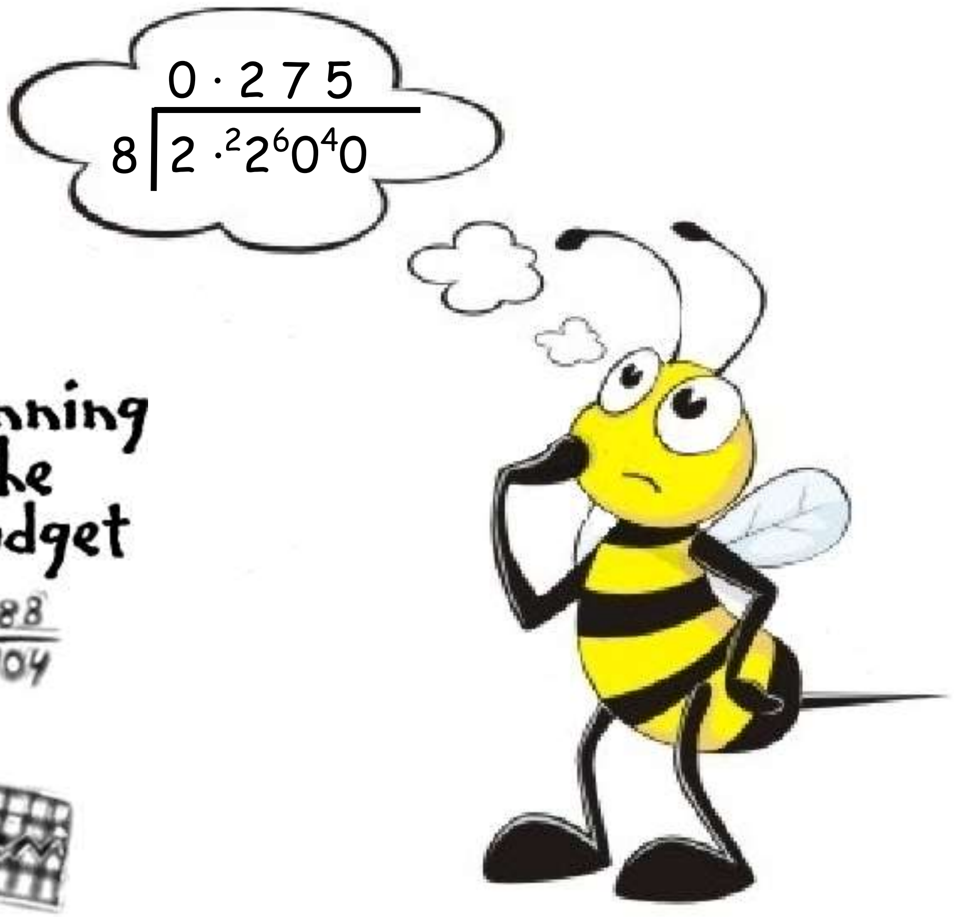
Shawlands Academy

Numeracy Booklet

A guide for pupils, parents and staff



Introduction

What is the purpose of the booklet?

This booklet has been produced to give guidance to pupils, parents and staff on how certain common Numeracy topics are taught in mathematics and throughout the school. Staff from all departments have been consulted during its production and will be issued with a copy of the booklet. It is hoped that using a consistent approach across all subjects will make it easier for pupils to progress.

How can it be used?

If you are helping your child with their homework, you can refer to the booklet to see what methods are being taught in school. Look up the relevant page for a step by step guide. Pupils have been issued with their own copy and can use the booklet in school to help them solve number and information handling questions in any subject.

The booklet includes the Numeracy skills useful in subjects other than mathematics.

Why do some topics include more than one method?

In some cases , for example percentages, the method used will be dependent on the level of difficulty of the question, and whether or not a calculator is permitted.

For mental calculations, pupils should be encouraged to develop a variety of strategies so that they can select the most appropriate method in any given situation.

For more information and a detailed description of the numeracy outcomes visit http://www.educationscotland.gov.uk

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Estimation & Rounding MNU 2-01a

Numbers can be rounded to give an approximation.

When rounding numbers which are exactly in the middle, convention is to **round up**. 78**6**5 rounded to the nearest 10 is 78**7**0.

The same principle applies to rounding decimal numbers.

In general, to round a number, we must first identify the place value to which we want to round. Then look at the next digit, the check digit - if it is 5 or more round up and if it is below 5 round down.

Example 1 Round 46 753 to the nearest thousand.

6 is the digit in the thousands column - the check digit, in the hundreds column is a 7, so round up.

4<u>6</u> 753
= <u>47 000</u> to the nearest thousand

Example 2 Round 1·57359 to 2 decimal places

The second number after the decimal point is a 7 - the check digit is a 3, so round down.

1·5<u>7</u>359
= <u>1·57</u> to 2 decimal places

Estimation MNU 3-01a

We can use rounded numbers to give us an approximate answer to a calculation. This allows us to check that our answer is sensible.

Example 1 Tickets for a concert were sold over 4 days.
The number of tickets sold each day was recorded in the table below. How many tickets were sold in total ?

Monday	Tuesday	Wednesday	Thursday
486	205	197	321

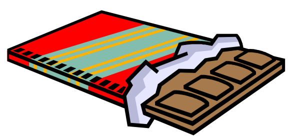
Estimate
500 + 200 + 200 + 300 = 1200

Calculate

$$\begin{array}{r} 486 \\ 205 \\ 197 \\ +\ \underline{321} \\ \underline{1209} \end{array}$$

Answer = 1209 tickets

Example 2 A bar of chocolate weighs 42g.



There are 30 bars of chocolate in a box.
What is the total weight of chocolate in the box?

Estimate
40 x 30 = 1200g

Calculate:

$$\begin{array}{r} 42 \\ \underline{\times\ 3} \\ \underline{126} \end{array} \rightarrow 126 \times 10 = 1260$$

Answer = 1260g

Number Processes MNU 2-02a

A decimal fraction can be used to write down the value of a part of a number. For example:

H	T	U	·	t	h	th	
2	4	1	·	**3**			The "3" means 3 tenths or $\frac{3}{10}$
	8	4	·	0	**5**		The "5" means 5 hundredths or $\frac{5}{100}$
1	0	6	·	2	9	**8**	The "8" means 8 thousandths or $\frac{8}{1000}$

These column headings help us when we carry out multiplication or division by 10 and 100.

H	T	U	·	t	h	th		**Examples**
		7	·	2	1			7·21 x 10
	7	2	·	1			=	<u>72·1</u>
5	2	0	·	8				520·8 ÷ 100
		5	·	2	0	8	=	<u>5·208</u>

Remember:

x 10	Numbers move one place to the right
x 100	Numbers move two places to the right
÷ 10	Numbers move one place to the left
÷ 100	Numbers move two places to the left

Addition MNU 2-03a

Mental strategies

There are a number of useful mental strategies for addition.
Some examples are given below.

Example Calculate 54 + 27

Method 1 Add tens, then add units, then add together.

50 + 20 = 70   4 + 7 = 11   70 + 11 = <u>81</u>

Method 2 Split up number to be added into tens and units and add separately.

54 + 20 = 74   74 + 7 = <u>81</u>

Method 3 Round up to nearest 10, then subtract.

54 + 30 = 84   but 30 is 3 too much so subtract 3

84 - 3 = <u>81</u>

Written Method

When adding numbers, ensure that the numbers are lined up according to place value. Start at right hand side, write down the units and carry the tens.

Example Add 3032 and 589

3032		3032		3032		3032
+589		+589		+589		+589
1	➔	21	➔	621	➔	**3621**
1		1 1		1 1		1 1
2 + 9 = 11		3 + 8 + 1 = 12		0 + 5 + 1 = 6		3 + 0 = 3

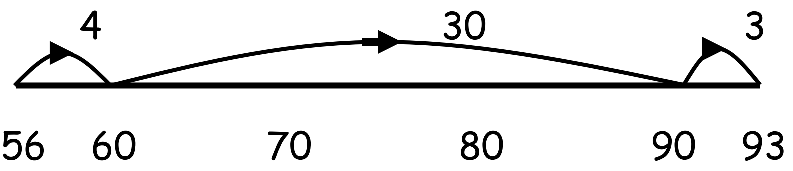
Subtraction MNU 2-03a Mental Strategies

We use decomposition as a written method for subtraction (see below). Alternative methods may be used for mental calculations.

Example Calculate 93 - 56

Method 1 Count on

Count on from 56 until you reach 93. This can be done in several ways

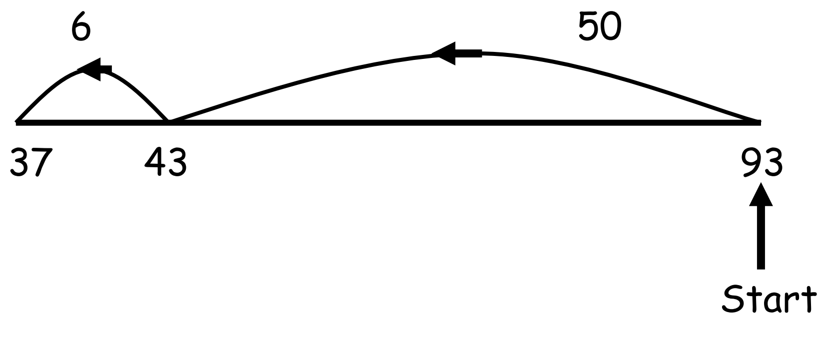


4 + 30 + 3 = <u>37</u>

Method 2 Break up the number being subtracted

Subtract 50, then subtract 6

93 - 50 = 43
43 - 6 = <u>37</u>



Written Method

Example 1 4590 – 386

$$\begin{array}{r} 45\overset{8}{\cancel{9}}{}^{1}0 \\ \underline{-\ 386} \\ \underline{\mathbf{4204}} \end{array}$$

We do not "borrow and pay back".

Example 2 Subtract 692 from 14597

$$\begin{array}{r} 1\overset{3}{\cancel{4}}{}^{1}597 \\ \underline{-\ \ 692} \\ \underline{\mathbf{13905}} \end{array}$$

Multiplication 1 MNU 2-03a

It is essential that you know all of the multiplication tables from 1 to 10.
These are shown in the tables square below.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Mental Strategies

Example Find 39 x 6

Method 1

30 x 6
= 180

9 x 6
= 54

180 + 54
= <u>234</u>

Method 2

40 x 6
= 240

40 is 1
too many

240 - 6
= <u>234</u>

Multiplication 2 MNU 2-03a / 2-03b

Multiplying by multiples of 10 and 100

To multiply by **10** you move every digit **one** place to the left.
To multiply by **100** you move every digit **two** places to the left.

Examples

1) (a) 354 x 10

= <u>3540</u>

1) (b) 50·6 x 100

= <u>5060</u>

TH	H	T	U	·	t	h	th
	3	5	4				
3	5	4	0				
		5	0	·	6		
5	0	6	0	·			

1) (c) **35 x 30**

To multiply by 30,
Multiply by 3, then by 10.
35 x 3 = 105
105 x 10 = 1050
so 35 x 30 = <u>1050</u>

1) (d) **436 x 600**

To multiply by 600,
Multiply by 6, then by 100.
436 x 6 = 2616
2616 x 100 = 261600
so 436 x 600 = <u>261600</u>

We may also use these rules for multiplying decimal numbers.

2) (a) **2·36 x 20**

2·36 x 2 = 4·72
4·72 x 10 = 47·2
so 2·36 x 20 = <u>47·2</u>

2)(b) 38·4 x 50

38·4 x 5 = 192·0
192·0 x 10 = 1920
so 38·4 x 50 = <u>1920</u>

Division MNU 2-03a / 2-03b

You should be able to divide by a single digit or by a multiple of 10 or 100 without a calculator.

Written Method

Example 1 There are 192 pupils in first year, shared equally between 8 classes. How many pupils are in each class?

$$\begin{array}{r} 0\;2\;4 \\ 8\overline{)\,1\;{}^{1}9\;{}^{3}2} \end{array}$$

There are 24 pupils in each class

Example 2 Divide 4·74 by 3

$$\begin{array}{r} 1\cdot 5\;8 \\ 3\overline{)\,4\cdot{}^{1}7\;{}^{2}4} \end{array}$$

When dividing a decimal number by a whole number, the decimal points must stay in line.

Example 3 A jug contains 2·2 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

$$\begin{array}{r} 0\cdot 2\;7\;5 \\ 8\overline{)\,2\cdot{}^{2}2\;{}^{6}0\;{}^{4}0} \end{array}$$

If you have a remainder at the end of a calculation, add a zero onto the end of the decimal and continue with the calculation.

Each glass contains 0·275 litres

Order of Calculation (BODMAS) MNU 2-03c

What is the answer to $2 + 5 \times 8$?

Is it $7 \times 8 = 56$ or $2 + 40 = 42$?

The correct answer is <u>42</u>.

Calculations which have more than one operation need to be done in a particular order. The order can be remembered by using the mnemonic **BODMAS**

The **BODMAS** rule tells us which operations should be done first.

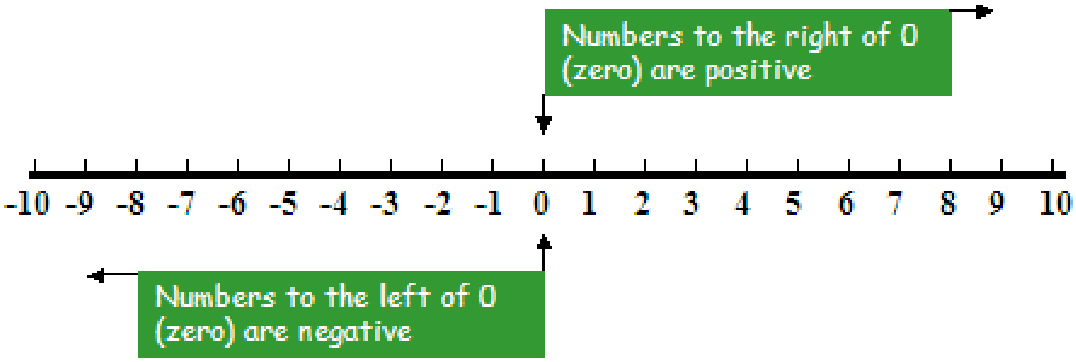
Scientific calculators use this rule, some basic calculators may not, so take care in their use.

BODMAS represents:

(B)rackets
(O)f
(D)ivide
(M)ultiply
(A)dd
(S)ubract

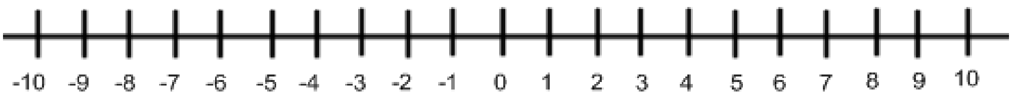
Example 1	$15 - \mathbf{12 \div 6}$	BODMAS tells us to divide first
	$= 15 - 2$	
	$= \underline{13}$	
Example 2	$\mathbf{(9 + 5)} \times 6$	BODMAS tells us to work out the
	$= 14 \times 6$	brackets first
	$= \underline{84}$	
Example 3	$18 + 6 \div \mathbf{(5 - 2)}$	Brackets first
	$= 18 + \mathbf{6 \div 3}$	Then divide
	$= 18 + 2$	Now add
	$= \underline{20}$	

Negative Numbers MNU 2-04a



A thermometer is the most obvious place to see negative numbers but we also use them for money and to describe depths.

To order negative numbers start with the **lowest value.**
You can place them on a number line like the one below.



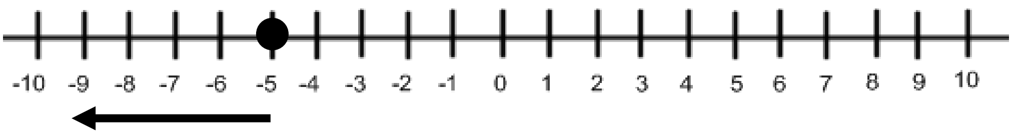
Example 1

Write these in order from lowest to highest: -6, 4, -8, 0, 1, -5, 3, 7

Lowest ⇨ Highest : -8, -6, -5, 0, 1, 3, 4, 7

Example 2

One winter's day in Glasgow the temperature was -5°C.
In Aberdeen it was 4°C colder. What was the temperature in Aberdeen?



Temperature in Aberdeen = <u>-9°C</u>.

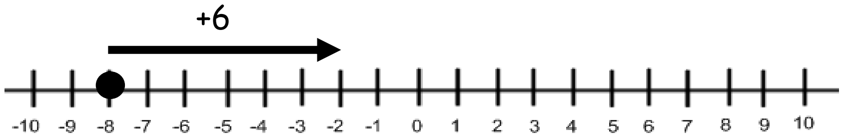
Negative Numbers MNU 3-04a

Adding and subtracting

Subtracting a negative is the same as adding a positive.

Example 1 $-8 + 6$

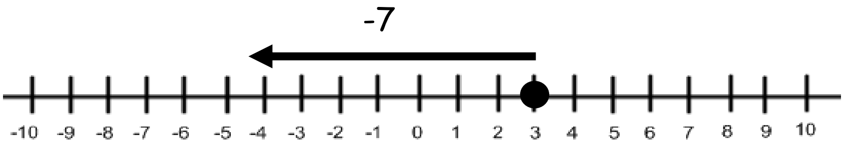
$= \underline{-2}$



Example 2 $3 + (-7)$

$= 3 - 7$

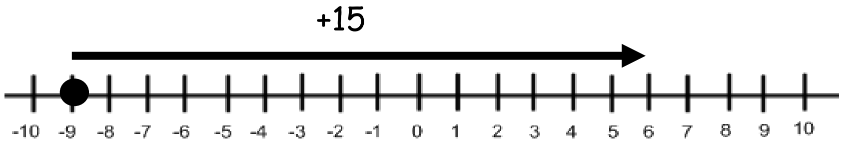
$= \underline{-4}$



Example 3 $-9 - (-15)$

$= -9 + 15$

$= \underline{6}$



Multiplying and dividing

The same rules apply for dividing

Rules

+	×	+	=	+
+	×	-	=	-
-	×	+	=	-
-	×	-	=	+

Example 1 $3 \times (-5)$

$= \underline{-15}$

Example 2 $(-9) \times 8$

$= \underline{-72}$

Example 3 $(35) \div (-7)$

$= \underline{-5}$

Example 4 $(-54) \div (-6)$

$= \underline{9}$

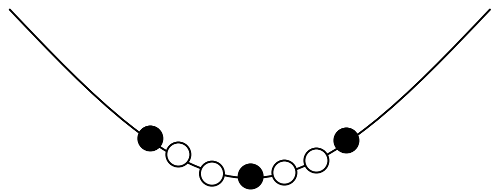
Fractions 1 MNU 2-07a

Understanding Fractions

Addition, subtraction, multiplication and division of fractions are studied in mathematics. However, the examples below may be helpful in all subjects.

Example

A necklace is made from black and white beads.



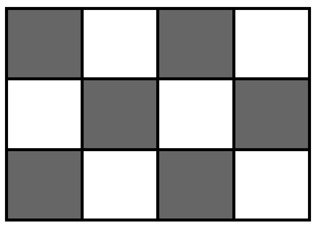
What fraction of the beads are black?

There are 3 black beads out of a total of 7, so $\frac{3}{7}$ of the beads are black.

Equivalent Fractions

Example

What fraction of the flag is shaded?



6 out of 12 squares are shaded. So $\frac{6}{12}$ of the flag is shaded.

It could also be said that $\frac{1}{2}$ the flag is shaded.

$\frac{6}{12}$ and $\frac{1}{2}$ are **equivalent fractions**.

Fractions 2 MNU 2-07a

Simplifying Fractions

The top of a fraction is called the **numerator**.
The bottom is called the **denominator**.
To simplify a fraction, divide the **numerator** and **denominator** by the same number.

Example 1

(a) $\frac{20}{25} \overset{\div 5}{\underset{\div 5}{=}} \frac{4}{5}$

(b) $\frac{16}{24} \overset{\div 8}{\underset{\div 8}{=}} \frac{2}{3}$

We can keep doing this until the numerator and denominator cannot be divided any further. The fraction is then said to be in its **simplest form**.

Example 2 Simplify $\frac{72}{84}$

$\frac{72}{84} \overset{\div 2}{\underset{\div 2}{=}} \frac{36}{42} \overset{\div 2}{\underset{\div 2}{=}} \frac{18}{21} \overset{\div 3}{\underset{\div 3}{=}} \frac{6}{7}$ (simplest form)

Calculating Fractions of a Quantity

To find the fraction of a quantity, divide by the denominator.
To find $\frac{1}{2}$ divide by 2, to find $\frac{1}{3}$ divide by 3, to find $\frac{1}{7}$ divide by 7...

Example 1 Find $\frac{1}{5}$ of £150

$\frac{1}{5}$ of £150 = £150 ÷ 5 = <u>£30</u>

Example 2 Find $\frac{3}{4}$ of 48

$\frac{1}{4}$ of 48 = 48 ÷ 4 = 12

$\frac{3}{4}$ of 48 = 3 x 12 = <u>36</u>

To find $\frac{3}{4}$ find $\frac{1}{4}$ then multiply by 3.

Fractions 3 MNU 3-07a

Adding and subtracting

Always remember to add the top and not the bottom.

Example 1 $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

Example 2 $\frac{8}{9} - \frac{5}{9} = \frac{3}{9} = \mathbf{\frac{1}{3}}$

Simplify

If the bottom numbers are different, we must find a **common denominator**.

Example 3

$$\frac{3}{8} + \frac{1}{6}$$

$$\times 3 \qquad \times 4$$

$$= \frac{?}{24} + \frac{?}{24}$$

$$= \frac{9}{24} + \frac{4}{24}$$

$$= \frac{13}{24}$$

We need to find the lowest common multiple of 6 and 8. Find the lowest number in both of these times tables.

Mixed numbers and top-heavy (improper) fractions

$4\frac{1}{2}$ is a mixed number. $\frac{23}{4}$ is a top heavy fraction. It is useful to be able to change between mixed numbers and top-heavy fractions.

Example 1

Change $3\frac{7}{8}$ into a top-heavy fraction

$3 = \frac{24}{8}$ so $3\frac{7}{8} = \frac{24+7}{8} = \frac{31}{8}$

Example 2

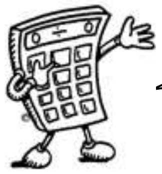
Change $\frac{44}{7}$ into a mixed number.

How many times does 7 go into 44?

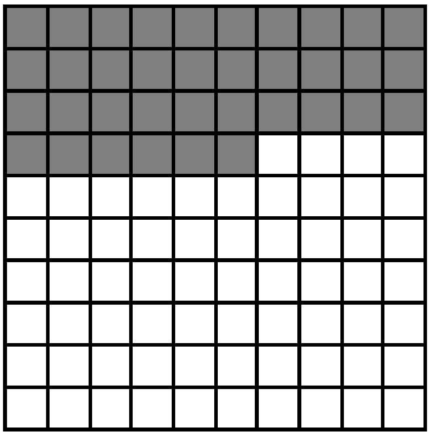
6 times with a remainder of 2

So $\frac{44}{7} = 6\frac{2}{7}$

Percentages MNU 2-07b



Percent means out of 100. A percentage can be converted to an equivalent fraction or decimal.



36% means $\frac{36}{100} \underset{\div 4}{\overset{\div 4}{=}} \frac{9}{25}$ and

36% means $\frac{36}{100}$ = 36 ÷ 100 = 0·36

Therefore 36% = $\frac{9}{25}$ = 0.36

Common Percentages

Some percentages are used very frequently.

It is very useful to know these as fractions and decimals.

Percentage	Fraction	Decimal Fraction
1%	$\frac{1}{100}$	0·01
10%	$\frac{1}{10}$	0·1
20%	$\frac{1}{5}$	0·2
25%	$\frac{1}{4}$	0·25
$33^{1}/_{3}$%	$\frac{1}{3}$	0·333…
50%	$\frac{1}{2}$	0·5
$66^{2}/_{3}$%	$\frac{2}{3}$	0·666…
75%	$\frac{3}{4}$	0·75
100%	$\frac{100}{100}$	1 OR 1·00

Percentages MNU 2-07b

Non-Calculator Methods

There are many ways to calculate percentages of a quantity. Some of the common ways are shown below.

Method 1 Using Equivalent Fractions

Example Find 25% of £640

25% of £640 = $\frac{1}{4}$ of £640 = £640 ÷ 4 = <u>£160</u>

Method 2 Using 1%

In this method, first find 1% of the quantity (by dividing by 100), then multiply to give the required value.

Example Find 9% of 200g

1% of 200g = $\frac{1}{100}$ of 200g = 200g ÷ 100 = 2g

9% of 200g = 9 x 2g = <u>18g</u>

Method 3 Using 10%

This method is similar to the one above.

First find 10% (by dividing by 10), then multiply to give the required value.

Example Find 70% of £35

10% of £35 = $\frac{1}{10}$ of £35 = £35 ÷ 10 = £3·50

70% of £35 = 7 x £3·50 = <u>£24·50</u>

Percentages MNU 2-07b / 3-07a

Non-Calculator Methods (continued)

The previous 2 methods can be combined to calculate any percentage.

Example Find 23% of £15000

10% of £15000 = £1500
20% = £1500 x 2 = £3000

1% of £15000 = £150
3% = £150 x 3 = £450

23% of £15000 = £3000 + £450
= <u>£3450</u>

Example An auction house charges commission of 15% on all purchases.

Calculate the **total price** of a painting bought for £650.

10% of £650 = £65 (divide by 10)
5% of £650 = £32·50 (divide previous answer by 2)

15% of £650 = £65 + £32·50
= £97·50

Total price = £650 + £97·50
= <u>£747·50</u>

Percentages MNU 2-07b / 3-07a

Calculator Method

To find the percentage of a quantity using a calculator, change the percentage to a decimal, then multiply.

Example 1

Find 23% of £15000 $= \frac{23}{100} \times 15000$

$= \underline{£3450}$

OR

23% = 0.23

so 23% of £15000 = 0·23 x £15000

$= \underline{£3450}$

We do not use the % button on calculators. The methods taught are all based on converting percentages to decimals.

Example 2

House prices increased by 19% over a one year period. What is the new value of a house which was valued at £236000 at the start of the year?

Increase $= \frac{19}{100} \times 236\,000$

= £44 840

Value at end of year = original value + increase

= £236 000 + £44840

= £280840

The new value of the house is $\underline{£280840}$

Percentages MNU 3-07a

Finding the percentage

To find a percentage of a total, first make a fraction, then convert to a decimal by dividing the top by the bottom. This can be changed to a percentage by multiplying by 100.

Example 1 There are 30 pupils in Class 3A3. 18 are girls.

What percentage of Class 3A3 are girls?

Fraction = $\frac{18}{30}$

Percentage = $18 \div 30 \times 100 = 60\%$

Therefore <u>60%</u> of 3A3 are girls

Example 2 James scored 36 out of 44 his biology test.

What is his percentage mark?

Fraction = $\frac{36}{44}$

Percentage = $36 \div 44 \times 100$

= 81·818..%

= <u>81·8 %</u> (rounded to 1 d.p.)

Example 3 In class 1X1, 14 pupils had brown hair, 6 pupils had blonde hair, 3 had black hair and 2 had red hair.

What percentage of the pupils were blonde?

Total number of pupils = 14 + 6 + 3 + 2 = 25

Fraction = $\frac{6}{25}$

Percentage = $6 \div 25 \times 100$ = <u>24%</u>

Ratio MNU 3-08a

Writing Ratios

When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

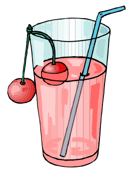
Example 1

To make a fruit drink, 4 parts water is mixed with 1 part of cordial.

The ratio of water to cordial is 4 : 1 which is said "4 to 1".

The ratio of cordial to water is 1 : 4.

Order is important when writing ratios.



Example 2

In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.

The ratio of red : blue : green is 5 : 7 : 8

Simplifying Ratios

Ratios can be simplified in the same way as fractions.

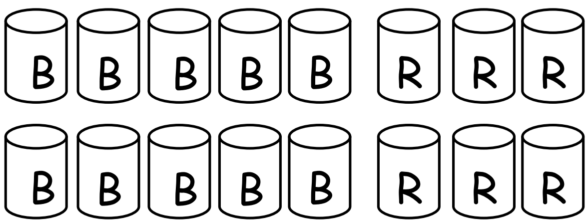
To simplify a ratio, **divide** each figure in the ratio by a common factor.

Example 1

Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red. The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, as it is possible to split up the tins into 2 groups, each containing 5 tins of blue and 3 tins of red.

Blue : Red
= 10 : 6
= 5 : 3



Ratio MNU 3-08a

Simplifying Ratios (continued)

Example 2

Simplify each ratio:

(a) 4 : 6	(b) 24 : 36	(c) 6 : 3 : 12
Divide each figure by 2	Divide each figure by 12	Divide each figure by 3
= 2 : 3	= 2 : 3	= 2 : 1 : 4

Example 3

Concrete is made by mixing 20 kg of sand with 4 kg cement.

Write the ratio of sand : cement in its simplest form

Sand : Cement

= 20 : 4

Divide each figure by 5

= 5 : 1

Using ratios

Example

The ratio of fruit to nuts in a chocolate bar is 3 : 2.

If a bar contains 15g of fruit, what weight of nuts will it contain?

	Fruit	**Nuts**	
	3	2	
×5			×5
	15	**10**	

So the chocolate bar will contain 10 g of nuts.

Ratio MNU 3-08a

Sharing in a given ratio

Example

Lauren and Sean earn money by washing cars.
By the end of the day they have made £90.
As Lauren did more of the work, they decide
to share the profits in the ratio 3:2.
How much money did each receive?

Step 1 **Add** up the numbers to find the **total** number of parts
$3 + 2 = 5$

Step 2 **Divide** the total by this number to find the value of **each** part
$90 \div 5 = £18$

Step 3 **Multiply** each figure by the value of each part
$3 \times £18 = £54$
$2 \times £18 = £36$

Step 4 Check that the total is correct
$£54 + £36 = £90$ ✓

Lauren received <u>£54</u> and Sean received <u>£36</u>

Proportion MNU 4-08a

Two quantities are said to be in direct proportion if when one doubles the other doubles.
We can use proportion to solve problems.

It is useful to make a table when solving problems involving proportion.

Example 1

A car factory produces 1500 cars in 30 days.
How many cars would they produce in 90 days?

	Days	Cars	
×3	30	1500	×3
	90	**4500**	

The factory would produce <u>4500</u> cars in 90 days.

Example 2

5 adult tickets for the cinema cost £27.50.
How much would 8 tickets cost?

Tickets	Cost
5	£27·50
1	£5·50
8	£44·00

Find the cost of 1 ticket →

Working

$$5\overline{)\text{£}27{\cdot}50} = \text{£}5{\cdot}50$$

$$\text{£}5{\cdot}50 \times 8 = \text{£}44{\cdot}00$$

The cost of 8 tickets is <u>£44</u>.

Money MNU 2-09a-c / 3-09a

There are 100 pence in a pound. £1.89 = 189p
349p = £3.49
Amounts should have a £ or p sign but not both!

Profit and loss

To calculate profit or loss: Profit = Selling price – cost price
Loss = Cost price – selling price

Example

Rory bought a car for £15 475 and sold it two years later for £8 995.
Calculate his loss.

Loss = 15 475 – 8995
= <u>£6 480</u>

Hire purchase

This can be an affordable method of buying an item.
However, you often end up paying a lot more than the value of the item.

Example

Lisa sees this advert for a motorbike.
How much more would hire purchase cost her than paying cash?

H.P. cost = 350 x 48 + 1000
= £17 800
Difference = 17 800 – 14 395
= <u>£3 405</u>

Time MNU 2-10a

Time Facts

It is essential to know the number of months, weeks and days in a year, and the number of days in each month.

In 1 year, there are:

- 365 days (366 in a leap year)
- 52 weeks
- 12 months

The number of **days** in each **month** can be remembered using the rhyme:

"30 days hath September,
April, June and November,
All the rest have 31,
Except February alone,
Which has 28 days clear,
And 29 in each leap year."

Example

This is part of a train timetable from Dundee to Aberdeen.

Dundee	0635	0656	0724	0828
Carnoustie	---	0708	0736	0844
Arbroath	0651	0715	0743	0859
Montrose	0705	0729	0757	0920
Stonehaven	0726	0751	0819	---
Portlethen	---	0800	0827	0940
Aberdeen	0750	0813	0840	0955

Adam caught the 0656 train from Dundee to Aberdeen.
How long was his journey?

$\boxed{0656} \xrightarrow{\text{4 mins}} \boxed{0700} \xrightarrow{\text{1 hr 13 mins}} \boxed{0813}$

Total journey time = 1 hour 13 minutes + 4 minutes = <u>1 hour 17 minutes</u>

Time MNU 2-10b / 2-10c

We use time calculations to plan our everyday activities.

Example

Angus is making a chocolate cake for his mum's birthday.

The cake takes 25 minutes to prepare, 30 minutes to cook and it is recommended to leave for 1 hour to cool before eating.

If Angus's cake is to be ready at 2:30pm, at what time must he start preparing it?

$$\boxed{\text{12:35 pm}} \xleftarrow{-\text{ 25 mins}} \boxed{\text{1 pm}} \xleftarrow{-\text{ 30 mins}} \boxed{\text{1:30 pm}} \xleftarrow{-1\text{ hr}} \boxed{\text{2:30 pm}}$$

Total time = 25 minutes + 30 minutes + 1 hour

= 1 hour 55 minutes

He must start making the cake at 12:35 pm

Distance, Speed and Time

For any given journey, the distance travelled depends on the speed and the time taken.

distance = speed x time or $d = s\,t$

Example

Owen rides his bike at an average speed of 10 miles per hour.

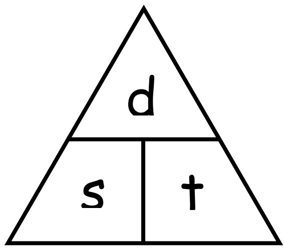
How far will he have travelled in $2\frac{1}{2}$ hours?

$d = s\,t$

$d = 10 \times 2{\cdot}5$

$d = \underline{25\text{ miles}}$

Time MNU 3-10a



Distance, Speed and Time.

This triangle helps us remember the formulae for calculating distance, speed and time.

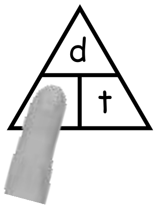
Cover up the one you are trying to find and what's left is the formula.

Formula in words	**Formula in symbols**
distance = speed x time	$d = s\,t$
speed = $\frac{\text{distance}}{\text{time}}$	$s = \frac{d}{t}$
time = $\frac{\text{distance}}{\text{speed}}$	$t = \frac{d}{s}$

In Physics this is given as: $v = \frac{d}{t}$

Example 1

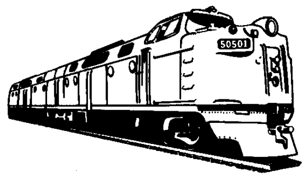
Calculate the speed of a train which travelled 450 km in 5 hours.



$s = \frac{d}{t}$

$s = \frac{450}{5}$

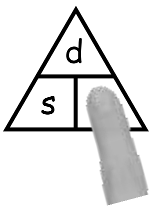
$s = \underline{90 \text{ km/h}}$



Example 2

Greig travels 250 miles at an average speed of 60 mph.

How long does this journey take?



$t = \frac{d}{s}$

$t = \frac{250}{60}$

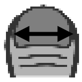
$t = 4\frac{1}{6}$ hours

$t = \underline{4 \text{ hours } 10 \text{ minutes}}$

Let's try this without a calculator

Measurement MNU 2-11a / 2-11b

When measuring we must decide on an appropriate unit dependent on the size of the object. The following can help us to estimate the size of different objects.

 1 cm 

1 kg  1 *l*

Length can be measured in millimetres, centimetres, metres and kilometres.

Rules 1 cm = 10 mm 1 m = 100 cm 1 km = 1000 m

Weight can be measured in grams, kilograms and metric tonnes.

Rules 1 kg = 1000 g 1 tonne = 1000 kg

Volume can be measured in millilitres and litres.

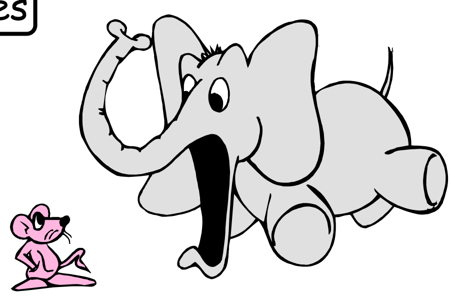
Rules 1 ***l*** = 1000 ***ml***

Converting between units

$$\boxed{\text{mm}} \underset{\div 10}{\overset{\times 10}{\rightleftarrows}} \boxed{\text{cm}} \underset{\div 100}{\overset{\times 100}{\rightleftarrows}} \boxed{\text{m}} \underset{\div 1000}{\overset{\times 1000}{\rightleftarrows}} \boxed{\text{km}}$$

$$\boxed{\text{g}} \underset{\div 1000}{\overset{\times 1000}{\rightleftarrows}} \boxed{\text{kg}} \underset{\div 1000}{\overset{\times 1000}{\rightleftarrows}} \boxed{\text{tonnes}}$$

$$\boxed{ml} \underset{\div 1000}{\overset{\times 1000}{\rightleftarrows}} \boxed{l}$$



Examples Convert the following:

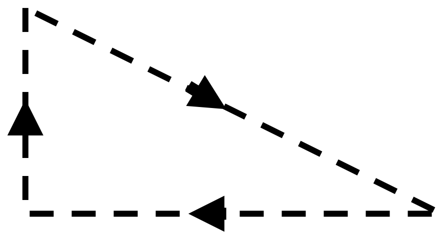
1) 89 mm into cm ⇨ 89 ÷ 10 = <u>8·9 cm</u>

2) 4·76 kg into g ⇨ 4·76 x 1000 = <u>4760 g</u>

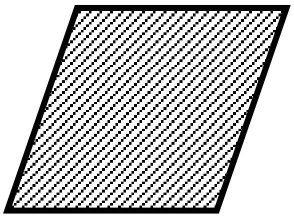
3) 1400 ***ml*** into ***l*** ⇨ 1400 ÷ 1000 = <u>1·4 *l*</u>

Measurement MNU 2-11c / 3-11a

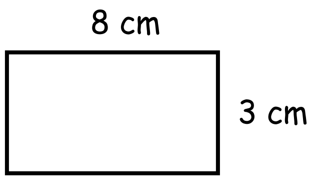
Perimeter
Total distance round a shape



Area
Space inside a shape

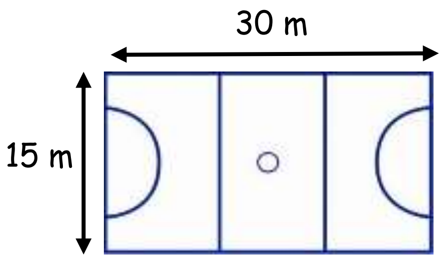


Example 1
Calculate the perimeter of this rectangle

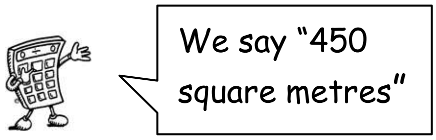


Perimeter = 8 + 3 + 8 + 3
= <u>22 cm</u>

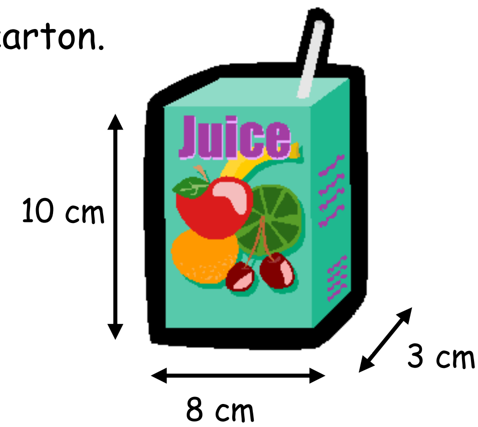
Example 2
Calculate the area of this netball court.



Area = $l\,b$
Area = 30 x 15
Area = <u>450</u> m^2



Example 3
Calculate the volume of orange juice in the carton.

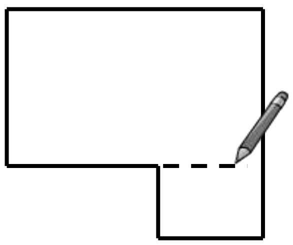


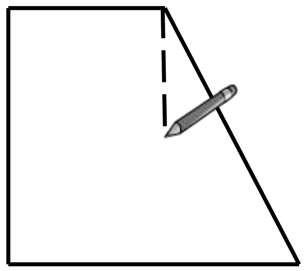
Volume = $l\,b\,h$
= 8 x 3 x 10
= <u>240</u> cm^3

Measurement MNU 3-11b

Compound areas

For more complicated shapes we can split them up into smaller parts.

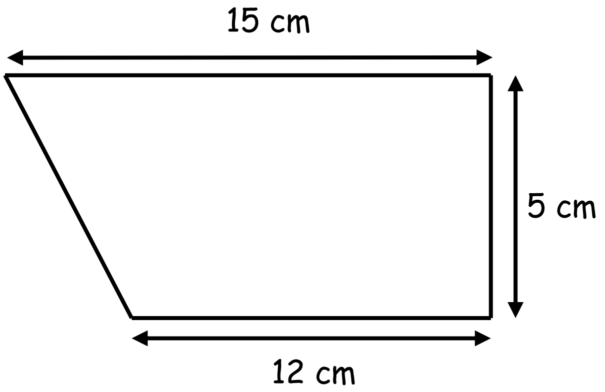


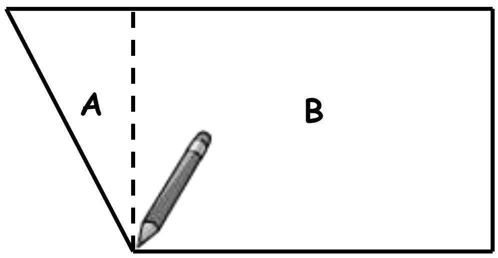


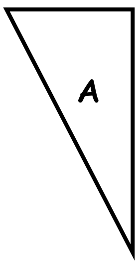
Example

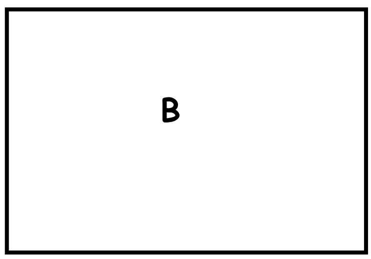
Find the area of the following shape.

Split up the shape and calculate the areas separately









Area A $= \frac{1}{2}bh$

$= \frac{1}{2} \times 3 \times 5$

$= 7{\cdot}5 \text{ cm}^2$

Area B $= lb$

$= 12 \times 5$

$= 60 \text{ cm}^2$

Total area $= 7{\cdot}5 + 60 = \underline{67{\cdot}5} \text{ cm}^2$

Data and Analysis MNU 2-20a / 3-20a

It is sometimes useful to display information in graphs, charts or tables.

Example 1 The table below shows the average maximum temperatures (in degrees Celsius) in Barcelona and Edinburgh.

	J	F	M	A	M	J	J	A	S	O	N	D
Barcelona	13	14	15	17	20	**24**	27	27	25	21	16	14
Edinburgh	6	6	8	11	14	17	18	18	16	13	8	6

The average temperature in June in Barcelona is <u>24°C</u>

Frequency Tables are used to present information.
Often data is grouped in intervals.

Example 2 Homework marks for Class 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

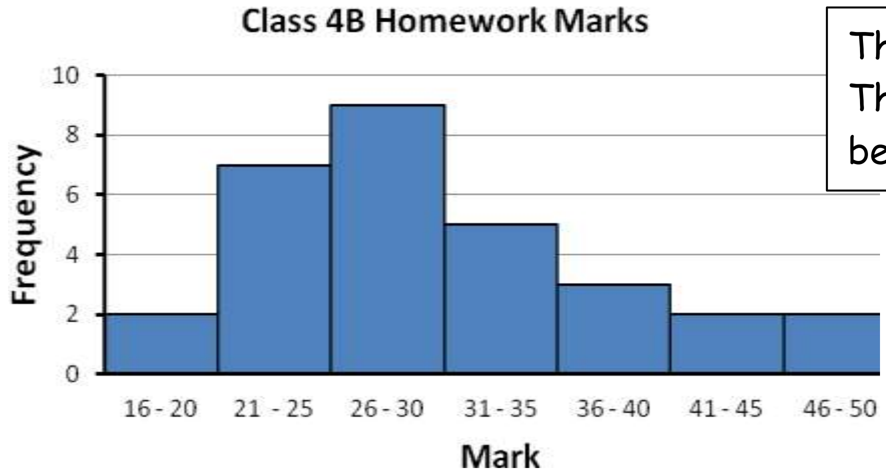
Mark	Tally	Frequency
16 - 20	||	2
21 - 25	~~||||~~ ||	7
26 - 30	~~||||~~ ||||	9
31 - 35	~~||||~~	5
36 - 40	|||	3
41 - 45	||	2
46 - 50	||	2
	Total	**30**

Each mark is recorded in the table by a tally mark.
Tally marks are grouped in 5's to make them easier to read and count.

Data and Analysis MNU 2-20a / 3-20a

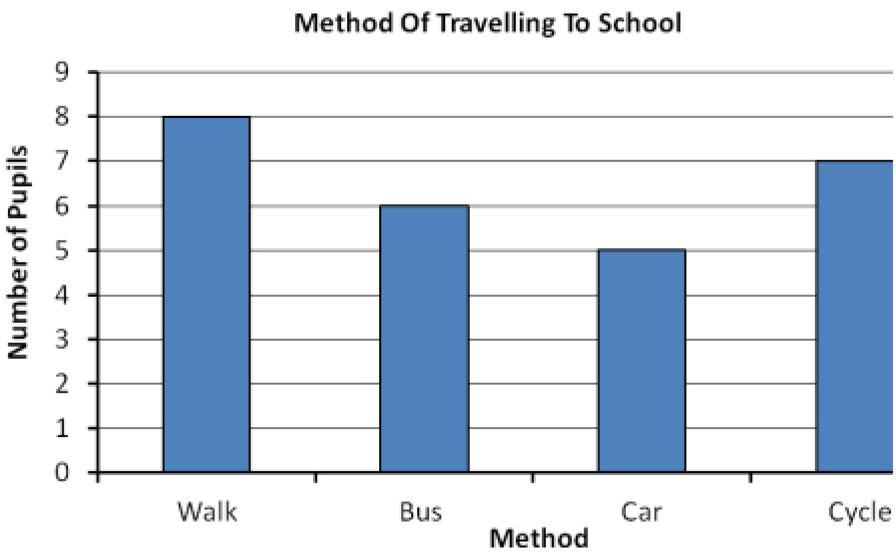
Bar graphs and histograms are often used to display data. The horizontal axis should show the categories or class intervals, and the vertical axis the frequency. All graphs should have a title, and **each axis must be labelled**.

Example 1 The graph below shows the homework marks for Class 4B.



This is a **histogram**. There are no spaces between the bars

Example 2 The graph below shows how class M13 travel to school?



This is a **bargraph**.

When the horizontal axis shows categories, rather than grouped intervals, it is common practice to leave gaps between the bars.

Data and Analysis MNU 2-20a / 3-20a

Line graphs consist of a series of points which are plotted, then joined by a line. All graphs should have a title, and **each axis must be labelled**.

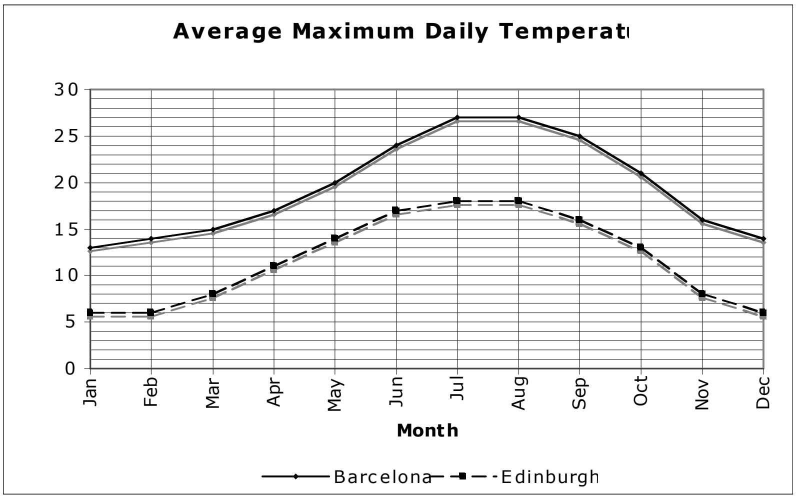
Example 1

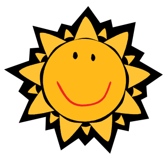
The graph below shows the effect of exercise on the body's heart rate. John pedalled at different work rates and measured his heart rate.

Effect of exercise on heart rate

The **trend** of the graph is that as the work rate increases so does his heart rate.

Example 2 Graph of temperatures in Edinburgh and Barcelona.





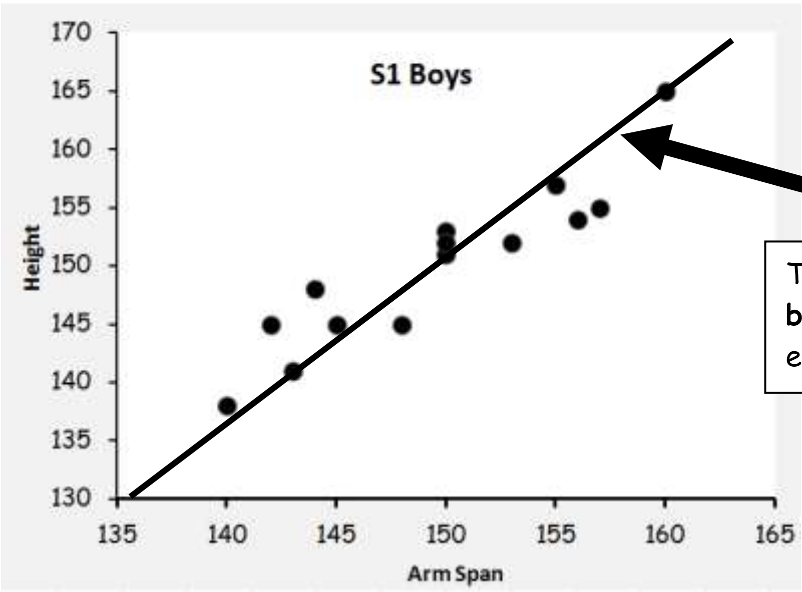
The **trend** of the graph is in both cities the temperature rises to a maximum in July and the drops.

Data and Analysis MNU 2-20a / 3-20a

A scatter diagram is used to display the relationship between two variables. A pattern may appear on the graph. This is called a **correlation**.

Example The table below shows the height and arm span of a group of S1 boys. This is plotted as a series of points on the graph below.

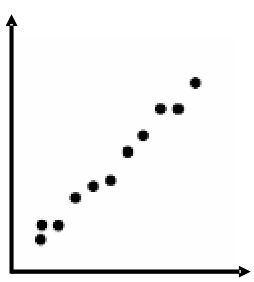
Arm Span (cm)	150	157	155	142	153	143	140
Height (cm)	153	155	157	145	152	141	138
Arm Span (cm)	145	144	150	148	160	150	156
Height (cm)	145	148	151	145	165	152	154



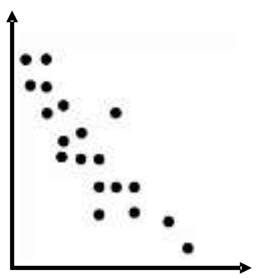
This line is called the **line of best fit**. It can be used to estimate different values.

The graph shows a general trend, that as the arm span increases, so does the height. This graph shows a positive correlation.

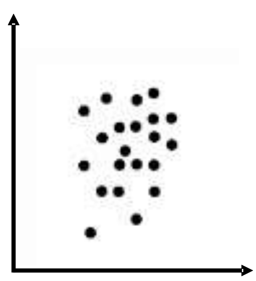
Types of correlation:



Positive correlation



Negative correlation



No correlation

Data and Analysis MNU 2-20a / 3-20a

A pie chart can be used to display information. Each sector (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

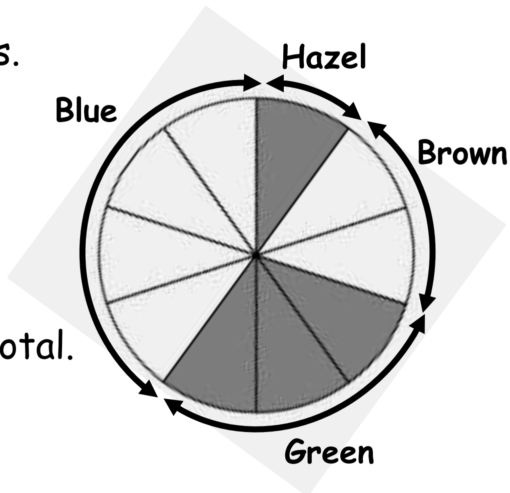
Example

30 pupils were asked the colour of their eyes.

The results are shown in the pie chart.

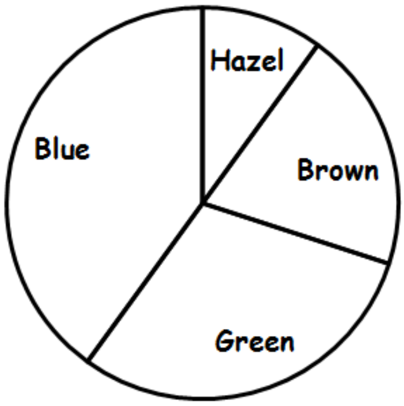
How many pupils had green eyes?

The pie chart is divided up into ten parts.



Pupils with green eyes represent $\frac{3}{10}$ of the total.

$\frac{3}{10}$ of 30 = 9, so 9 pupils had green eyes.



If no divisions are marked, we can work out the fraction by measuring the angle of each sector.

The angle in the green sector is 108°.

So the number of pupils with green eyes = $\frac{108}{360}$ x 30 = 9 pupils.

If finding all of the values, you can check your answers.
The total should be 30 pupils.

Drawing Pie Charts

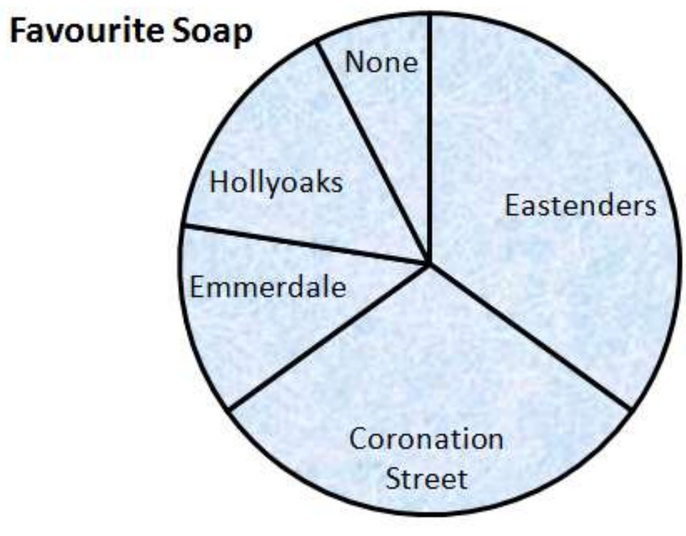
In a pie chart, the size of the angle for each sector is calculated as a fraction of 360°.

Example

In a survey about television programmes, a group of people were asked to name their favourite soap. Their answers are given in the table below. Draw a pie chart to illustrate the information.

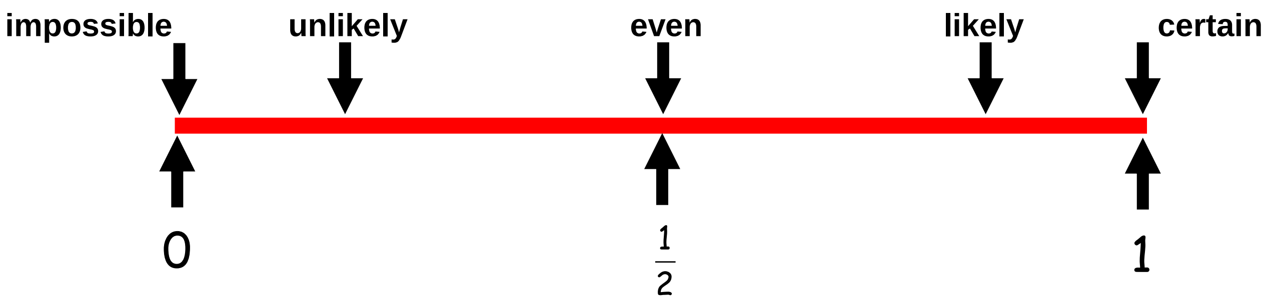
Soap	**Number of people**
Eastenders	28
Coronation Street	24
Emmerdale	10
Hollyoaks	12
None	6
Total	**80**

Fraction	**Angle**
$\frac{28}{80}$	$\frac{28}{80} \times 360° = 126°$
$\frac{24}{80}$	$\frac{24}{80} \times 360° = 108°$
$\frac{10}{80}$	$\frac{10}{80} \times 360° = 45°$
$\frac{12}{80}$	$\frac{12}{80} \times 360° = 54°$
$\frac{6}{80}$	$\frac{6}{80} \times 360° = 27°$
Total	**360°**



Chance and Uncertainty MNU 2-22a / 3-22a

Probability is the likelihood that an event will happen.
We can use words to describe the chance of something happening.



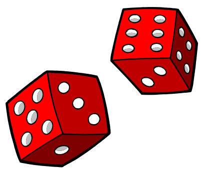
However to be more accurate, we can determine the probability of an event using fractions, decimals or percentages. To calculate the probability of an event:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Example 1 Roll a die.

What is the probability of getting an even number?

P(even) = $\frac{3}{6} = \frac{1}{2}$



Example 2

A survey in a car park shows how many cars of each colour there are.

Colour	Red	Blue	Black	Silver
Number of cars	30	15	20	35

Based on this information, what is the probability that the next car to come into the car park is black?

Total = 30 + 15 + 20 + 35 = 100 cars

P(black car) = $\frac{20}{100} = \frac{1}{5}$

We could also write 0·2 or 20%

Mathematical Dictionary (Key words):

Add Addition (+)	To combine 2 or more numbers to get one number. 12+76 = 88
a.m.	Any time in the morning. (ante meridiem -between midnight and 12 noon)
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Average	A typical or middle value of a set of numbers. Mean, median and mode are all measures of average.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Data	A collection of information. (may include facts, numbers or measurements)
Denominator	The bottom number in a fraction.
Difference (-)	The amount between two numbers (subtraction). The difference between 50 and 36 is 14 ➔ 50 – 36 = 14
Division (÷)	Sharing a number into equal parts. 24 ÷ 6 = 4
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Equivalent fractions	Fractions which have the same value. $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions.
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. The factors of 15 are 1, 3, 5 and 15.

Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Greater than (>)	Is bigger or more than. 10 is greater than 6 → 10 > 6
Least	The lowest number in a group (minimum).
Less than (<)	Is smaller or lower than. 15 is less than 21 → 15 < 21
Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers.
Median	Another type of average. The middle number of an ordered set of data.
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.
Mode	Another type of average. The most frequent number or category.
Most	The largest or highest number in a group (maximum).
Multiply (x)	To combine an amount a particular number of times. 6 x 4 = 24
Negative Number	A number less than zero. -5 is a negative number.
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1, 3, 5, 7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done - BODMAS.

Place value	The value of a digit in a number. In the number 1573.4, the 5 has a place value of 500.
p.m.	Any time in the afternoon or evening. (post meridiem - between 12 noon and midnight).
Product	The answer when two numbers are multiplied together. The product of 5 and 4 is 20.
Remainder	The amount left over when dividing a number.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Total	The sum of a group of numbers (found by adding).

Addition

increase
together
sum
more
plus

Subtraction

difference between
subtract
fewer
decrease
minus
take from
reduce

Multiplication

multiplied by
multiply
product
groups of
lots of
times table
times

Division

divided by
share
divide
share equally
divisible by
divide into
group