1:** Determine the nature of the roots of the quadratic $y = x^2 + 5x + 4$ **Solution:** a = 1 , b = 5, c = 4 $b^2 – 4ac = 5^2 – 4 \times 1 \times 4 = 25 – 16 = 9$ Since $b^2 – 4ac > 0$ the quadratic has two real roots. **Example 2:** Determine p, where $x^2 + 8x + p$ has equal roots **Solution:** $b^2 – 4ac = 0$ $8^2 – 4 \times 1 \times p = 0$ $64 – 4p = 0$ $64 = 4p$ $P = 16$				
Properties of Shapes					
Circles					
Pythagoras	Use Pythagoras Theorem to solve problems involving circles and 3D shapes. **e.g.** Find the depth of water in a pipe of radius 10cm.  *r* is the radius $x^2 = 10^2 - 9^2$ $x^2 = \ldots$ $x = 4.4$cm Depth = 10 – 4.4 = 5.6cm				
Similar Shapes					
Linear Scale Factor	$\text{Linear.Scale.Factor} = \frac{\text{New.Length}}{\text{Original.Length}}$				

Area Scale Factor	$\text{Area.Scale.Factor} = \left(\frac{\text{New.Length}}{\text{Original.Length}}\right)^2$				
Volume Scale Factor	$\text{Volume.Scale.Factor} = \left(\frac{\text{New.Length}}{\text{Original.Length}}\right)^3$				
Trigonometry					
Trig Graphs – Sine Curve	**y = *a*sin *b*x + *c*** ***a*** = maxima and minima of graph ***b*** = no. of waves between 0 and 360° ***c*** = movement of graph vertically				
	y = sin *x* maxima and minima 1 and -1, period = 360°  **y = 2sin x**  **y = sin 3x**  **y = 2sin x + 2**  **y = -sin x**  **y = sin (x - 30°)** 				

Trig Graphs – Cosine Curve	**$y = a\cos bx + c$** ***a*** = maxima and minima of graph ***b*** = no. of waves between 0 and 360^{o} ***c*** = movement of graph vertically				
	y = cos x maxima and minima 1 and -1, period = 360^{o}  The same transformations apply for Cosine as Sine (above)				
Trig Graphs – Tan Curve	**y = tan x** no maxima or minima, period = 180^{o} 				
Solving Trig Equations	Know the CAST diagram  **Memory Aid:** **A**ll **S**tudents **T**ake **C**are				
	Use the diagram above to solve trig equations: **Example 1:** Solve $2\sin x – 1 = 0$ $2\sin x = 1$ $\sin x = ½$ $x = \sin^{-1}(½)$ $x = 30^{\circ}, 180^{\circ} - 30^{\circ}$ $x = 30^{\circ}, 150^{\circ}$ **Example 2:** Solve $4\tan x + 5 = 0$ $4\tan x = -5$ $\tan x = -5/4$ **NB:** tan x is negative so there will be solutions in the second and fourth quadrant $x = = \tan^{-1}(5/4)$ $x_{acute} = 51.3^{\circ}$ $x = 180^{\circ} - 51.3^{\circ}, 360 – 51.3^{\circ}$ $x = 128.7^{\circ}, 308.7^{\circ}$				
Trig Identities	Know: $\sin^2 x + \cos^2 x = 1$ $\therefore \sin^2 x = 1 - \cos^2 x$ *and* $\cos^2 x = 1 - \sin^2 x$ and $\tan x = \frac{\sin x}{\cos x}$				
	Use the above facts to show one trig function can be another. Start with the left hand side of the identity and work through until it is equal to the right hand side.				

National 5 Learning Checklist - Applications

Topic	**Skills**	**Extra Study / Notes**	🟥	🟨	🟩
Trigonometry					
Triangle	Label Triangle 				
Area of a Triangle	$A = \frac{1}{2}ab\sin C$				
Sine Rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$				
	Use Sine Rule to find a side				
	Use Sine Rule to find an angle. **NB:** $\sin A = \ldots$ $A = \sin^{-1}(\ldots)$				
Coosine Rule	Use $a^2 = b^2 + c^2 - 2bc\cos A$ to find a side				
	Use $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ to find an angle **NB:** $\cos A = \ldots$ $A = \cos^{-1}(\ldots)$				
Bearings	Use knowledge of bearings to solve trig problems. Including knowledge of Corresponding, Alternate and Supplementary angles.				
Vectors					
2D Line Segments	Add or subtract 2D line Segments • Vectors end-to-end • Arrows in same direction 				
3D Vectors	Determine coordinates of a point from a diagram representing a 3D object				
Vector Components	Add and Subtract 2D and 3D vector components. $\mathbf{a} = \begin{pmatrix}1\\1\\4\end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix}3\\2\\5\end{pmatrix}$ $\mathbf{a} + \mathbf{b} = \begin{pmatrix}1+3\\1+2\\4+5\end{pmatrix}$				
	Multiply vector components by a scalar $2\mathbf{a} = 2\begin{pmatrix}1\\1\\4\end{pmatrix} = \begin{pmatrix}2\\2\\8\end{pmatrix}$				
	Find the magnitude of a 2D or 3D vector: For vector $\begin{pmatrix}x\\y\end{pmatrix}$ magnitude = $\sqrt{x^2 + y^2}$ For vector $\begin{pmatrix}x\\y\\z\end{pmatrix}$ magnitude = $\sqrt{x^2 + y^2 + z^2}$				
Percentages					
Compound Interest	Calculate multiplier from percentage: e.g. 5% increase **100% + 5% = 105% = 1.05**				
	Use multiplier to calculate compound interest / depreciation. e.g. £500 with 5% interest for 3 years **1.05^3 x 500**				

Percentages Contd.					
Percentage increase/decrease	% Increase/decrease = $\frac{\textit{difference}}{\textit{original}}\times 100$				
Reverse the Change	Find initial amount. e.g. Watch reduced by 30% to £42. **70% = £42, 1% = £0.60, 100% = £60** or **42 ÷ 0.7 = £60**				
Fractions					
Add and Subract Fractions	Find a common denominator $\frac{2}{3}+\frac{4}{5}=\frac{10}{15}+\frac{12}{15}$				
Add and Subract Mixed Numbers	Add or subtract whole numbers, or make an improper fraction: $2\frac{2}{3}+3\frac{4}{5}=5\frac{10}{15}+\frac{12}{15}$ or $2\frac{2}{3}+3\frac{4}{5}=\frac{8}{3}+\frac{19}{5}$				
Multiply Fractions	Multiply top with top, bottom with bottom: $\frac{3}{7}\times\frac{4}{5}=\frac{12}{35}$				
Multiply Mixed Numbers	Make top heavy fraction then as above: $3\frac{3}{7}\times\frac{4}{5}=\frac{23}{7}\times\frac{4}{5}=\frac{92}{35}$				
Divide Fractions	Invert second fraction and multiply: $\frac{6}{7}\div\frac{2}{3}=\frac{6}{7}\times\frac{3}{2}=\frac{18}{14}=\frac{9}{7}$				
Statistics					
Comparing data	Calculate the mean: $\bar{x}=\frac{sum\cdot of\cdot data}{number\cdot of\cdot terms}$				
	Find five figure summary: L = lowest term, Q1 = lower quartile, Q2 = Median, Q3 = upper quartile, h = highest term				
	Interquartile range: IQR = Q3 – Q1 middle 50% of data				
	Semi-Interquartile range: SIQR = $\frac{Q3-Q1}{2}$				
	Calculate Standard Deviation: $s=\sqrt{\frac{\Sigma x^2-\frac{(\Sigma x)^2}{n}}{n-1}}$ or $s=\sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$				
	Know that IQR, SIQR and standard deviation are a measure of the *spread* of data. Lower value means more *consistent* data.				
Line of Best Fit	Use knowledge of straight line to find the equation of a line of best fit: **y = mx + c** or **y – b = m(x – a)**				
	Use equation of line of best fit to find estimate for new value. Usually do so by substituting value for *x* into equation.				
	Draw best fitting line: • In line with direction of points • Roughly the same number of points above and below line.				