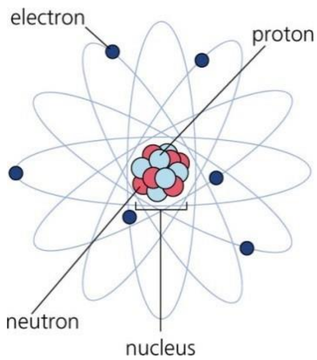
Advanced Higher Physics



Rotational Motion and Astrophysics

Data

Common Physical Quantities

QUANTITY	SYMBOL	VALUE
Gravitational acceleration	g	9.8 m s^{-2}
Radius of Earth	R_E	6.4×10^{6} m
Mass of Earth	M_E	6.0×10^{24} kg
Mass of Moon	M_M	7.3×10^{22} kg
Mean radius of Moon orbit		3.84×10^{8} m
Universal constant of gravitation	G	6.67×10^{-11} m^{3} kg^{-1} s^{-2}
Speed of light in vacuum	c	3.0×10^{8} m s^{-1}
Speed of sound in air	v	3.4×10^{2} m s^{-1}
Mass of electron	m_e	9.11×10^{-31} kg
Charge on electron	e	-1.60×10^{-19} C
Mass of neutron	m_n	1.675×10^{-27} kg
Mass of proton	m_p	1.673×10^{-27} kg
Planck's constant	h	6.63×10^{-34} J s
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}

Astronomical Data

Planet or satellite	Mass/ kg	Density/ kg m^{-3}	Radius/ m	Grav. accel./ m s^{-2}	Escape velocity/ m s^{-1}	Mean dist from Sun/ m	Mean dist from Earth/ m
Sun	1.99×10^{30}	1.41×10^{3}	7.0×10^{8}	274	6.2×10^{5}	--	1.5×10^{11}
Earth	6.0×10^{24}	5.5×10^{3}	6.4×10^{6}	9.8	11.3×10^{3}	1.5×10^{11}	--
Moon	7.3×10^{22}	3.3×10^{3}	1.7×10^{6}	1.6	2.4×10^{3}	--	3.84×10^{8}
Mars	6.4×10^{23}	3.9×10^{3}	3.4×10^{6}	3.7	5.0×10^{3}	2.3×10^{11}	--
Venus	4.9×10^{24}	5.3×10^{3}	6.05×10^{6}	8.9	10.4×10^{3}	1.1×10^{11}	--

Motion of a Particle

1. For the first ten seconds of a rocket's motion its acceleration function can be described by

$$\frac{dv}{dt} = 4t$$

(a) Derive the velocity function if at t = 0 the instantaneous velocity is also zero.
(b) Derive the displacement function if at t = 0 the displacement is zero.
(c) On graph paper plot the acceleration/time and velocity/time graphs of the rocket's motion during the first 10 seconds.

2. The motion of a rocket, already in flight, can be described by the following function for the first 5 seconds, after an alteration of the rocket motor's thrust,

$$\frac{dv}{dt} = 10 - 2t$$

(a) Derive the velocity function if at t = 0, the instant the thrust is changed, the instantaneous velocity is 40 ms^{-1}.
(b) On graph paper plot the acceleration and velocity/time graphs for the first 5 seconds.

3. A ball is thrown up vertically with a speed of 30 ms^{-1}. Its displacement in metres from its starting point is given by the function:

$x = 30t - 5t^2$ where t is in seconds.

(a) Deduce the ball's velocity function.
(b) Find the ball's acceleration.
(c) Find the time the ball takes to return to its starting point.
(d) Find the maximum height reached by the ball.

4. A marble rolls down a slope 6 metres long. Its distance in metres from the top of the slope is given by the function:

$x = 5t + t^2$ where t is in seconds.

(a) Find its velocity function.
(b) Find the marble's acceleration.
(c) Find the time the marble takes to reach the foot of the slope.
(d) Find the marble's speed at the foot of the slope.

5. The displacement of a pendulum bob measured relative to its rest position can be approximated by the function:

$$x = 0.1\cos(6t) \text{ where } x \text{ is in metres and } t \text{ is in seconds.}$$

(a) Find the bob's velocity function.
(b) Find the bob's acceleration function.
(c) What are the displacement, velocity and acceleration of the bob when:

(i) $t = 0$s (ii) $t = \pi/12$s (iii) $t = \pi/6$s (iv) $t = \pi/4$s (v) $t = \pi/3$s
(d) What is the period of the pendulum?

6. The location of a projectile relative to the start position at the origin is given by:

Horizontal displacement $x = 60t$
Vertical displacement $y = 30t - 5t^2$ where x and y are in metres and t is in seconds.

(a) Show that the horizontal velocity is constant at $60ms^{-1}$.
(b) Show that the acceleration is $10ms^{-2}$ downwards.
(c) Calculate the maximum height reached.
(d) Calculate the horizontal range of the projectile.
(e) Show that the instantaneous speed of the projectile during its flight is given by the function:

$$v = 10\sqrt{(45 - 6t + t^2)}$$

7. The position of a mark on a rolling bicycle tyre relative to a fixed point on the ground is given by the functions:

$x = 0.5\sin(12t) + 6t$ and $y = 0.5 - 0.5\cos(12t)$, with x and y in metres and t in seconds.

(a) Calculate the velocity and acceleration functions for the x and y directions.
(b) Calculate the sizes of these velocities and accelerations at maximum and minimum values of y.

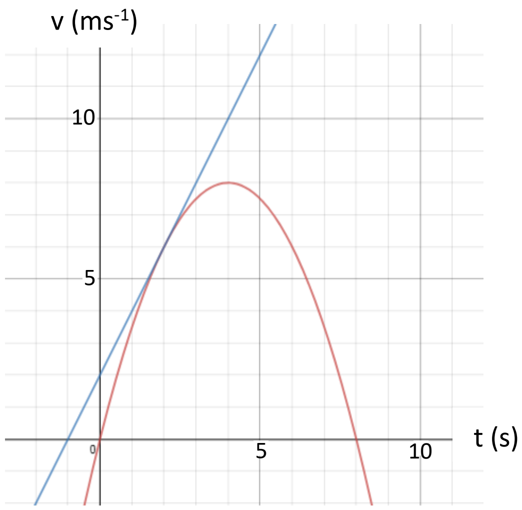
8. The velocity of a body is described by the function $v = 4t - 0.5t^2$.

(a) Using integration, calculate the displacement between 4s and 8s.

(b) Find the gradient of the tangent to the curve at t = 2s.

(c) Determine the acceleration function of the body.

(d) Calculate the acceleration after 2s.



Circular Motion

1. A disc rotating at a constant angular velocity rotates through an angle of 60^o in 0.2 seconds.

(a) What is the angular displacement of the disc, in radians, in 0.2s?
(b) What is the angular velocity of the disc?
(c) What is the periodic time of rotation of the disc?

2. A ball moves steadily in a circle of radius 0.5 metre with a period of 2 seconds.

(a) What is the circumferential speed of the ball?
(b) What is the angular velocity of the ball?
(c) How long does it take the ball to move through an angular displacement of $\pi/4$ radians?

3. An object moves in a circular path of radius 100m at a steady speed of $10ms^{-1}$ around the circumference.

(a) Calculate the periodic time,
(b) Find the object's angular velocity.

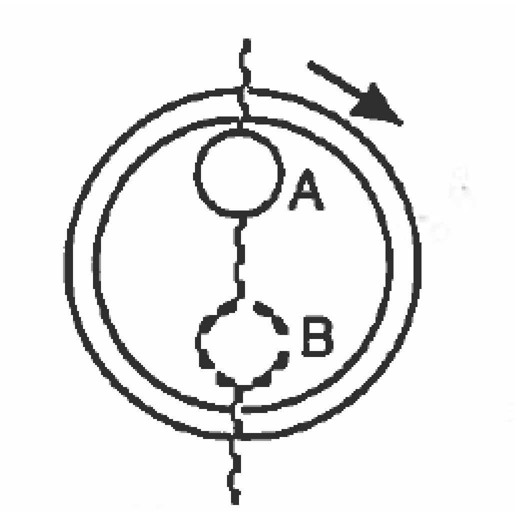
4. A rotating display platform in an exhibition rotates through 90^o in 30 seconds.

(a) Find its angular displacement in 30 seconds in radians.
(b) Calculate the platforms angular velocity.
(c) How long will it take for an angular displacement of 4π radians?

5. A wheel turns at 4800 revolutions per minute (r.p.m.).

(a) Calculate its angular velocity in radians per second.
(b) What is the wheel's periodic time?

6. If a wheel is turning at 600 $rads^{-1}$, how many rotations does it make in 1 minute?



7. A car's engine is running at 6000 r.p.m.

(a) What is the engine's angular velocity?

(b) How long does it take for a piston rod end to rotate from A to B?

Constant Angular Acceleration

1. A wheel starts rotating from rest and 20 seconds later has an angular velocity of $120rads^{-1}$. Find its angular acceleration.

2. A wheel starts rotating from rest with an angular acceleration of $0.2\ rads^{-2}$ and after an interval of time its angular velocity has increased to $10rads^{-1}$.

(a) Find the time taken.
(b) Find the total angular displacement of the wheel in this time

3. A disc makes 25 revolutions in the time taken for its angular velocity to increase from $10rads^{-1}$ to $20rads^{-1}$.

(a) What is this angular displacement in radians?
(b) Find the disc's angular acceleration.
(c) Find the time taken for the 25 revolutions.

4. A point on the rim of a wheel of diameter 1 metre has its speed of $25ms^{-1}$ reduced to $20ms^{-1}$ in 10 seconds.

Find the angular deceleration of the wheel in $rads^{-2}$.

5. A wheel starts rotating from rest and attains an instantaneous angular velocity of 900 revolutions per minute in 20 seconds.

(a) State the final instantaneous angular velocity in $rads^{-1}$.
(b) Find the angular acceleration.
(c) Find the angular displacement of the wheel at the end of the first second of motion.
(d) Find the instantaneous angular velocity after the wheel has rotated through 10 revolutions after starting.

6. The angular velocity of a flywheel is reduced from $100rads^{-1}$ to $40rads^{-1}$ in 5s.

(a) Calculate the angular deceleration of the flywheel.
(b) Find the number of revolutions made by the wheel in this time.

7. The wheel of a steam locomotive is 2 metres in diameter.

(a) Find the angular velocity of the wheel when the locomotive is running at $36kmh^{-1}$.
(b) The locomotive is brought to rest In 10 seconds; what is the angular deceleration of the wheel?
(c) How many revolutions does the wheel make during the 10 seconds of slowing down?
(d) How far does the locomotive travel in these 10 seconds?
(e) State any assumptions made about the motion of the wheel.

Radial Acceleration and Centripetal Force

1. A mass, tied to the end of a string 2 m long is made to rotate in a horizontal circle with a uniform speed. The mass makes 90 revolutions in one minute.

(a) What is the frequency of rotation in hertz?
(b) What is the period of rotation?
(c) What is the angular velocity?
(d) What is the speed of the mass around the circumference?
(e) What is the radial acceleration of the mass?

2. A point on the circumference of a rotating disc is moving with a constant speed of $2ms^{-1}$. The disc has a radius of 10 cm.

Find (a) the angular velocity
(b) the radial acceleration of the point
(c) the period of rotation of the disc
(d) the frequency of rotation in hertz
(e) how long the disc takes to rotate through an angle of $\pi/2$ radians.

3. A car having a mass of 1000 kg moves around a circular track of radius 100 metres at a steady speed of $20ms^{-1}$.

Find (a) the car's angular velocity
(b) the car's acceleration towards the centre
(c) the centripetal force acting on the car.

4. A ball of mass 2.5×10^{-2}kg moves in a horizontal circular path of radius 2.0 metres with a steady speed of $5.0ms^{-1}$. Calculate:

(a) the radial acceleration of the ball,
(b) the centripetal force on the ball.

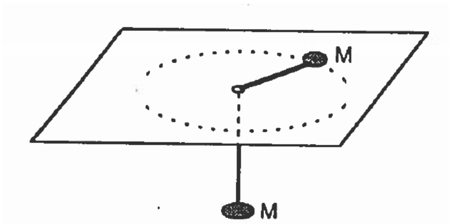
5. A force of 10N is required to keep a 100g mass rotating in a horizontal circle of radius 1 metre.

Find (a) the speed of the mass
(b) the angular velocity
(c) the frequency of rotation of the mass.

6. A mass of 2 kg is whirled in a horizontal circle by a string 0.5 metres long. This string has a breaking strain of 400N. What is the maximum possible speed at which the mass can be whirled without breaking the string?

7. If an electron moves in a circular orbit with a speed of $2.4\ Mms^{-1}$ and the centripetal force acting on the electron is 0.1μN, calculate the radius of the electron's orbit.

8. Two bodies of equal mass M, are connected by a string which passes through a hole in a smooth horizontal table top so that one mass sits on the table and the other hangs vertically below the hole. The mass on the table top is set rotating as shown.



What is the angular velocity of the mass on the table top if, when it rotates in a circle of 0.2 m radius, it can support the other mass?

9. A 5kg mass is whirled in a vertical circle of radius 1m at a steady speed of $10ms^{-1}$.

(a) Calculate the centripetal force needed to keep the 5kg mass rotating at this speed and at this radius.
(b) Calculate the tension in the string at the bottom of the circle.
(c) Calculate the tension in the string at the top of the circle.

(Use $g = 10Nkg^{-1}$)

10. A hump-backed bridge is in the form of a circular arc of radius 40 metres.

What is the greatest speed with which a car can cross the bridge without leaving the ground at the highest point? (g = 10 N/kg)

11. A bucket with 0.5 kg of water is whirled in a vertical circle of radius 1 metre, at a constant speed of 4 ms^{-1}.

(a) Calculate the reaction of the bucket on the water at the highest point.
(b) At what speed will the water just leave the bucket at the highest point?

12. A sphere having a mass of 1 kg is suspended by a string which is 5 metres long. The sphere is drawn to one side and set into circular motion in a horizontal plane. When the radius of rotation of the sphere is 3 metres, find:

(a) the distance of the plane of rotation below the point of suspension.
(b) the angle which the string makes with the vertical,
(c) the tension in the string,
(d) the centripetal force acting on the sphere, and
(e) the angular velocity and period of rotation of the sphere.

13. A conical pendulum has a string 1 metre long with a breaking strain of 3N. If the bob has a mass of 0.1 kg, find the maximum angle the string can make with the vertical and the maximum angular velocity of the bob before the string breaks.

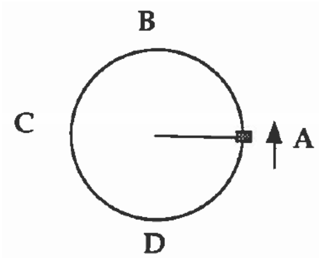
14. A small mass on a string 40cm long is whirled in a vertical circle with a frequency of 5Hz. If the string is released when horizontal and the mass is moving upwards, how high will the mass go above the point of release?

15. Part of a car racing track has a radius of 100m at a semi-circular bend. The track is banked in such a way that the angle of the banking Increases from the inside of the track towards the outside, as shown.

Ignoring any difference in radius between the inside and outside of the track, that is, taking 100m to be the average radius, find the angles at which cars will have to position themselves to avoid side-slip when they are travelling around the curve at (a) $15ms^{-1}$, and (b) $50ms^{-1}$.

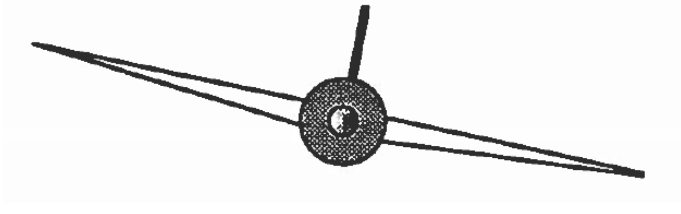
16. A train travels round a curve of radius 1000m at $15ms^{-1}$. The distance between the rails is 1.5 metres. How far must the outer rail be raised above the inner rail so that there will be no thrust on the flanges of the wheels?

17. The diagram shows a rubber stopper attached to a string which is held at **O**, and is being swung round in a vertical circle.



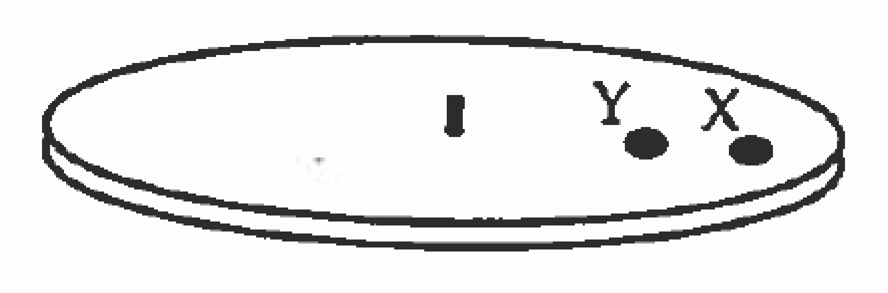
(a) What force is keeping the stopper moving in a circle when it is at **A** or at **C**?
(b) What two forces are keeping the stone moving in a circle when it is at **B**?
(c) If the string is weak, it is most likely to break when the stopper is near **D**. Why is this?
(d) Draw a diagram showing the path of the stopper if the string breaks at **D**. Assume that ground level is at a distance equal to **BD** below **D**.
(e) Draw a diagram showing the path of the stopper if, in the portion **AB**, the stopper is "not moving fast enough".

18. Two forces acting on an aeroplane in flight are its weight and the aerodynamic (lift) force. The aerodynamic force acts in a direction perpendicular to the wings. The pilot banks the aeroplane so that it flies at $150ms^{-1}$ in a horizontal circle of radius 4.0 km.



At what angle must the aeroplane be banked?

19. The following observations are made in three experiments with a record player turntable and a coin.



A. The coin is placed on the turntable at **X**. When the turntable rotates at 33 revolutions per minute (rpm) the coin remains at **X**. When the turntable rotates at 45 rpm the coin slides away from **X**.
B. The same coin is placed at **Y**. It remains at **Y** when the turntable rotates at 33 rpm and at 45 rpm.
C. A record is now put on the turntable and the coin placed on the record above **Y**. At 45 rpm the coin slides off.

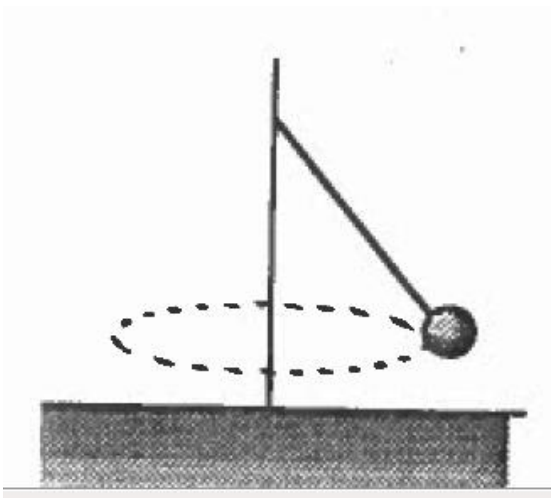
Explain the behaviour of the coin in these three experiments.

Angular Momentum of a Particle

1. An aircraft of mass 2.0 x 10^5kg travels at a constant speed of 100ms^{-1} in a horizontal circle of radius 2000m. Taking the aircraft's mass to be concentrated at its centre of mass, i.e. taking the aircraft as a particle, calculate its angular momentum about the centre of the circle.

2. A rubber bung (mass 100g) is being rotated in a horizontal circle of radius 1 metre with a constant speed of 1.2ms^{-1} when suddenly the string is shortened by being pulled in at the centre of the circle and the radius of rotation is reduced to 0.8m.

(a) Calculate the angular momentum of the bung at the start.
(b) By using the Principle of Conservation of Angular Momentum calculate the new speed of the bung after the radius has been reduced.

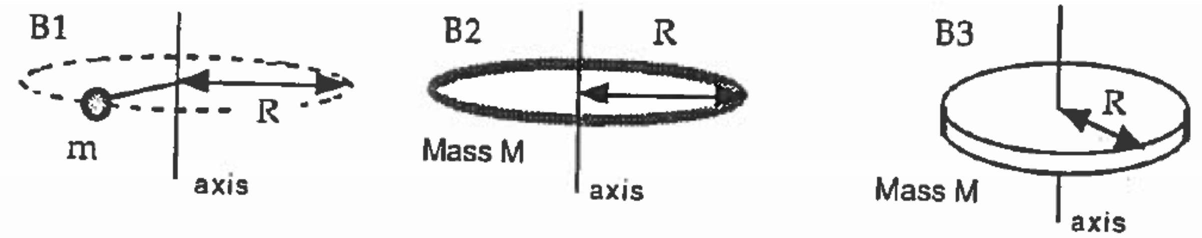


3. In a game a 0.2kg ball is attached by a string to a pole, as shown. The ball is hit by a bat and rotates around the pole in a horizontal circle.
On one occasion, the speed of the ball was 2ms^{-1}, when its radius of rotation was 1.3m. As the ball rotated, the string wound around the pole and the radius decreased to 1m
Calculate the new speed of the ball at this smaller radius.

Moments of Inertia

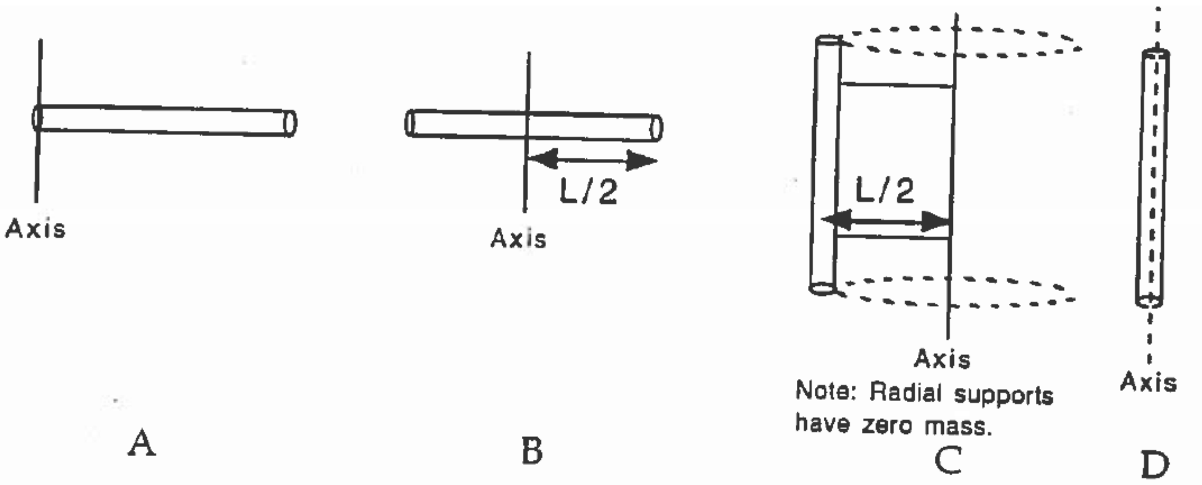
1. (a) Write down the general expression for the moment of inertia of a rigid body about an axis.

(b) Write down the expressions for the moments of inertia of the three rigid bodies about their axes shown in the diagrams below.



B1: Small sphere of mass m rotating about the axis shown, with a radius R.
B2: Hoop of mass M and radius R. rotating about the axis through its centre and perpendicular to its plane.
B3: Uniform disc of mass M and radius R rotating about the axis through its centre and perpendicular to the plane of the disc.

2. A uniform rod of length L is made to rotate about different axes, as shown below.



(a) Identify the arrangement which has the smallest moment of inertia, and explain why it is smallest.
(b) Which of the arrangements A or B has the larger moment of inertia?
(c) Using the general expression for moment of inertia [answered in Q1(a)] deduce the moment of inertia of the rod in arrangement C, if the rod has a mass M.
(d) Which arrangement has the largest moment of inertia? Explain why you think this is so.

3. A flywheel on a piece of machinery is in the form of a uniform metal disc. It has a mass of 10kg and radius 0.15 metres. Calculate its moment of inertia about the axis through its centre and perpendicular to the plane of the disc.

4. Which of the following arrangements has the larger moment of inertia about its axis shown?

Moments of Inertia and Conservation of Angular Momentum

5. The drum of a spin-dryer has a moment of inertia of 5 x 10^{-2} kgm^2 about its axis of rotation. What is its kinetic energy of rotation when it is rotating at 100 $rads^{-1}$?

6. The moment of inertia of the blades and rotor of an electric fan is 0.004 kgm^2. What is the rotational kinetic energy when the fan is rotating at 3600 rpm?

7. A uniform circular disc of 2kg mass and radius 0.4 metres rolls along a horizontal surface at 1.6ms^{-1}.
(a) Find its kinetic energy of rotation.
(b) Calculate the total kinetic energy of the disc as it is rolling.

8. Both wheels of a bicycle have a moment of inertia of 0.20kgm^2, and a radius of 0.30m. When the bicycle is travelling at 6ms^{-1} calculate:
(a) the kinetic energy of rotation of a wheel
(b) the angular momentum of a wheel.

9. A gramophone record of mass 0.1kg and radius 0.15m is dropped onto a turntable which has a moment of inertia of 1.5 x $10^{-2}$$kgm^2$ and is rotating at 3.5$rads^{-1}$.
(a) Find the common angular velocity of the record and the turntable immediately after they interact and move together, assuming that the turntable was rotating freely, having been disconnected from the motor.
(b) Calculate any changes in the kinetic energy of the system as a result of the interaction.

10. A figure skater has a moment of inertia of 1.2kgm^2 when her arms are pulled into her sides and the axis of rotation is the vertical axis which runs centrally down through her body. Her moment of inertia about this axis becomes 8.0kgm^2 when both arms are stretched out horizontally and one leg is raised into a horizontal position.
(a) If she spins at 25$rads^{-1}$ in the first position, calculate her angular momentum and angular velocity when she changes into the second position.
(b) What is the change in her kinetic energy of rotation as a result of this change in position and why is there a change?

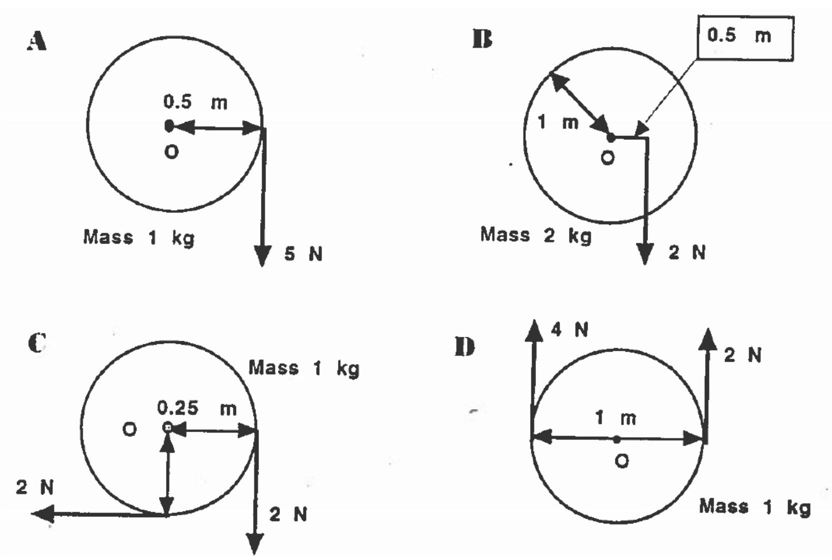
11. In a large piece of machinery, a shaft has a moment of inertia about its axis of $30kgm^2$ and an angular velocity of $600rads^{-1}$. It locks end-on to a similar shaft of moment of inertia $20kgm^2$ which is stationary. Find the angular velocity of the combination assuming that frictional effects are negligible.

12. A playground roundabout has a moment of inertia of $200kgm^2$ and it is rotating so that its rim is moving at $1.5ms^{-1}$. A reckless youth of 50kg mass jumps on to the rim from a stationary position. If the radius of the roundabout is 2 metres, calculate the new speed of the rim when the youth is at rest relative to the roundabout.

13. A man with a mass of 80kg is standing 2.5 metres from the centre of a horizontal circular platform. The platform has a moment of inertia about the vertical axis through its centre of $1000kgm^2$ and is rotating with an angular velocity of $0.8rads^{-1}$ while the man stands 2.5 metres from the centre. The man then walks towards the centre until he is 0.5 metres from the centre.
(a) Calculate the man's moment of inertia when he is 2.5m from the axis.
(b) Calculate the man's moment of inertia when he is 0.5m from the axis.
(c) Assuming that the platform was rotating freely on friction-free bearings, find the new angular velocity after the man changes his position.

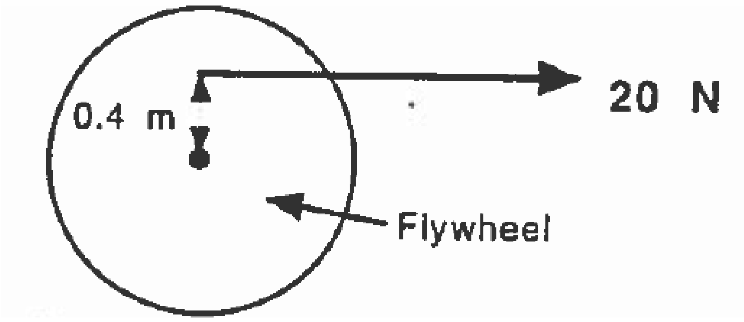
Rotating Bodies

1. The diagrams below show a series of discs free to rotate about an axis through 0, their centre, and normal to their plane. For each disc find:
(a) the moment of inertia
(b) the unbalanced torque acting on it
(c) the resulting angular acceleration.



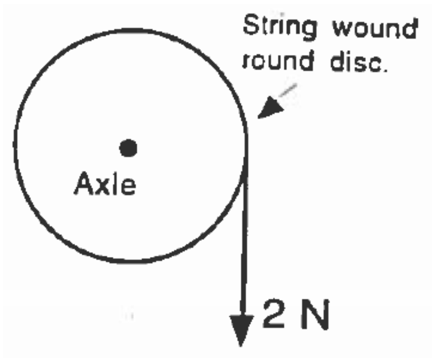
2. A 1kg hoop of radius 2 metres is free to rotate about an axis through its centre, normal to its plane. Calculate the unbalanced torque which must have acted on the hoop to cause angular accelerations of:-
(a) $5rads^{-2}$ (b) $0.25rads^{-2}$ (c) $10rads^{-2}$

3. A flywheel has a moment of inertia about its axle of $2kgm^2$. A constant force of 20N is applied tangentially to the wheel at a distance of 0.4 metres from the axle. Calculate,
(a) the torque acting
(b) the angular acceleration
(c) the angular velocity 4 seconds after starting from rest.
(d) the kinetic energy. of rotation 4 seconds after starting from rest.



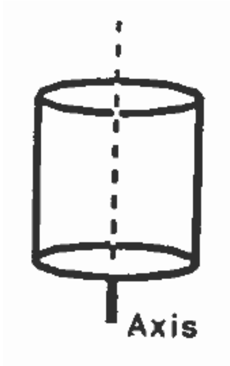
(e) What will the angular velocity of the flywheel be 10 seconds after starting from rest?
(f) How many rotations has this flywheel made 10 seconds after starting from rest?
(g) What braking torque would be required to stop the flywheel in a further 2s?

4. A string is wound around a disc of mass 2kg and radius 0.5m. The string is pulled with a steady force of 2N.



The disc is free to rotate about a horizontal axle through its centre and at right angles to the plane of the disc.
(a) Calculate the moment of inertia of the disc.
(b) Find the unbalanced torque acting on the disc.
(c) The angular acceleration caused.
(d) 5 seconds after the unbalanced torque starts the disc rotating, the string comes off the disc.
(i) Find the angular velocity at this instant.
(i i) Find the angular displacement during these 5 seconds.

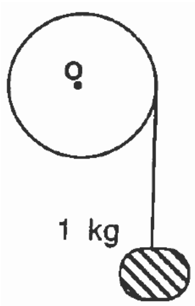
5. The drum of a spin dryer has a moment of inertia of $0.18kgm^2$.



When it is rotating at 600 rpm the braking mechanism is triggered and in 2 seconds the drum's rotation rate is 240 rpm.
(a) Calculate the constant retarding torque required to cause this uniform reduction in speed.
(b) What is the decrease in rotational kinetic energy of the drum ?
(c) How many rotations did the drum make during these 2 seconds of slowing down?

6. A large flywheel, having a moment of inertia of 100kgm^2 is driven by a motor which develops a torque of 1.1kNm. The frictional torque on the flywheel is 100Nm.
(a) Calculate the angular acceleration of the flywheel.
(b) Find the angular velocity 50 seconds after starting from rest.
(c) The motor is now switched off and disconnected from the flywheel after 50 seconds. Find how long it will take the flywheel to come to rest.

7. A 1kg mass is attached to string wound around the circumference of a metal disc which has a moment of inertia about axis O of 0.25kgm^2.



The radius of this disc is 0.5 metres. The disc is free to rotate about the axis at O with negligible friction.
If the system is initially at rest, calculate the acceleration of the falling mass and the tension in the string.
[Use g = 10 Nkg^{-1}]
{Hint: Remember that the force of gravity on the 1kg mass accelerates the 1kg mass linearly and accelerates the disc rotationally, at the same time.}

Energy and Rotating Bodies

1. How much work is done by a 5Nm torque when it gives a rotating body the following angular displacements;
(a) 20rad (b) 5krad (c) 0.1rad (d) 10mrad?

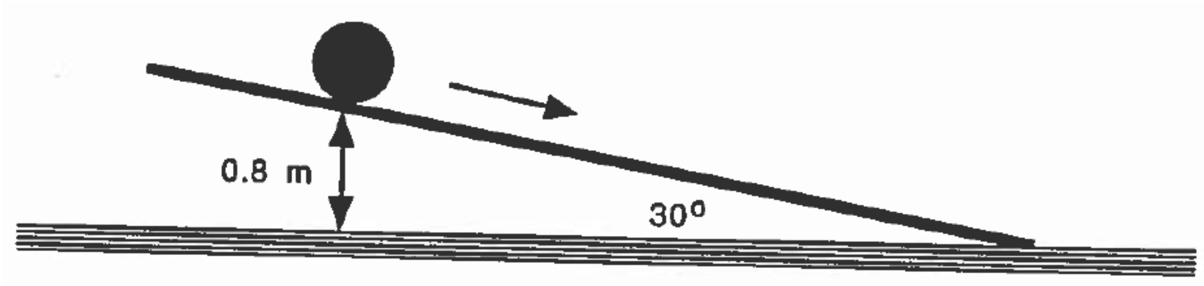
2. A torque of 2Nm acts on a disc of moment of inertia 0.5kgm^2 which is initially at rest.
(a) Find the work done by the torque when the disc has rotated through an angular displacement of 50rad.
(b) What is the kinetic energy of rotation of the disc at the end of this angular displacement, assuming negligible friction?

3. A spin dryer drum of moment of inertia 5 x 10^{-2}kgm^2 is rotating at 100rads^{-1}.
(a) What is the kinetic energy of rotation of the drum?
(b) If it is brought to a stop in 5 seconds, at what rate is its energy dissipated?

4. A flywheel having a moment of inertia of 2kgm^2 is driven by a 2kW electric motor. Assuming negligible friction, what will be,
(a) the kinetic energy of rotation of the flywheel, and
(b) its angular velocity, 10 seconds after the motor has started up the flywheel from rest?

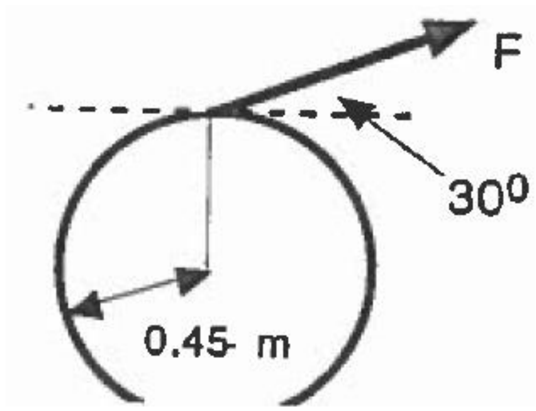
5. A wheel has a moment of inertia of 0.5kgm^2 and is driven by a 100W motor. If the friction acting on the wheel converts energy into heat at the rate of 20Js^{-1}, calculate the kinetic energy of rotation and angular velocity of the wheel 20 seconds after starting from rest.

6. A solid metal cylinder having a mass of 2kg a radius of 0.04m rolls down the slope, shown below, without slipping. The cylinder is started from vertical height 0.8 metres above the foot of the slope which is at 30 degrees to the horizontal.



(a) Calculate the moment of inertia of the cylinder assuming that it can be treated like a uniform disc ..
(b) The loss in gravitational potential energy between the starting point and the foot of the slope.
(c) State clearly what happens to this potential energy by the time the cylinder reaches the foot of the slope, assuming that a negligible amount of energy is converted into unwanted heat.
(d) Find the linear speed and angular velocity of the cylinder at the foot of the slope.

7. A force of 200N is applied, for a short time, at the angle shown, to the rim of a disc of radius 0.45 metres and moment of inertia 1.2kgm^2.



(a) What torque acts on the disc?
(b) If the disc is freely pivoted at its centre, find the initial angular acceleration of the disc.

Gravitation and Satellite Motion

1. Calculate the force of gravitational attraction between:
(a) A 60kg boy and a 50kg girl, 1 metre apart.
(b) Two 1kg mass, 50cm apart.
(c) The Earth and the Sun.
(d) The Earth and the Moon.

2. Calculate from Newton's Law of Gravitation the force of gravity acting on a 1kg mass:
(a) on the surface of the Earth
(b) on the surface of the Moon
(c) at a point 10km above the Earth's surface.
(d) What is the value of "g" in each of these positions?

3. What is the value of "g" at the surface of the Sun?

4. At what height above the Earth's surface would the force of gravity on a rocket be half the value it is at the surface of the Earth?

5. (a) Neglecting air resistance, with what speed would a projectile have to be launched horizontally just above the Earth's surface so that it would orbit the Earth in a circular path?
(b) What would be the period of the projectile's orbit?

6. A 200kg satellite orbits the Earth at a height of 230km.
(a) What is the force of gravity on this satellite?
(b) What is its orbital speed?
(c) What is the periodic time of the orbit?

7. A geostationary satellite is placed in orbit over the equator.
(a) What is the periodic time of the orbit?
(b) What is its orbital speed?
(c) What is its height above the equator?
(d) What is its centripetal acceleration?

8. Taking the value of "g" at the Earth's surface to be $10Nkg^{-1}$ and using the value of the Earth's radius given in the Data List, together with the value of G, but no other Information, calculate
(a) the Earth's mass
(b) the Earth's volume and
(c) the mean density of the Earth.
[Volume of a sphere, $V = (4/3)\pi R^3$]

9. (a) Calculate the gravitational force on an electron which is moving in a circular orbit of radius 5.3×10^{-11}m about a proton.
(b) If the speed of the electron in orbit is about $2 \times 10^6 ms^{-1}$, calculate the centripetal force acting on the electron.
(c) Is the centripetal force acting on the electron provided by the gravitational force? If not, what force does provide the centripetal?

Gravitational Field Strength

1. Calculate the gravitational field strength at the following distances from a 10kg point mass. Ignore the contribution which other masses such as the Earth would make, that is, find the gravitational field strength caused by the 10kg mass only.
(a) 1 metre (b) 2 metres (c) 4 metres (d) 10 metres.

2. Two 1kg point masses are placed 10 metres apart.
(a) Calculate the gravitational field strength (size and direction) caused by these two masses only, at points 1m, 2m, 3m, 4m, and 5m from either mass, along the line joining them.
(b) Sketch a graph showing how the gravitational field strength changes as you move from one mass to the other.

3. A 5kg mass is placed 0.1m from a point mass of unknown size. The 5kg mass experiences a force of gravitational attraction of 6.7×10^{-7}N caused by the unknown mass only.
(a) What is the gravitational field strength at 0.1 metres from the unknown mass?
(b) What is the size of the unknown mass?

4. A 1kg point mass and a 2kg point mass are placed 1 metre apart.
Calculate the gravitational field strength halfway between them. (Size and direction!)

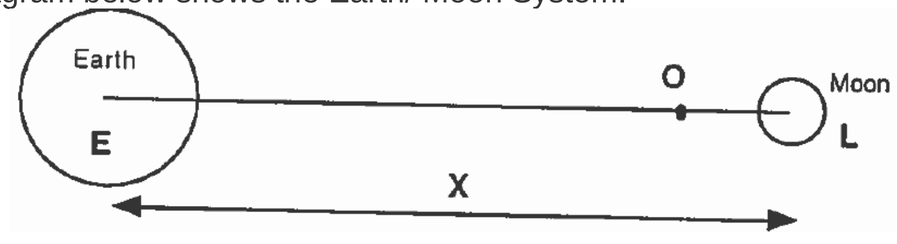
5. (a) Show that the gravitational field strength on the surface of any planet, caused by the planet is

$$\frac{GM}{R^2}$$

where M and R are the mass and radius of the planet respectively. You may assume that the planet is symmetrical and can be treated as a point mass.
(b) Calculate the gravitational field strength on the surface of (i) Mars and (ii) Jupiter.
(Jupiter: Mass= 2.0×10^{27}kg, Radius = 7.2×10^{7}m)

6. Calculate the gravitational field strength caused by the Earth at the following altitudes:
(a) 10km above the surface (jet airliners),
(b) 600km above the surface (orbiting satellites).

7. The diagram below shows the Earth/ Moon System.



Earth/Moon distance = X, Earth Mass = E, Moon Mass = L
Show that the gravitational field strength at the mid-point between the Earth and the Moon is

$$\frac{4G}{X^2}(E - L)$$, directed towards the Earth.

Gravitational Potential

1. A 10kg mass is moved from a point in the gravitational field of a planet where the gravitational potential is $-5Jkg^{-1}$ to another point where the potential is $-25Jkg^{-1}$.
(a) What is the potential difference between the two points?
(b) Calculate the gain in the gravitational potential energy of the 10kg mass.
(c) How much work was done to move the 10 kg mass from the first to the second point?
(d) Was this work done by the gravity of the planet or by an external agency?

2. Calculate the gravitational potential at the following distances from a 1kg point mass:
(a) 1m
(b) 2m
(c) 4m
(d) 10m.

3. Two 1kg mass are placed 1 metre apart. How much work must be done (against gravitational attraction) to double their separation ?

4. Calculate the gravitational potential at the Earth's surface. (Use Data List).

5. What is the gravitational potential at an infinite distance from a 100000 kg mass?

6. Calculate the gravitational potential caused by a mass of 1×10^5kg at distances of,
(a) 0.01 m
(b) 1m
(c) 10m
(d) 1km

7. At what distance from a 5×10^5kg point mass would there be a gravitational potential of
(a) $-3.335 \times 10^{-5} Jkg^{-1}$
(b) $-1.6675 \times 10^{-6} Jkg^{-1}$
(c) -6.67 x 10-9 Jkg^{-1}
(d) $-3.335 \times 10^{-12} Jkg^{-1}$?

8. An asteroid has a mass of 1×10^{14}kg. It is estimated that the surface of the asteroid is 50 metres from the centre of mass. How much energy is needed to remove a 100kg space probe from the surface to infinity?

9. A satellite of mass 200kg in orbit 30 000km above the surface of the Earth is transferred to an orbit 20 000km above the surface of the Earth.
(a) Calculate the change in gravitational potential energy.
(b) Calculate the change in kinetic energy.
(c) Suggest what operation must be carried out to transfer the satellite from a given circular orbit to one of a smaller radius.

10. A spacecraft on a deep space mission is in circular orbit of radius 5.0×10^8km around the Sun. The mass of the spacecraft is 8.0×10^2kg. The mass of the Sun is 2.0×10^{30}kg.
(a) Calculate the gravitational force between the spacecraft and the Sun.
(b) Find the speed of the spacecraft in this orbit.
The potential of the Sun's gravitational field at this distance from the Sun is -2.7×10^8 Jkg^{-1}.
(c) Explain what this quantity means and the significance of the negative sign.
(d) Calculate the gravitational potential energy when the spacecraft is at this distance from the Sun.
(e) Calculate the speed the spacecraft must attain to escape from the solar system from this orbit.

11. Show that the escape velocity, v, from any planet of mass M and radius R is

$$V = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

where g is the gravitational field strength on the planet's surface.

Calculate the escape velocity from the following planets:
Mercury, mass 3.28×10^{23}kg, radius 2.57×10^6m
Saturn, mass 5.67×10^{26}kg, radius 6.03×10^7m.

12. Calculate the escape velocity from the Sun.

13. (a) Write an expression for the gravitational potential at a height h above the Earth's surface, caused by the Earth itself.
(b) Show that, in general, the work done against gravity in moving a mass m a distance h from the Earth's surface is

$$\frac{GMmh}{R(R+h)}$$

where M and R are the Earth's mass and radius respectively.
Hence show that if $h << R$ the work done is mgh,

17. If you were to move closer and closer to an ultra-dense object, a position would be reached where the gravitational potential would require that the escape velocity be greater than the speed of light. The closest approach to such an object which will still allow escape is given by the following expression

$$r = \frac{2GM}{c^2}$$

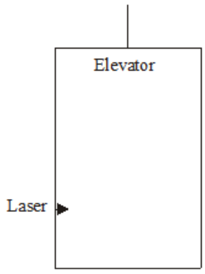
where M is the mass of the object, c is the speed of light and r is the distance of closest approach.
Calculate r for an ultra-dense object having a mass of (i) 10kg (ii) equal to the Sun.

Space and Time

<u>Equivalence principle</u>

1. State Einstein's principle of equivalence.

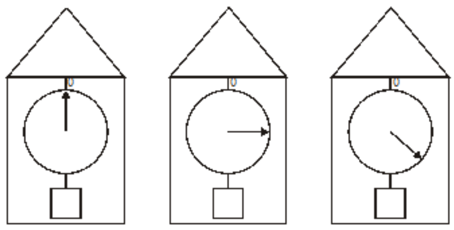
2. In a thought experiment, a laser is attached to the wall of a high-speed lift as shown in the diagram below. The laser is set to emit a beam parallel to the floor of the stationary elevator.



The elevator is set in motion from the top of a tall building. While it is still accelerating downward the laser beam is fired (shot 1), it is fired again (shot 2) once the elevator is moving with constant velocity, and again (shot 3) as the elevator slows down.

(a) On the diagram, draw the path of the laser beam as seen by an observer in the elevator for
(i) constant speed, label this line C;
(ii) constant downward acceleration, label this line D;
(iii) constant upward acceleration, label this line U.
(b) Explain how the paths you have drawn in (a) are consistent with the principle of equivalence.

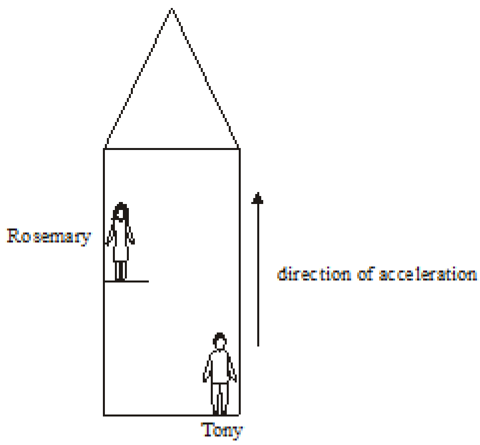
3. A spacecraft is initially at rest on the surface of the Earth. It then accelerates away from Earth into deep space where it then moves with constant velocity. There is a spring balance supporting a mass from the ceiling.
The diagrams show the readings on the spring balance at these different stages of the motion.



A B C

(a) Identify and explain, in each case, the motion of the spacecraft that could give rise to the reading shown
(i) at rest on the Earth's surface.
(ii) moving away from Earth with acceleration.
(iii) moving at constant velocity in deep space.
(b) The spacecraft now accelerates in deep space with an acceleration equal to the acceleration of free fall at the Earth's surface.
State and explain which of the readings A, B or C would now be observed on the spring balance.

4. Tony and Rosemary are in an accelerating spaceship as shown below.



Rosemary throws a ball in a direction horizontal to the floor that Tony is standing on.

(a) On the diagram, draw the path of the ball as seen by Tony.

(b) Describe how this situation relates to Einstein's principle of equivalence.

<u>Spacetime</u>

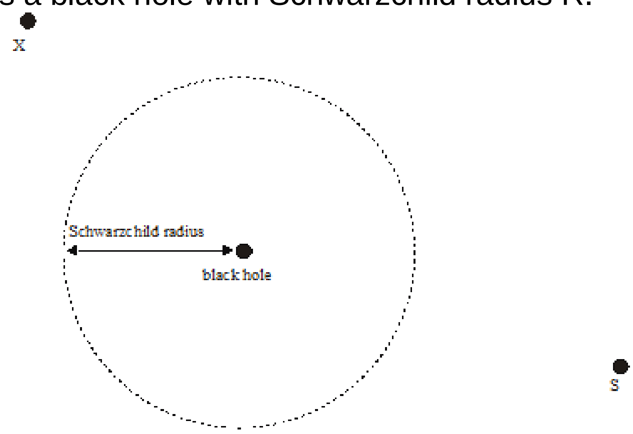
5. Use the concept of spacetime to
(a) explain the gravitational attraction between the Earth and an orbiting satellite;
(b) describe what is meant by a black hole.

6. A satellite is in orbit about Earth.
(a) Outline how the concept of spacetime is used to account for the orbital motion of the satellite.
(b State the reason why the gravitational force of attraction between the satellite and Earth decreases with distance from Earth.
Black Holes and the Schwarzschild radius

7. (a) Describe, by reference to space-time, what is meant by a black hole.
(b) After a particular star has become a supernova, its mass is 2 x 10^{31}kg. Determine the radius of the black hole it subsequently forms.

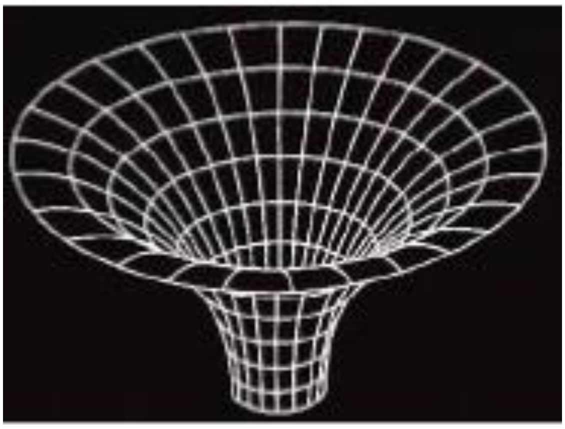
8. (a) Describe what is meant by a black hole.
(b) Estimate the radius of the Sun for it to become a black hole. ($M_{Sun} \approx 2 \times 10^{30}$ kg)

9. (a) Define the Schwarzchild radius of a black hole.
(b) The diagram shows a black hole with Schwarzchild radius R.



An observer at X sends a signal that is received by the spacecraft S.
(i) On the diagram, draw a line to indicate a possible path of the radio signal.
(ii) Explain the path you have drawn.

10. The diagram below illustrates the distortion of space by the gravitational field of a black hole.



(a) Describe what is meant by the centre and the surface of a black hole.
(b) With reference to your answer in (b) define the Schwarzschild radius.
(c) Calculate the Schwarzschild radius for an object having a mass of 2.0×10^{31}kg (ten solar masses).
(d) A spacecraft approaches the black hole in Q11(c). If it were to continue to travel in a straight-line it would pass within 10^{6}m of the black hole.
(i) Suggest what effect the black hole would have on the motion of the spacecraft.
(ii) Explain gravitational attraction in terms of the warping of space-time by matter.

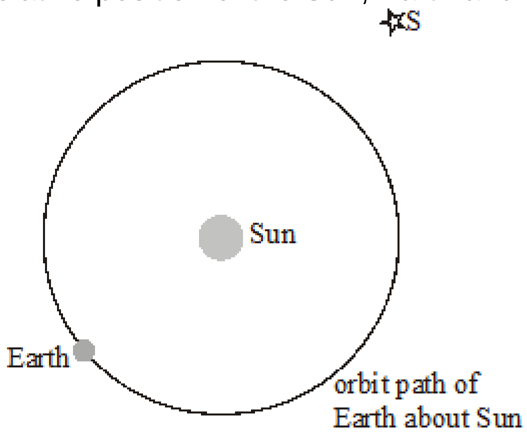
Gravitational lensing

11. In 1979, Wahl, Carswell and Weymann discovered "two" very distant quasars separated by a small angle. Spectroscopic examination of the images showed that they were identical.
Outline how these observations give support to the theory of General Relativity.

12. One prediction of Einstein's general theory of relativity is the effect of "gravitational lensing". This effect can be predicted from the principle of equivalence. Use the principle of equivalence to explain gravitational lensing.

13. (a) How are photons affected by a massive object such as the Sun?
(b) Explain, using a sketch, why light from a distant star passing close to the Sun may suggest that the star is at a different position from its 'true' position.

14. On 29 March 1919, an experiment was carried out by Eddington to provide evidence to support Einstein's General Theory of Relativity. The diagram below (not to scale) shows the relative position of the Sun, Earth and a star S on this date.



This particular date was chosen because at the place where the experiment was carried out, there was a total eclipse of the Sun.

Eddington measured the apparent position of the star and six months later, he again measured the position of the star from Earth.

(a) State why it was necessary for there to be a total solar eclipse to carry out the experiment.

(b) Explain why it was necessary to measure the position of the star six months later.

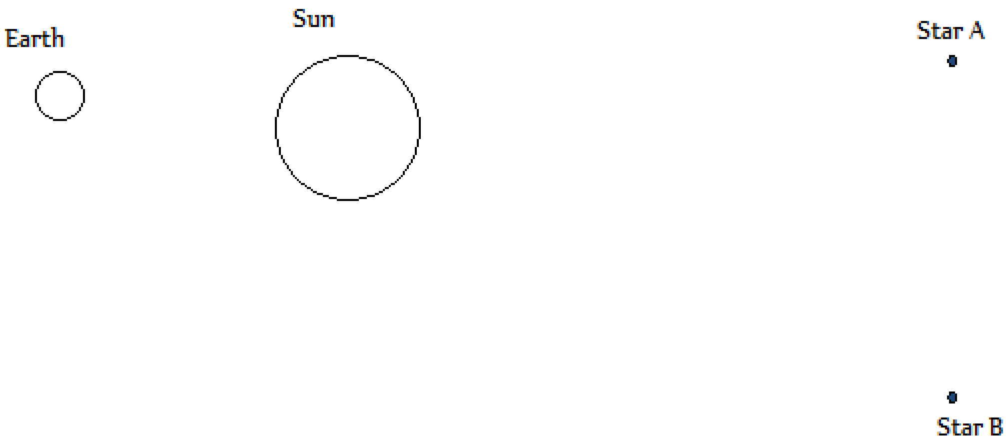
(c) On the diagram, draw the path of a ray of light from S to the Earth as suggested by Einstein's theory.

(d) Explain how Einstein's theory accounts for the path of the ray that you have drawn.

(e) On the diagram, label with the letter A, the apparent position of the star as seen from Earth.

(c) Suggest two reasons why the planet Jupiter cannot become a black hole.

15. The diagram below shows Earth, Sun and two distant stars A and B.

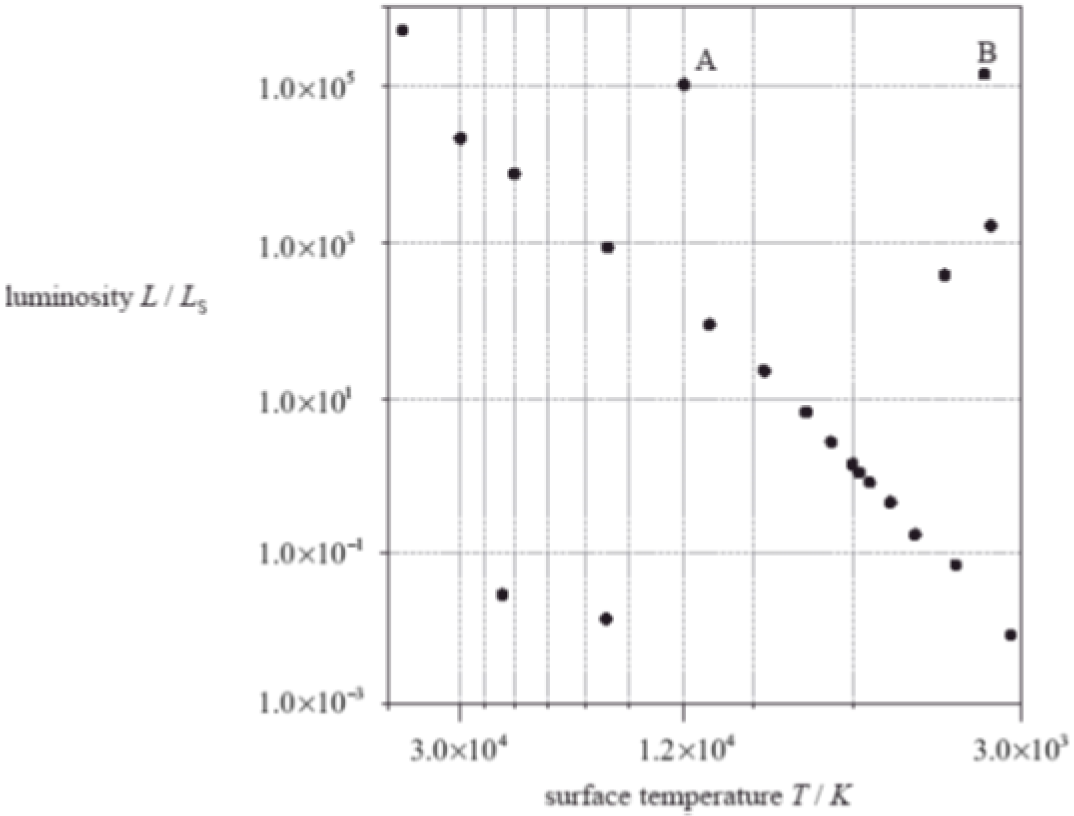


(a) Add rays to the diagram to show the path of light from star A and star B to Earth.

(b) Describe briefly how Eddington's observations provided evidence for the paths you have drawn in (i).

Stellar Physics

1. The diagram below shows the grid of an HR diagram, on which the positions of selected stars are shown. (LS = luminosity of the Sun.)



(a) (i) Draw a circle around the stars that are red giants. Label this circle R.
(ii) Draw a circle around the stars that are white dwarfs. Label this circle W.
(iii) Draw a line through the stars that are main sequence stars.
(b) Explain, without doing any calculation, how astronomers can deduce that star B has a larger diameter than star A.
(c) Using the following data and information from the HR diagram, show that star A is at a distance of about 1.7×10^8 AU from Earth.
Apparent brightness of the Sun = $1.4 \times 10^3 Wm^{-2}$.
Apparent brightness of star A = $4.9 \times 10^{-9} Wm^{-2}$
Mean distance of Sun from Earth = 1.0 AU
(d) Explain why the distance of star A from Earth cannot be determined by the method of stellar parallax.

2. Becrux is a main sequence star and is one of the stars that make up the Southern Cross. The following data are available for Becrux.
Magnitude = 1.25 Distance from Earth = 2.21×10^7 AU
Apparent brightness = 7.00×10^{-12} bSun
bSun is the apparent brightness of the Sun.
(a) Show that the luminosity of Becrux is 3.43×10^3 LSun where LSun is the luminosity of the Sun.
(b) The mass of Becrux is about 10MSun where MSun is the mass of the Sun. State the differences between the eventual fate of the Sun and Becrux after they leave the main sequence.

3. Describe the final evolutionary state of
(a) a low-mass star (of the order of 1 solar mass).
(b) a high-mass star (of approximately 15 solar masses).

4. Betelgeuse and Rigel are two super giants in the constellation of Orion.
The table below gives some information about the types and magnitudes of Betelgeuse and Rigel.

Star	Type	Magnitude	Apparent brightness
Betelgeuse	M	−0.04	2.0×10^{-7} W m^{-2}
Rigel	B	0.12	3.4×10^{-8} W m^{-2}

(a) Complete the above table for the colours of the stars.
(b) State why Betelgeuse has a lower apparent magnitude than Rigel.
(c) Given that the distance of Betelgeuse from Earth is 4.0×10^{18}m, calculate the luminosity of Betelgeuse.
(d) The luminosity of Rigel is 2.3×10^{31}W. Without any further calculation, explain whether Rigel is closer or further than Betelgeuse from Earth.

5. Eta Carinae is a main sequence star whose mass is about 100 times larger than that of the Sun. The star will evolve to become a neutron star.
Outline the evolution of Eta Carinae from when it leaves the main sequence until the neutron star stage.

6. State the two quantities that need to be measured in order to use a Cepheid variable as a “standard candle” to determine the distance to the galaxy in which the Cepheid is located.

7. The star Antares is a red supergiant star in the constellation Scorpius.
(a) Describe three characteristics of a red supergiant star.
(b) The apparent brightness of Antares is 4.3×10^{-11} times the apparent brightness of the Sun.
(i) Define apparent brightness.
(ii) The distance of Antares from Earth is 3.9×10^{7} AU. Show that Antares is 6.5×10^{4} times more luminous than the Sun.

8. This question is about the relative population density of stars and galaxies.
The number of stars around the Sun, within a distance of 17 ly, is 75. The number of galaxies in the local group, within a distance of 4.0×10^{6} ly from the Sun, is 26.
(a) Calculate the average population density, per ly^3, of stars and galaxies.
(b) Use your answer to (a) to determine the ratio

$$\frac{average\ population\ density\ of\ stars}{average\ population\ density\ of\ galaxies}$$

9. The table gives information on three stars, Achernar, EG 129 and Mira.

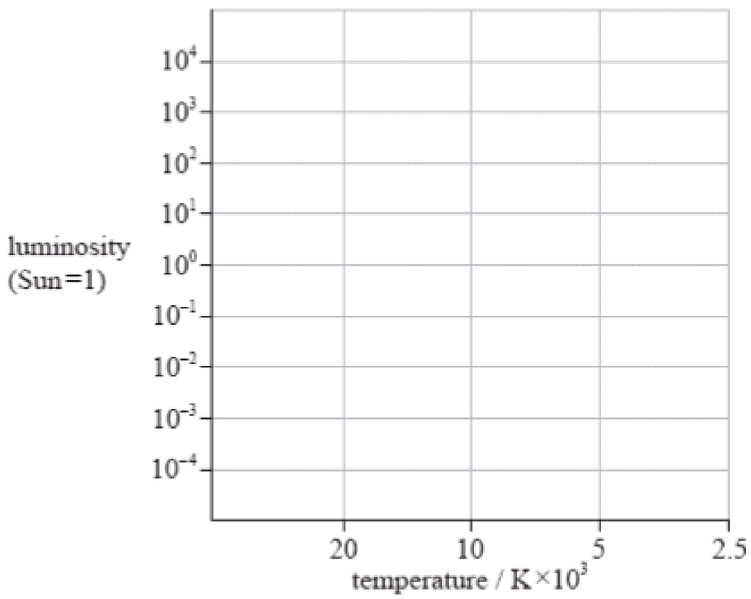
	Magnitude	**Spectral class**
Achernar	+0.50	B
EG 129	+14.0	B
Mira	+5.0	M

(a) State which one of the three stars appears brightest from Earth.
(b) The luminosities of Mira and Achernar are approximately the same. EG 129 has a much lower luminosity. The surface temperature of Mira is 5 times lower than that of Achernar.
Estimate the ratio RM/ RA where RM is the radius of Mira and RA is the radius of Achernar.
(c) State and explain which of the stars in the table above is a white dwarf

10. The stars Procyon A and Procyon B are both located in the same stellar cluster in the constellation Canis Minor. The two stars are approximately the same distance from Earth.
The table shows some data for Procyon A and Procyon B.

	Magnitude	**Apparent brightness / W m^{-2}**
Procyon A (P_A)	+0.400	2.06×10^{-8}
Procyon B (P_B)	+10.7	1.46×10^{-12}

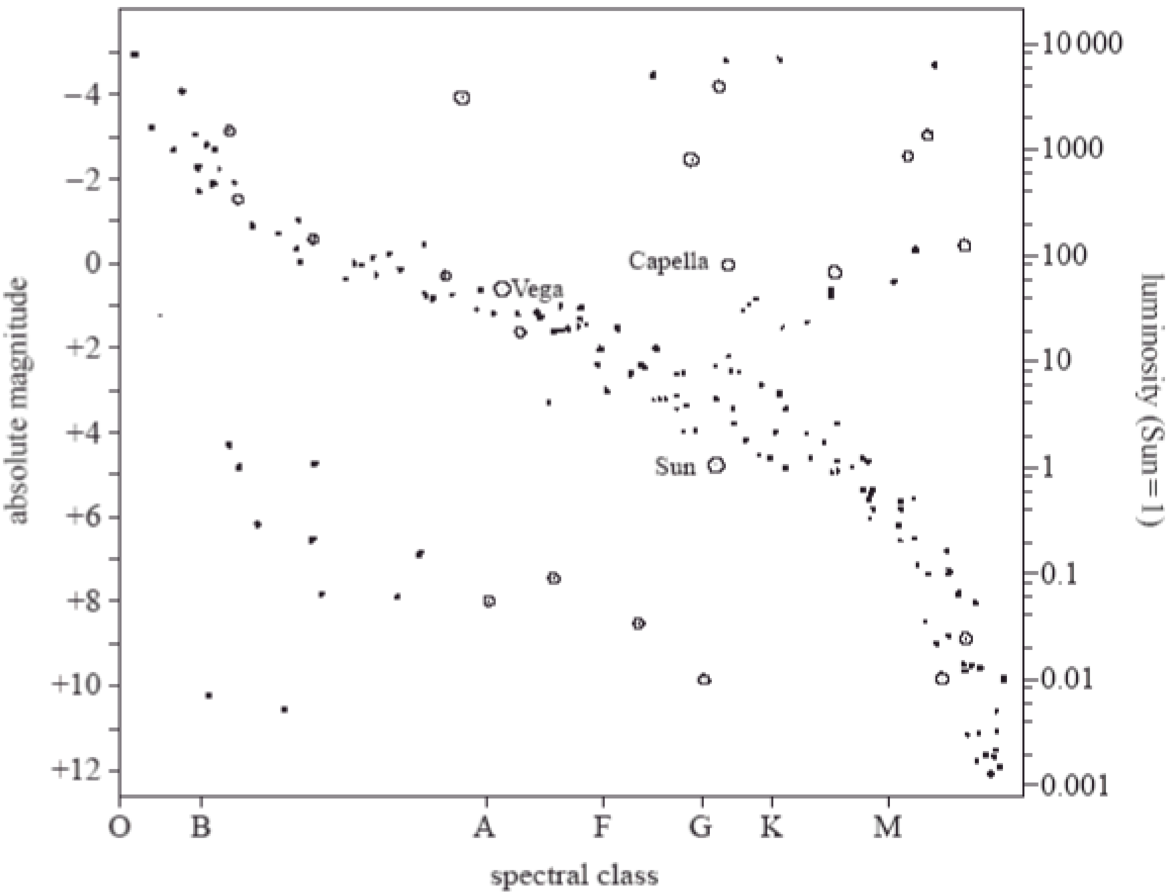
(a) Explain, using data from the table, why as viewed from Earth, PA is much brighter than PB.
(b) Calculate, using data from the table above, the ratio LA/LB where LA is the luminosity of PA and LB is the luminosity of PB.
(c) The surface temperature of both PA and PB is of the order of 104 K. The luminosity of PA is of the order of 10LS, where LS is the luminosity of the Sun. The diagram shows the grid of a Hertzsprung–Russell diagram.



Copy the grid above, and on it label the approximate position of
(i) star PA with the letter A.
(ii) star PB with the letter B.
(d) Identify the nature of star PB.
(e) On the grid provided in (f), draw the evolutionary path of the star PA.

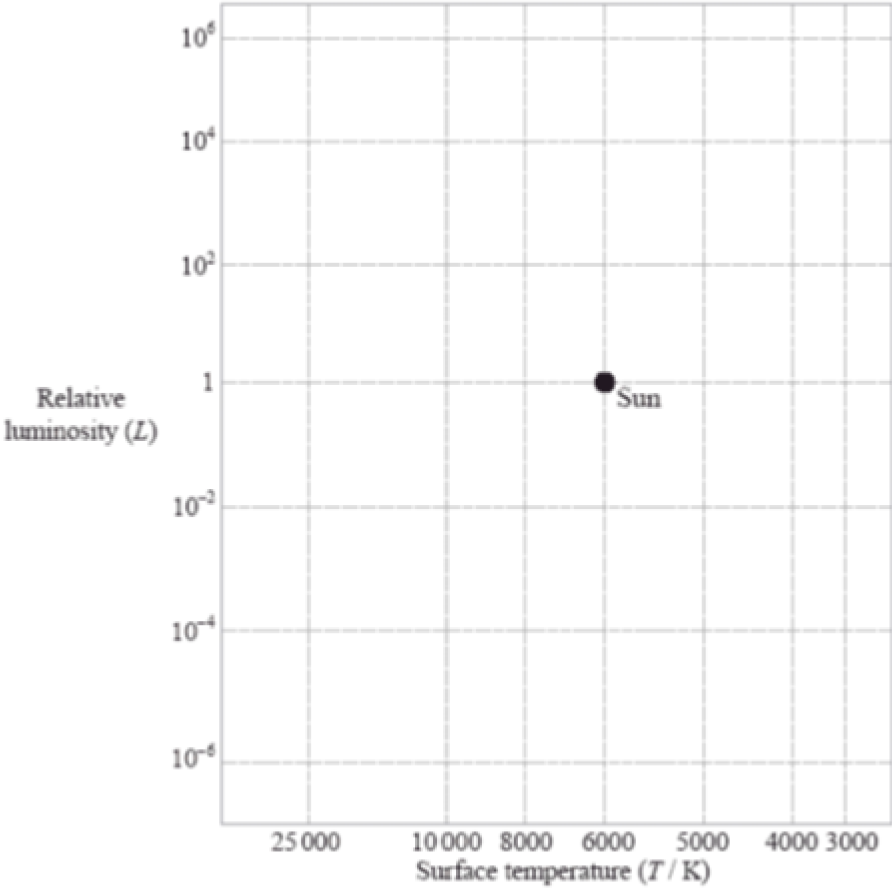
11. (a) Describe how a large cloud of hydrogen gas can lead to conditions that initiate a fusion reaction.
(b) State the property of a main sequence star that determines for how long hydrogen in its core fuses into helium.
(c) State the end product of nuclear fusion processes in the core of
(i) a red giant.
(ii) the largest red super giants.

12. The Hertzsprung–Russell (HR) diagram shows the variation with spectral class of the absolute magnitude of stars.



The star Capella and the Sun are in the same spectral class (G). Using the HR diagram,
(a) (i) suggest why Capella has a greater surface area than the Sun.
(ii) estimate the luminosity of Capella in terms of that of the Sun.
(iii) calculate the radius of Capella in terms of that of the Sun.
(b) Light from Vega is absorbed by a dust cloud between Vega and Earth. Suggest the effect, if any, this will have on determining the distance of Vega from Earth.
(c) Vega is a very massive star. State why Vega does not undergo gravitational collapse.

13. Some data for the variable star Betelgeuse are given below.
Average magnitude = + 0.60 Average apparent brightness = $1.6 \times 10^{-7} Wm^{-2}$
Radius = 790 solar radii
The luminosity of the Sun is 3.8×10^{26}W. It has a surface temperature of 5700K.
The distance from Earth to Betelgeuse is about 4.0×10^{18}m.
(a) Determine, in terms of the luminosity of the Sun, the luminosity of Betelgeuse.
(b) Calculate the surface temperature of Betelgeuse.

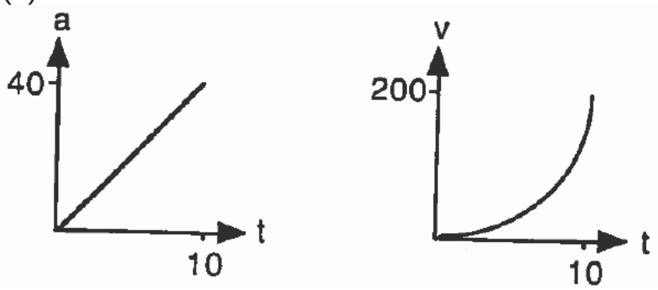


(c) Copy the Hertzsprung–Russell diagram above,
(i) On it label the position of Betelgeuse with the letter B.
(ii) sketch Betelgeuse's likely evolutionary path.

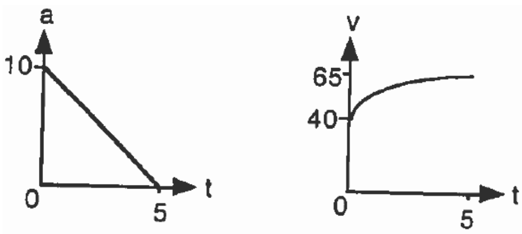
ANSWERS

Motion of a particle

1 (a) $v = 2t^2$ (b) $s = \frac{2t^3}{3}$
(c)



2 (a) $v = 10t - t^2 + 40$
(b)



3 (a) v = 30-10t (b) a= $-10ms^{-2}$ (c) t=6s (d) x=45m

4 (a) v = 5 + 2t {b) a = $2ms^{-2}$ (c) t = 1 s (d) v = 7 ms^{-1}

5 (a} v = -0.6sin(6t) (b) a= -3.6cos(6t)
(c) (i) x = 0.1m v = $0ms^{-1}$ a = $-3.6ms^{-2}$
(ii) x = 0m v = $-0.6ms^{-1}$ a = $0ms^{-2}$
(iii) x = -0.1m v = $0ms^{-1}$ a = $3.6ms^{-2}$
(iv) x = 0m v = $0.6ms^{-1}$ a = $0ms^{-2}$
(v) x = 0.1m v = $0ms^{-1}$ a = $-3.6ms^{-2}$
(d) T = $\frac{\pi}{3}$s

6 (a) $v_x = 60ms^{-1}$ (b) y = 45m (c) x =360m

7 (a) $v_x = 6\cos(12t) + 6$ $v_y = 6\sin(12t)$ $a_x = -72\sin(12t)$ $a_y = 72\cos(12t)$
(b) **$Y_{(max)}$**
$v_x = 0ms^{-1}$ $v_y = 0ms^{-1}$ $a_x = 0ms^{-2}$ $a_y = -72\ ms^{-2}$
$Y_{(mln)}$
$v_x = 12ms^{-1}$ $v_y = 0ms^{-1}$ $a_x = 0ms^{-2}$ $a_y = 72ms^{-2}$

8 (a) s = 21.3m (b) m = 2 (c) a = 4 – t (d) a = $2ms^{-2}$

Circular motion

1 (a) $\Theta = \frac{\pi}{3}$ rad (b) $\omega = \frac{5\pi}{3}$ $rads^{-1}$ (c) T = 1.2s

2 (a) $v = \frac{\pi}{2} ms^{-1}$ (b) $\omega = \pi rads^{-1}$ (c) t = 0.25s

3 (a) T = 20πs (b) $\omega = 0.1 rads^{-1}$

4 (a) $\Theta = \frac{\pi}{2}$ rad (b) $\omega = \frac{\pi}{60}$ $rads^{-1}$ (c) t = 240s

5 (a) $\omega = 160\pi rads^{-1}$ (b) T = 12.5ms

6 $\Theta = \frac{18000}{\pi}$ revs

7 (a) $\omega = 200\pi rads^{-1}$ (b) t = 5ms

Constant Angular Acceleration

1 $\alpha = 6 rads^{-2}$

2 (a) t = 50s (b) Θ = 250rad

3 (a) Θ = 50πrad (b) $\alpha = \frac{3}{\pi} rads^{-2}$ (c)) $t = \frac{10\pi}{3}$ s

4 $\alpha = -1 rads^{-2}$

5 (a) $\omega = 30\pi rads^{-1}$ (b) $\alpha = \frac{3\pi}{2} rads^{-2}$ (c) $\Theta = \frac{3\pi}{4}$ rad (d) $\omega = 7.75\pi rads^{-1}$

6 (a) $\alpha = -12 rads^{-2}$ (b) $\Theta = \frac{175}{\pi}$ revs

7 (a) $\omega = 10 rads^{-1}$ (b) $\alpha = -1 rads^{-2}$ (c) $\Theta = \frac{25}{\pi}$ revs (d) s =50m (e) α is constant

Centripetal force

1 (a) f = 1.5Hz (b) T = 0.67s (c) $\omega = 3\pi rads^{-1}$ (d) $v = 6\pi ms^{-1}$ (e) $a = 18\pi^2 ms^{-2}$

2 (a) ω = 20rads-1 (b) $a = 40 ms^{-2}$ (c) $T = \frac{\pi}{10}$s (d) $f = \frac{10}{\pi}$ Hz (e) $t = \frac{\pi}{40}$ s

3 (a) $\omega = 0.2 rads^{-1}$ (b) $a = 4 ms^{-2}$ (c) F = 4kN

4 (a) $a = 12.5 ms^{-2}$ (b) F = 0.31N

5 (a) $v = 10 ms^{-1}$ (b) $\omega = 10 rads^{-1}$ (c) $f = \frac{5}{\pi}$ Hz

6 (a) $v = 10 ms^{-1}$

7 $r = 5.25 \times 10^{-11}$ m

8 ω = 7rads^{-1}

9 (a) F = 0.5kN (b) T= 550N (c) T = 450N

10 v = 20ms^{-1}

11 (a) R = 3N (b) v = Ö10ms^{-1}

12(a) s = 4m (b) Θ = 36.87^{o} (c) T = 12.5N (d) F = 7.5N (e) ω = 1.58rads^{-1}, T= 3.97s

13 Θ = 70.53^{o}, ω = Ö30rads^{-1}

14 h = 0.8π^2m

15 (a) Θ = 12.68^{o} (b) Θ = 68.2°

16 h = 3.37cm

17 (a) Tension (b) Tension and weight (c) Tension is greatest

18 Θ = 29.36^{o}

19 A) $F_r = mr\omega^2$ so as ω increases F_r must increase
B) $F_r = mr\omega^2$ so if r is reduced less friction is needed
C) The record has a lower coefficient of friction than the turntable

Angular momentum of a particle

1 L = 4 x 10^{10} kgm^{2}s^{-1}

2 (a) L = 0.12kgm^{2}s^{-1} (b) v = 1.5ms^{-1}

3 v_2 = 2.6ms^{-1}

Moments of inertia

1 (a) I=Σmr^2 (b) B1 : I = mR^2, B2 : I = MR^2, B3: I = $\frac{MR^2}{2}$

2 (a) D, it has the smallest radius of rotation (b) A (c) I = $\frac{ML^2}{4}$ (d) A, largest radius

3 I = 0.1125kgm^2

4 Mass M, Radius 2R

5 E_{rot} = 250J

6 E_{rot} = 28.8π^2J

7 (a) E_{rot} = 1.28J (b) E = 3.84J

8 (a) E_{rot} = 40J (b) L = $4kgm^2s^{-1}$

9 (a) ω_2 = $3.26rads^{-1}$ (b) 6.2mJ lost

10 (a) L = $30kgm^2s^{-1}$, ω_2 = $3.75rads^{-1}$ (b) 318.75J lost energy is converted to heat in the muscles.

11 ω_2 = $360rads^{-1}$

12 v = $0.75ms^{-1}$

13 (a) I_{m1} = $500kgm^2$ (b) 1_{m2}= $20kgm^2$ (c) ω_2 = $1.18rads^{-1}$

Rotating bodies

1 A (a) I = $0.125kgm^2$ (b) T = 2.5Nm (c) α = $20rads^{-2}$

B (a) I = $1kgm^2$ (b) T = 1Nm (c) α = $1rads^{-2}$

C (a) I = $\frac{1}{32}$ kgm^2 (b) T = 1Nm (c) α = $32rads^{-2}$

D (a) I = 0.125 kgm^2 (b) T = 1Nm (c) α = $8rads^{-2}$

2 (a) T = 20Nm (b) T = 1Nm (c) T = 40Nm

3 (a) T = 8Nm (b) α = $4rads^{-2}$ (c) ω =$16rads^{-1}$ (d)E_{rot} = 256J (e) ω = $40rads^{-1}$ (f) $\frac{100}{\pi}$ revs (g) T = -40Nm

4 (a) I = 0.25 kgm^2 (b) T = 1Nm (c) α = $4rads^{-2}$ (d) (i) ω = $20rads^{-1}$ (ii) Θ = 50rads

5 (a) T = -1.08πNm (b) E = $30.24\pi^2$J (c) 14 rotations

6 (a) α = $10rads^{-2}$ (b) ω = $500rads^{-1}$ (c) t = 500s

7 T = 5N, a= $5ms^{-2}$

Energy and rotating bodies

1 (a) E=100J (b) E = 25kJ (c) E = 0.5J (d) E = 50mJ

2 (a) E = 100J (b) E_{rot} = 100J

3 (a} E_{rot} = 250J (b) P = 50W

4 E_{rot} = 20kJ, ω = $141rads^{-1}$

5 E_{rot} = 1.6kJ, ω = 80rads^{-1}

6 (a) I = 1.6 x 10^{-3}kgm^2 (b) E_p = 16J (c) converted into E_{rot} and E_k (d) v = 3.26ms^{-1}

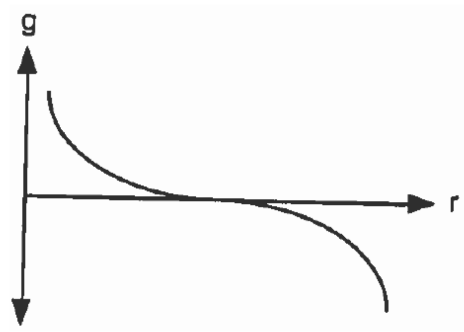
7 (a) T = 77.9Nm (b) α = 64.9rads^{-2}

Gravitation and satellite motion

1 (a) F = 0.2μN (b) F = 26pN (c) F = 3.54 x 10^{22}N (d) F = 1.98 x 1 0^{20}N
2 (a) F = 9.77N (b) F = 1.68N (c) F = 9.74N (d) g is numerically equal to F
3 g = 271Nkg^{-1}
4 h = 2.65 x 10^{6}m
5 (a) v = 7.91kms^{-1} (b) T = 1.41h
6 (a) F = 1.82kN (b) v = 7.77 kms^{-1} (c) T = 1.49h
7 (a) T = 24h (b) v = 3.08 kms^{-1} (c) h = 35.9 x 10^{3}km (d) a = 0.224ms^{-2}
8 (a) m_E = 6.14 x 10^{24}kg (b) V = 1.1 x 10^{21}m^3 (c) ρ = 5582kgm^{-3}
9 (a) F = 3.62 x 10^{-47}N (b) F = 68.8nN (c) Electromagnetic force
10 g_E/g_m = 81/16

Gravitational field strength

1 (a) g = 6.67 x 10^{-10}Nkg^{-1} (b) g = 1.67 x 10^{-10}Nkg^{-1} (c) g = 4.17 x 10^{-11} Nkg^{-1}
(d) g = 6.67 x 10^{-12}Nkg^{-1}
2 (r =1m), g = 6.59 x 10^{-11}Nkg^{-1}
(r =2m), g = 1.56 x 10^{-11}Nkg^{-1}
(r =3m), g = 6.05 x 10^{-12}Nkg^{-1}
(r = 4m), g = 2.32 x 10^{-12}Nkg^{-1}
(r = 5m), g = 0Nkg^{-1}



3 (a) g = 1.34 x 10^{-7}Nkg^{-1} (b) m = 20.1kg
4 g = 2.67 x 10^{-10}Nkg^{-1} towards 2kg mass
5 (b) (i) g = 3.69Nkg^{-1} (ii) g = 25.7Nkg^{-1}
6 (a) g = 9.74Nkg^{-1} (b) g = 8.17Nkg^{-1}

Gravitational potential

1 (a) ΔV = -20Jkg^{-1} (b) ΔE_p = -200J (c) E_w = 200J (d) gravity (E_p was lost)
2 (a) V = -6.67 x 10^{-11}Jkg^{-1} (b) V = -3.34 x 10^{-11}Jkg^{-1} (c) V = -1.67 x 10^{-11}Jkg^{-1}
(d) V = -6.67 X 10^{-12}Jkg^{-1}

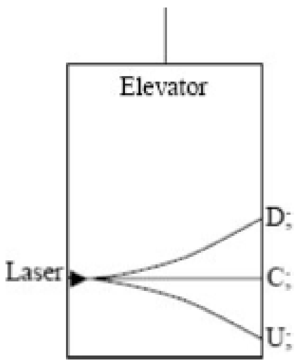
3 $E_W = 3.335 \times 10^{-11}J$
4 $V = -62.5MJkg^{-1}$
5 $V = 0Jkg^{-1}$
6 (a) $V = -6.67 \times 10^{-4}Jkg^{-1}$ (b) $V = -6.67 \times 10^{-6}Jkg^{-1}$ (c) $V = -6.67 \times 10^{-7}Jkg^{-1}$
(d) $V = -6.67 \times 10^{-9}Jkg^{-1}$
7 (a) r =1m (b) r = 20m (c) r = 5km (d) r = 10,000km

8 $E_W = 13.34kJ$
9 (a) $\Delta E_p = -800MJ$ (b) $\Delta E_k = 400MJ$ (c) lose energy by slowing down
10 (a) F = 427mN (b) $v = 16.3kms^{-1}$ (d) $E_p = -2.13 \times 10^{11}J$ (e) $v_e = 23.1kms^{-1}$
11 Mercury, $v_e = 4.13kms^{-1}$ Saturn, $v_e = 35.4kms^{-1}$
12 $v_e = 616kms^{-1}$
14 (i) $r = 1.48 \times 10^{-26}m$ (ii) r = 2.95km

Space and Time

1. It is not possible to distinguish between an accelerating frame and a stationary / inertial frame in a gravitational field;
OR a stationary / inertial frame in a gravitational field is equivalent to an accelerating frame;

2. (a)



(b) C zero acceleration represents zero gravitational field;
D represents a stationary lift in an upward gravitational field
U represents a stationary lift in a downward gravitational field

3. (a) (i) B; because the scale reads the weight of the mass;
(ii) C; because the scale reads a force F where F = mg + ma;
(iii) A; because there is no acceleration and no gravitational force on the mass;
(b) B; because by the equivalence principle the accelerating frame is equivalent to a frame at rest on Earth's surface;

4. (a) curved line (striking the wall) below Rosemary;
(b) Einstein's principle states that it is impossible to distinguish between a system that is accelerating and one that is at rest in a gravitational field. (do not accept "gravity and acceleration are the same/ indistinguishable")
If the spaceship were at rest on the surface of a planet the ball would follow the same path

5. (a) Earth causes warping of spacetime;
satellite follows shortest path in spacetime curve;
(b) black hole causes extreme warping of space in its vicinity;
extreme warping causes photons / light to curve back into the black hole;

6. (a) all particles/objects take the shortest path between two points in spacetime;
Earth warps spacetime; the satellite follows the shortest path in this warped spacetime;
(b) the closer to Earth the greater the degree of warping of spacetime
or
the further from Earth the less the degree of warping of spacetime

7. (a) if object is dense / massive enough it will cause severe warping of space-time;
such that light entering the space-time surrounding the object cannot escape;
Do not accept "light cannot escape".

(b) $R_{SCH} = \frac{2GM}{c^2}$

$= \frac{2 \: x \: 6.67 \: x \: 10^{-11} \: x \: 2 \: x \: 10^{31}}{(3 \: x \: 10^8)^2}$

$= 3 \text{ x } 10^4\text{m}$

8. (a) if an object is dense enough it will cause extreme warping of spacetime; such that any light leaving the surface will not be able to escape the spacetime surrounding the object;

(b) $R_{SCH} = \frac{2GM}{c^2}$

$= \frac{2 \: x \: 6.67 \: x \: 10^{-11} \: x \: 2 \: x \: 10^{30}}{(3 \: x \: 10^8)^2}$

= 3000m

9. (a)the radius from within which nothing can escape to the outside / the distance from the black hole where the escape speed is equal to the speed of light;
(b) (i) any curved path from observer to spacecraft that does not cross the event horizon; Do not accept paths that start straight and then curve around event horizon.
(ii) the black hole curves / "warps" spacetime; Radio signal follows shortest distance / geodesic of the curved spacetime;

10. (a) centre is single point to which all mass would collapse;
surface is where the escape speed is equal to c;
within this surface, mass has "disappeared" from the universe;
(b) distance from point of singularity to the event horizon;

(c) $R_{SCH} = \frac{2GM}{c^2}$

$= \frac{2 \: x \: 6.67 \: x \: 10^{-11} \: x \: 2 \: x \: 10^{31}}{(3 \: x \: 10^8)^2}$

$= 3.0 \text{ x } 10^4\text{m}$

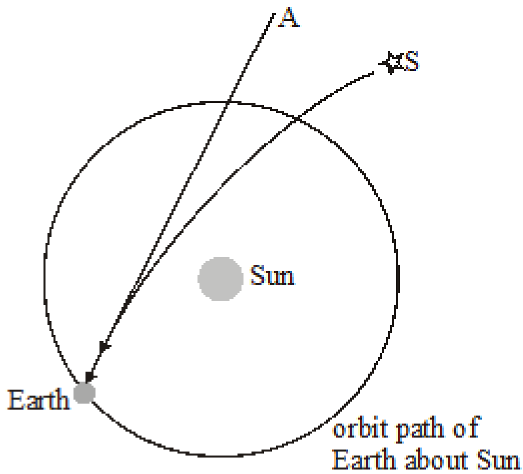
(d)(i) deflect the path of spacecraft / send into orbit round black hole / "attracted" by black hole;
(ii) matter distorts space;
particles in space-time follow shortest path;
path curved as mass approaches black hole / matter;
this is interpreted as a force;

11. theory suggests that light is affected by gravitational fields; diagrams or "words" to explain formation of two images;

12. any correct argument to show that light would be expected to be bent in an accelerating frame (eg observer in lift / rocket etc); application of principle of equivalence to show that light must also be bent in a gravitational field; gravitational lensing is the bending of light around a massive astronomical object;
to produce multiple images or magnified images of a region of space that is further away;

13(a) The sun curves/ "warps" space time, so that the light follows a curved path (known as a geodesic)
(b) sketch showing Earth, Sun and distant star.
Light from distant star being 'bent'.
Dotted line showing where an observer on Earth would 'see' the star.

14. (a) in order that the star could be seen;
(b) in order that the degree / amount of bending of the light by the Sun can be measured;
(c) path showing bending by the Sun;
Note that a correct diagram may also include rays from the other side of the Sun. Only accept rays from the star that at the point of closest approach to the Sun are no more than about 1 cm from the Sun's surface.

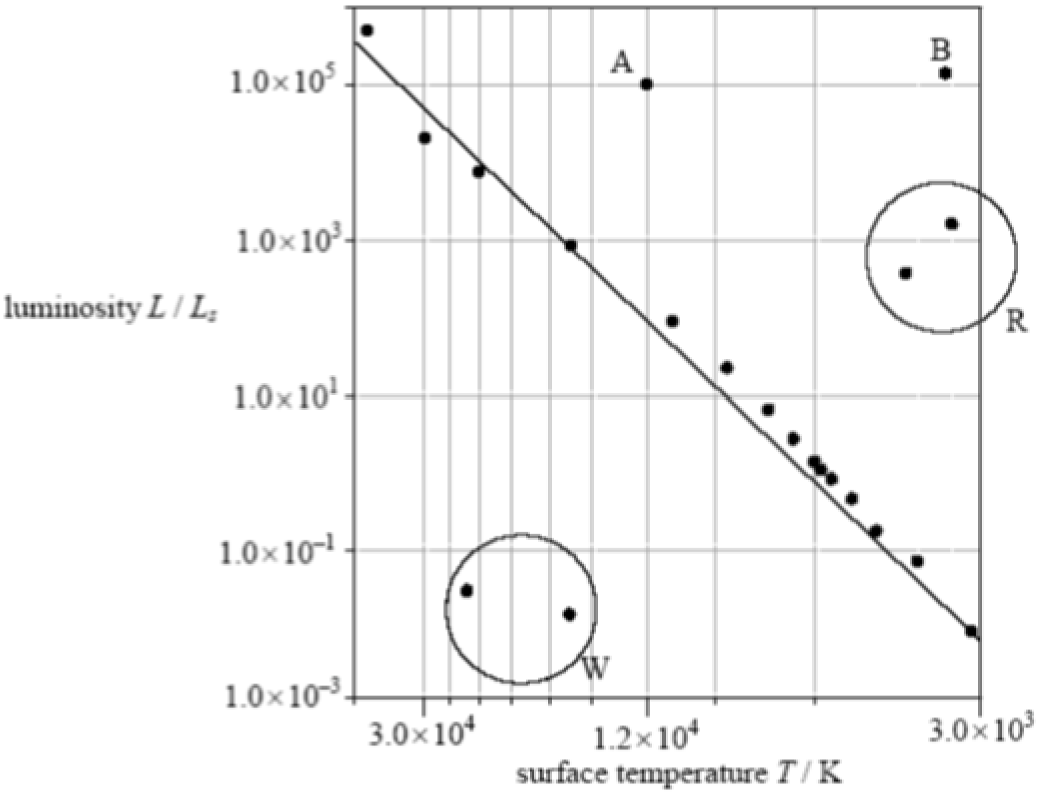


(d) the theory predicts that space-time is curved / warped by the presence of matter; the light ray takes the shortest path between the star and Earth in the curved / warped space;
To award [2], space-time must be mentioned. An answer such as "gravity bends light" would only receive [1].
(e) see diagram; Position must be consistent with bent ray.

15. (a) ray from star A to observer deviated when near Sun;
straight-line from star B to observer;
Do not award credit where curvature shown at distances greater than two Solar diameters from the Sun.
(b) observation made when no Sun and when Sun is eclipsed;
star A moves relative to background stars;

Stellar Physics

1. (a)



(i) circle labelled R as shown above;
(ii) circle labelled W as shown above;
(iii) any line (not necessarily straight) going from top left to bottom right, through or near all or most of stars;
(b) star B has lower temperature; star B has (slightly) larger luminosity / stars have approximately same luminosity; surface area calculated from L = σAT^4, so star B has larger surface area/diameter / to give the same/similar luminosity at lower temperature, star B must have bigger diameter/ surface area;
(c) (from HR diagram) LA = 10^5 LS;

$$b = \frac{L}{4\pi d^2} \text{ used to give ;}$$

$$\frac{d_A}{d_S} = \sqrt{\frac{L_A}{L_S} \times \frac{b_S}{b_A}} = \sqrt{10^5 \times \frac{1.4 \times 10^3}{4.9 \times 10^{-9}}}$$

hence $d_A = 1.7 \times 10^8$ AU.
(d) the parallax angle is too small to be measured accurately
Do not accept “it’s too far away”.

2. (a) $L = 4\pi d^2 b$

$\frac{L_B}{L_{Sun}} = \frac{d_B^2 b_B}{d_{Sun}^2 b_{Sun}}$

$L_B = (10^8 \times 2.05)^2 \times 10^{10} \times 7.00 \times 10^{-12} L_{Sun}$

$= 3.43 \times 10^3 L_{Sun}$

(b) Sun: white dwarf; Becrux: neutron star black hole;

3. (a) low mass stars will collapse to a white dwarf;
(b) high mass stars will collapse into a neutron star / black hole;

4. (a) red / red-orange; (not orange)
blue / blue-white / white;
(b) Betelgeuse looks brighter;
(c) $L = 4\pi b d^2$
$L = 4\pi \times 2.0 \times 10^{-7} \times (4.0 \times 10^{18})^2$
$L = 4.0 \times 10^{31} W$
(d) $L = 4\ \pi b d^2$
luminosity of Rigel is about half that of Betelgeuse;
brightness of Rigel is about 0.1 times that of Betelgeuse;
so Rigel is more distant.

5. the star will evolve to become a red super giant; nuclear reactions involving elements heavier than hydrogen take place / nuclear reactions produce heavier elements up to iron; will then explode in a supernova;

6. period/frequency with which luminosity varies; apparent brightness / apparent magnitude;

7. (a) appears red in colour; (has a very) large luminosity; (relatively) low (surface) temperature; (very) large mass; (very) large surface area;
(b) (i) the power per square meter received at the surface of Earth/observer;
(ii) use of $L = 4\pi d^2 b$

$\frac{L_{Antares}}{L_{Sun}} = \frac{d_{Antares}^2 b_{Antares}}{d_{Sun}^2 b_{Sun}}$

$L_{Antares}$ ($\times L_{Sun}$) $= 4.3 \times 10^{-11} \times 3.92 \times 10^{14}$

($= 6.5 \times 10^4$)

8. (a) stars: $\frac{75}{4.19(17)^3} = 3.6 \times 10^{-3}$ (ly^{-3})

galaxies: $\frac{26}{4.19(4.0 \times 10^6)^3} = 9.7 \times 10^{-20}$ (ly^{-3})

(b) $\frac{10^{-3}}{10^{-19}} = (3.8 \times)10^{16}$ or star population density greater than galaxies population density by an order of magnitude 16;

9. (a) Achernar;

(b) $\frac{L_M}{L_A} = 1$

$$1 = \frac{\sigma 4\pi R_M^2 T^4}{\sigma 4\pi R_A^2 (5T)^4}$$

$\frac{R_M}{R_A} = 25$

(c) it has to be hot star/a B star; low luminosity; hence EG129;

10. (a) the magnitude of PA is (much) smaller than that of PB; in the magnitude scale the smaller the magnitude the brighter the star;

or

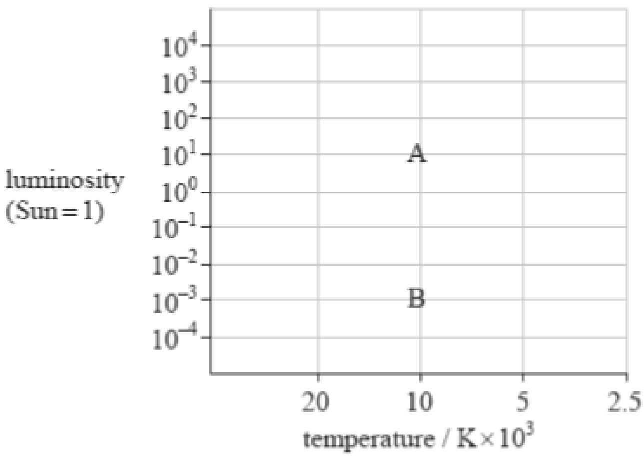
apparent brightness of PA is greater than PB; apparent brightness is intensity at surface of Earth;

(b) the ratio of the luminosities is the same as the ratio of apparent brightness;

$\frac{L_A}{L_B} = \frac{2.06 \times 10^{-8}}{1.46 \times 10^{-12}}$

$= 1.41 \times 10^4$

(c)



(i) PA → 10 000 K at 10; (labelled A)

(ii) PB → 10 000 K at 10^{-3}; (labelled B)

(d) white dwarf;

(e) to the red giant region (approximately either side of L = 10^2 and T = 2500K); to the white dwarf region (approximately either side of L = 10^{-2} and T = 10 000 K);

11. (a) gas cloud collapses under its own gravity; gravitational potential energy changes to kinetic energy of particles; eventually temperature/pressure at centre is so great that fusion occurs;

(b) (initial) mass;

(c) (i) carbon / oxygen / neon; (ii) iron;

12. (a)(i) luminosity is a function of surface and temperature (of star); (same class) same temperature (therefore greater surface area);

(ii) LC = 80 LS;

(iii) $\frac{L_C}{L_S} = (\frac{r_C}{r_S})^2 = 80$

$r_C^2 = 80 r_S^2$

$r_C = 8.9 r_S$

(b) Vega appears dimmer; hence distance over-estimated;
(c) the inward gravitational pressure is balanced by the outward radiative pressure;

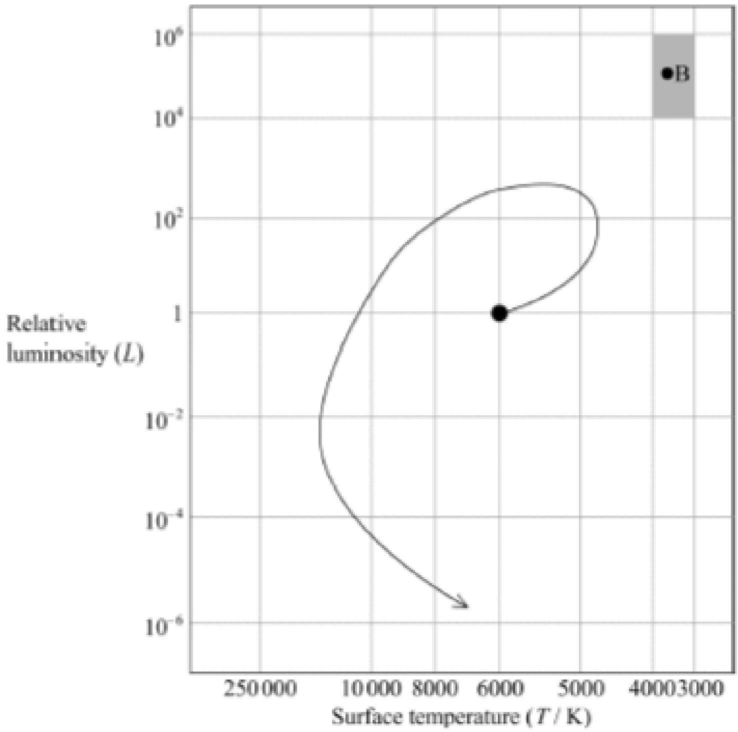
13. (a) L = $\frac{1.6 \ x\ 10^{-7} x\ 4\pi \ x\ (4.3\ x\ 10^{18})^2}{3.8\ x\ 10^{26}}$

9.8 × 10^4 LSun or 8.4 × 10^4 LSun (if 4.0 used);

(b) T = $T_{Sun}\sqrt[4]{9.8\ x\ 10^4\ x\ (\frac{1}{790})^2}$

3600K or 3500K;

(c)



(i) position labelled B within shaded area;
(ii) generally the correct shape.