# Subitising through the years


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**The importance and usefulness of building on perceptual subitising and the development of conceptual subitising is explained. A guide on how to continue to develop numerical ideas based on subitising is shared.**

Subitising, or to 'suddenly see' quantity (Clements, 1999), has appeared in classrooms of young students for many years through informal hand-made 'dot cards' used for early number identification tasks and through curricula such as *Right Start* [1](Griffin, 2005). In 1999 Doug Clements asked the question "Subitising: what is it? Why teach it?" More recently, subitising has been outlined as a vital component of student mathematical proficiency through work in learning trajectories (Clements and Sarama, 2009). Hurst and Hurrell (2014) identify subitising as part of 'trusting the count', which is one of the four 'big ideas' that are critical for elementary classroom instruction. Further-more, neuropsychological studies have demonstrated that the brain processes only very small quantities at a time, up to five or six (Dehaene, 1997). The brain does so through the 'analogue processor' responsible for making sense of number (Dehaene, 1997). Dehaene explains how this analogue processor measures quantity through comparative fullness (Figure 1).

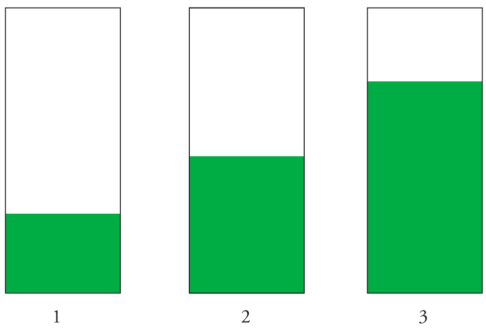



**Figure 1. Graphic interpretation of the brain's analogue processor for quantity.**

While humans have a limited ability to process larger quantities adeptly at a visual level, we also have other strengths within the brain, most notably an ability to abstract ideas into symbols, that make possible the creation of a highly sophisticated numeration system. The Arabic numeration system allows us to discuss and process quantities that are infinitesimally large and small. But even this sophisticated system requires a connection to the analogue processor. It is well-established that when people process quantities as written numerals, their brains route the information through the quantity processor: all math goes through this limited processor that 'sees' only five or six at a time (Dehaene, 1997). So how do we ensure that students are processing numerals with meaning? Instructionally, we must develop our students' abilities to make sense of mathematical ideas (addition, subtraction, algebraic properties, multiplication) by building on small quantities and we can do this through subitising. And yet, in spite of increased awareness of this critical psychological process for understanding number, subitising remains something that is addressed almost solely in the Foundation year, if it is addressed at all. Here we make the argument, and provide a detailed outline, about how teachers can leverage subitising to improve instruction throughout the primary years, and particularly in Year 3.

Our colleague asks an unexpected question to help adults understand that numerals are not quantities: "Is the word C-A-T a cat?" (Faulkner, Cain 2010). No, of course not. C-A-T is merely an orthography we use to communicate the idea of a cat to others. And so it goes for the numeral 3. The numeral 3 is no more 'three' than C-A-T is 'a cat'. As with the alphabet where words of letters can be strung into miraculous communications of truth and fiction that change our lives, so it is with digits and numerals.

1. Now known as Number Worlds A,B,C and published through SRA McGraw Hill.

Once understood, we can string numerals into miraculous communications of truth and imagination. Yet, with mathematics we tend to think that a manipulation at the symbolic level, such as 3 + 8 = 11 is the mathematics. In a story about a cat, the 'c' - 'a' - 't' is a way to communicate about our character the cat. Writing c-a-t is not the essence of understanding a cat. And so it is with numerals: 3 is merely a way to communicate to others about the actual idea of the quantity three. Dot cards allow us to represent the actual numerosity of 3 (* * *) to students and therefore allow us to teach through the numbers that are represented by the numerals. As described above, this also allows us to connect directly to the 'analogue' part of our brain that measures quantity through fullness because the dots themselves communicate directly as a quantity (Cain, Faulkner, 2011).

In order to understand this phenomenon of our analogue quantity processor consider which petrol gauge you would prefer on your car, a digital representation or an analogue representation of your petrol in relation to your tank's capacity (Figures 2 & 3): Why? Because in one instance you need to re-route to another part of the brain to process meaning and in the other you know immediately, with no extra processing, the relationship between your tank and the petrol you have in it. When you are driving you want dashboard information as immediately accessible as possible.

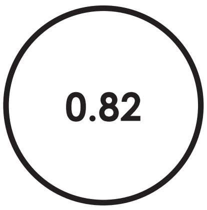



Figure 2. Digital petrol gauge.


Figure 3. Analogue petrol gauge.

To further emphasise this point, consider an aircraft cockpit display and an imaginary digital one (Figure 4). Which plane would you rather be on?

Most of us will say the plane with the real display because we can immediately see the relationships built into the gauges without translating the symbolic language. And so it is with subitising. Seeing dots allows us to access a number's value directly without needing to interpret through a numeral.

In the Subitising Instructional Guide (see Table 1, pp 31–36) we have outlined how a teacher can move students through important learning trajectories and mathematical understandings using subitising cards to support the talking points. We recommend that, no matter the age level of students with whom you are working, you move through these phases beginning at Perceptual 1 and progressing through the levels as students demonstrate mastery. A key to teaching subitising is to hold the cards up quickly! Students need to understand that they did not count, and a quick flash of the dot value is the only way for them to notice this about their own brain. We have also created an extensive presentation document that allows teachers to project these images on a wall for students, or to create cards based on the levels and learning objectives

Following the Subitising Level Instructional Guide you will see that the learning begins with numbers no larger than five or six because these values represent the limit of even an adult brain's quantity processor (Clements, 1999, Dehaene, 1997). Being able to process very small quantities quickly is called perceptual subitising and it involves little to no processing other than seeing and responding to the number value itself (Figure 5 & See Levels P1, P2 in Table 1).

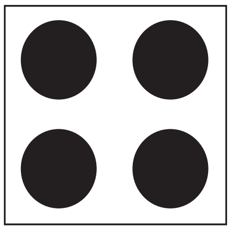
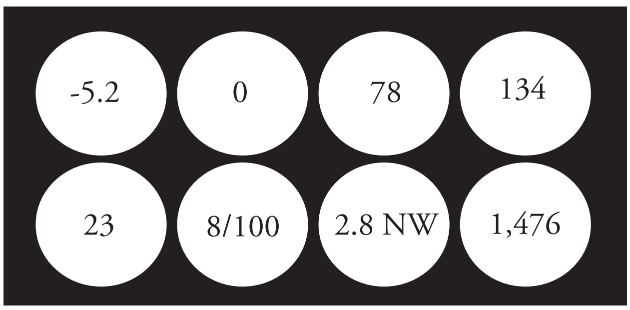


Figure 5. Perceptual subitising. Numbers 6 and below are perceptually recognised with no further mathematical processes needed.

Conceptual subitising builds on this and involves composing and decomposing small values into clumps in order to process values beyond five or six (Clements, 1999). (Figure 6 & See Levels C1, C2 in Table 1.)

Figure 4. Actual analogue versus imaginary digital pilot's gauges.



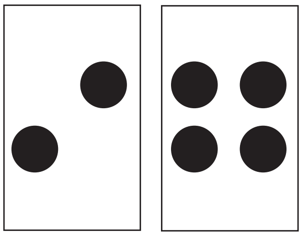


Figure 6. Conceptual subitising. Numbers 6 and above are processed by recognising smaller values and rapidly adding them together.

Through this quick daily practice in these early levels (See P1–C5, Table 1) the critical early number sense concepts of conservation (understanding that a quantity does not have to look the same to be equal) and cardinality ('trusting the count') become deeply understood. Furthermore, students develop the critical understanding that numbers are not merely a series of ones, but that numbers are composed of/decomposed into other numbers in a variety of ways (Grey & Tall, 2007; Ma, 1999).

Perhaps surprisingly, subitising allows us to have sophisticated conversations with students about algebraic properties and the concept of equality because the visuals support student attempts to put language on these mathematical ideas. See Levels C5, C7, C8, C9 (Table 1) and the suggested talking points for the commutative and associative properties and equality.

Teachers can support children in their number sense development by progressing through these levels, but it is important to understand the specific mathematical connections that can be made as students progress to encourage timely classroom discussions. For instance, as students move beyond ten it is critical that they understand the nature of the base-ten system and how it allows us to make sense of values that are larger than our brain can process at one time (See C6, Table 1). Showing students a group of disorganised dots (very quickly) (Figure 7) allows them to understand that we need help figuring out the value of those dots!

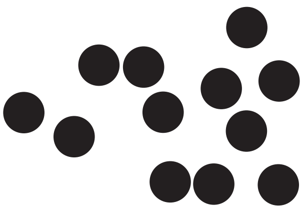


Figure 7. Disorganised array of 12.

We can communicate the power of a base system by showing the same amount of dots in an organised fashion (Figure 8) so that the brain can immediately process the value as a ten and two more: twelve!

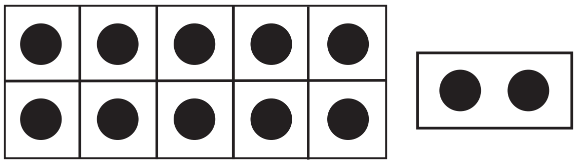


Figure 8. Base-ten organisation allows us to process 12 quickly.

Building on this, students are then supported in developing proficiency with 'break apart to make ten strategies' (BAMT) in early primary school years. Teachers must be very deliberate in moving students from 'getting answers' when simplifying expressions such as 8 + 5 towards teaching students to use the base-ten system to clump into tens. This is a critical development and one that subitising is particularly well-suited to address (See C7, C8, C9, Table 1). When students get stuck using counting-on strategies to the exclusion of other more sophisticated strategies we call this 'living-in-ones world' (Cain & Faulkner, 2011). Students living here seem to understand number as a continuous string of ones. This conceptualisation may work at the Foundation level, but it does not hold up as numbers get larger and as students move to multiplicative reasoning (Gray & Tall, 2007).

As students develop proficiency in 'seeing' what is underneath numerals they are better positioned to attack word problems because they better understand quantity and can build on this to make sense of comparative magnitudes. In Figure 9 you can see an example of subitising dots organised into a bar model where students can now discuss up to what point the two magnitudes are equal, they can decompose the larger value into the part that is equal, and they can see the 'missing amount'. Our work with bar models has been greatly informed by Julie Russo McKee, who has simplified bar models to three basic types (parts equal total, comparison, and repeated equal parts) to represent all four operations. For more on bar models also see Hoven and Garelick (2007).

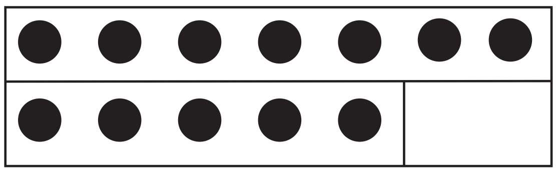


Figure 9. Bar models allow us to compare magnitudes.

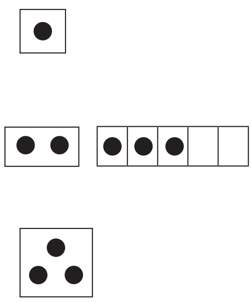
Through the work they have done with subitising, students are better prepared to compare values and make sense of comparisons and relationships between numbers. Connecting this competence with bar models is a great way to support students' transition from the concrete to the more abstract representations of number and the language used to convey these ideas (See C4, Table 1).

Even as students enter their fourth and fifth year in school, subitising remains relevant. For instance, as students learn multiplication they must understand that multiplication is a more powerful operation that tracks groups of numbers rather than merely combining or decomposing like units. Subitising supports important discussions about factors and products (See C10, Table 1) as it allows students to visualise factors in a meaningful way. Throughout their work with subitising, students are also able to process and utilise the critical mathematical properties of the commutative property and the associative property at the multiplicative level as well (See C8, C9, C10, Table 1). Subitising can even help students to have discussions about the different structures of division (See C11) and to better understand that division is not about 'making things smaller' but about considering relationships between parts (Faulkner, 2013; Ma, 1999).

For young students who begin subitising before they enter school these subitising levels represent-years-worth of work and will begin with the most basic ability to identify quantities (See P1, P2, Table 1) before moving to a more conceptual understanding that numbers are comprised of other numbers (See C1–4, Table 1). For a student in their second year of school the early subitising levels will likely be mastered in a few days and then play can linger in the levels that address year two standards such as break apart to make ten strategies (See C6, C7, C8, C9, Table 1). An older student might need to play with these same levels because they managed to get through earlier grade levels using 'counting on' strategies rather than leveraging the power of the base-ten system to group by tens. This older student can then also play with the levels dealing with multiplication and division and thereby connect to later primary standards.

The levels presented here include a visual representation of the types of dot cards to be used as well as key messages that need to be communicated to the learner and played with during that level. To connect to the literature that supports educators in conceptualising, and benchmarking, student progress, the levels also include connections to the work of Richardson (2013), Clements and Sarama (2011), Clements (1999), Ma (2010) and the Cognitively Guided Instruction school of thought (Carpenter, et al. ,1999). Key points with no referent are common and understood mathematical ideas (such as the commutative property). Connections to the *Australian Curriculum: Mathematics* are also provided. All levels are cumulative, meaning that you can and should continue to practise and deepen fluency with prior levels as you move up.

Table 1: Subitising instructional guide.

| Subitising level instructional guide | Example visuals | Key messages and student response expectations | Connections<br>Critical Learning Phases (KR)<br>Learning Tractories (C&S),<br>Clements, 1999 (DC)<br>Knowing and Teaching Elem Math (Ma) Carpenter, et al (CGI)<br>..........<br>Australian Curriculum: Mathematics |
|---|---|---|---|
| **Perceptual 1 (P1)**<br>0-3<br>One card at a time.<br>Various patterns. |  | How did you do that so fast?!<br>*I just saw it.*<br>There's no way you had time to count!<br>Is this still 3 even though it looks different?<br>*Numbers can look different and have same value.* | • Perceptual subitiser to 3 (C&S)<br>• Sees number patterns up to 3 (KR)<br>• Cardinality of a set can be accessed without counting (DC)<br>• Conservation of Number (C&S, KR) Different forms of a number (Ma)<br>..........<br>• (F) Connect number names, numerals and quantities (ACMNA002)<br>• (F) Subitise small collections (ACMNA003) |

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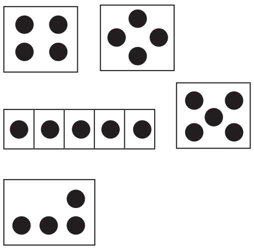
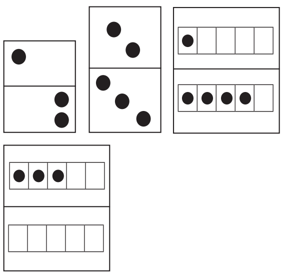
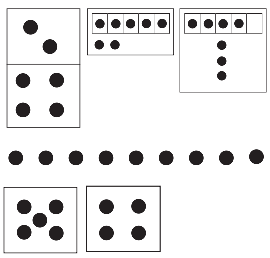
| **Perceptual 2 (P2)**<br>0–5<br>One card at a time.<br>Various patterns. | **P1+**<br> | How did you do that so fast?!<br>*I just saw it.*<br>There's no way you had time to count!<br>Is this still 3 even though it looks different?<br>*Numbers can look different and have s ame value.* | • Perceptual subitiser to 5 (C&S)<br>• Sees number patterns up to 5 (KR)<br>• Cardinality of a set can be accessed without counting (DC)<br>• Conservation of number (C&S, KR)<br>• Different forms of number (Ma)<br>............................................<br>• (F) Connect number names, numerals and quantities (ACMNA002)<br>• (F) Subitise small collections (ACMNA003) |
| **Conceptual 1(C1)**<br>0–5<br>Two cards or distinct separation.<br>Various combos and patterns up to 5. |  | How did you see that?<br>*3! I saw a 1 and a 2 and together that makes 3*<br>Did you count?<br>*No!*<br>What did you combine and what did you get altogether?<br>*I combined 1 and 2 and together that makes 3.* | • Conceptual subitiser to 5 (C&S)<br>• Cardinality of a set can be accessed by 'clumping' numbers (adapted from C&S)<br>• Numbers are contained within numbers<br>• Composer to 4 then 5 (C&S)<br>............................................<br>• (F) Connect number names, numerals and quantities (ACMNA002)<br>• (F) Subitise small collections (ACMNA003)<br>• (Y1) Represent and solve simple addition problems using parti-tioning and rearranging parts (ACMNA015 |
| **Conceptual 2 (C2)**<br>0–9<br>Two cards or distinct separation.<br>Various combos and patterns up to 9. | **C1+**<br> | The brain can't process more than 5 or 6 at a time— even adults need to see parts and combine. Look how hard it is for the brain to see so many dots! (Long string of 9 dots.)<br>*So hard I had to count to keep track!*<br>Why is this easier? (Clump of 5 and clump of 4 in familiar pattern.)<br>*Because I could see the two clumps/groups and didn't have to count by ones.*<br>How did you see that?<br>*9 —I saw the 5 and 4 and knew that was 9* | • Conceptual subitiser 5+ (C&S)<br>• Cardinality of a set can be accessed by 'clumping' numbers (adapted from C&S)<br>• Numbers are contained within numbers<br>• Composer to 7 (C&S)<br>• Composer to 10 (C&S)<br>............................................<br>• (F) Connect number names, numerals and quantities (ACMNA002)<br>• (F) Subitise small collections (ACMNA003)<br>• (Y1) Represent and solve simple addition problems using parti-tioning and rearranging parts (ACMNA015)<br>• Explore (Y2), recognise and explain (Y3) the connection be-tween addition and subtraction (ACMNA029 & 054)<br>• (Y2) Solve simple addition and subtraction problems using a range of efficient mental strategies (ACMNA030) |

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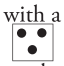
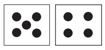
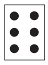
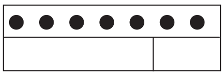
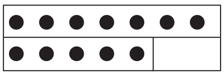
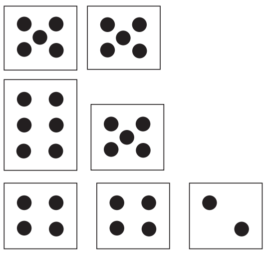
| **Conceptual 3 (C3)**<br>Introduce hidden addend or missing subtrahend or minuend. | Start with **C1** and introduce hidden addend. Move to **C2** only upon mastery of **C1.** | I have two cards with a total of 4, this is one of the cards. <br>How many dots on the other card? *One.*<br>How do you know? *That is 3, and 1 more makes 4.*<br>If have this many,<br><br>but only want 6, how many do I need to take away?<br>*Keep the 5 and one from the 4 and take away 3. That leaves 6.*<br>Vary language: I want to get to 8 and I only have this card.<br><br>What other card do I need? | • Composer to 4 then 5 (C&S)<br>• Find change +/- with subitising to 5 (adapted from C&S)<br>• Find change +/- with subitising to 10 (adapted from C&S)<br>• Changes a number to another number by adding a group (KR)<br>..............................................<br>• (Y2) Solve simple addition and subtraction problems using a range of efficient mental strategies (ACMNA030)<br>• (Y3) Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation (ACMNA055) |
| **Conceptual 4 (C4)**<br>Connect to bar models. | <br> | **Parts equal total:**<br>If I have 7 birds and 5 flew away, how many are left?<br><br>**Comparison:**<br>If you have 7 stuffed animals and I have 5, how many more do I have than you? | • Matching comparer (C&S)<br>• Find change +/- to 5 (C&S)<br>• Find change +/- to 10 (C&S)<br>• Arithmetic problem types (CGI)<br>..............................................<br>• (F) Represent practical situations to model addition and sharing (ACMNA004) |
| **Conceptual 5 (C5)**<br>0–12<br>Two cards or distinct separation. Various combos and patterns up to 12. | **C1 + C2 +**<br>(Building on from C1 and C2)<br> | **As above in C2 +**<br>What if I had put the cards up like this (reverse order of cards with two distinct values) would it still have the same value? Why?<br>*Yes, because it is still the same amount of dots.*<br>Press for reasoning: So numbers can be in different forms and still have the same value?<br>And/or: So I can put these values together in either order and it will give me the same total number of dots?Explain this to me.<br>*Even if you move the 5 dots to the other side it is still 5 dots and 5 dots plus 3 dots is going to equal 3 dots plus 5 dots. They are both 8 even if you put them in a different order.*<br>Did anyone see a double? Double + 1, double + 2 ?<br>*I saw five and six and knew that five and five is ten and 6 is one more than five so one more than ten is one ten and 1 one, 11.*<br>*I saw two fours to make 8 and then 2 more to make 10* | • Conceptual subitiser to 10 (C&S)<br>• Cardinality of a set can be accessed by 'clumping' numbers (C&S)<br>• Numbers are contained within numbers<br>• Doubles (CGI, KR)<br>• Doubles plus 1 (CGI, KR)<br>• Commutative property of addition<br>• Different forms of numbers (Ma)<br>..............................................<br>• (Y1) Count collections to 100 by partitioning numbers using place value (ACMNA014)<br>• (Y2) Group, partition, and rearrange collections…to facilitate more efficient counting (ACMNA028) |

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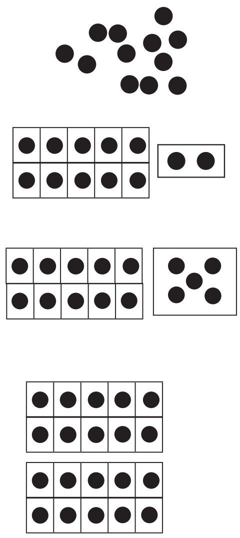
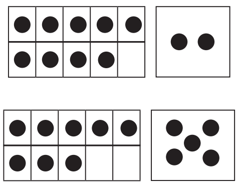
| **Conceptual 6 (C6)**<br>w/10s |  | Why was that so hard to see?<br>(disorganised array of 12 or more)<br>*The dots were all over the place!*<br>*Our brain can't see more than 5 or 6 at one time!*<br>Right, our brain doesn't process more than 5 or 6 dots at a time so we need to get organised to 'see' and understand more dots quickly! In our number system we organise quantity by tens. How about this? (Flash ten frame and some extras.)<br>Real name ? *One ten two ones!*<br>Nickname? *12!*<br>As students are ready, show two and three tens frames as well for responses such as: *2 tens! The nickname is 20!* [2] | • Conceptual subitiser to 20 (C&S)<br>• Conceptual subitiser with place value and skip counting (C&S)<br>• Composer with tens and ones (C&S)<br>..............................................<br>• (Y1) Skip count by twos, fives, and tens starting from zero (ACMNA012)<br>• (Y1) Count collections to 100 by partitioning numbers using place value (ACMNA014)<br>• (Y1) Investigate and describe number patterns formed by skip counting (ACMNA018)<br>• (Y2) Investigate number sequences, initially those increasing and decreasing by… tens (ACMNA026)<br>• (Y3) Apply place value to partition, rearrange and regroup numbers (ACMNA053) |
| **Conceptual 7 (C7)**<br>Break Apart to Make Ten (BAMT) with (9, 8). | <br>Note: Introduce symbols for students if students are ready as demonstrated by their discussions, but keep focus on the subitising.<br>8 + (5)<br>8 + (2 + 3)<br>(8 + 2) + 3<br>10 + 3<br>13 | Now we are going to learn to add numbers using what we know about decomposing and our facts to ten! We don't have to count! How do we break apart our second addend to make a ten and check for left over ones?<br>*1 to make ten and 1 left over*<br>Continue with 9 and up through 9 + 9. Have student answer with the decompositions and the 'real names' of the numbers.<br>Add in questions with the eight-frame. Continue to have students answer with their decompositions and real names.<br>As you move them to answering quickly with the nickname, continue to press for reasoning: Student response to what is the nickname for ten and three?: *13!*<br>What is the real name for that number? *10 and 3.*<br>How did you get 10 and 3?<br>*I decomposed the 5 into a 2 and a 3.*<br>Why did you decompose your 5 into a 2 and a 3? *Because I knew I needed two more to make a ten so I decomposed the 5 into 2 and 3.*<br>Include compositions to ten and have students respond *1 ten and 0 ones.*<br>Discuss associative property briefly:<br>So we have 8 + 5 and we can see that as 8 + (2 + 3) and then we can put our 2 over with the 8 so that becomes (8 + 2) + 3. Why can we do that?<br>*Value doesn't change/Same amount of dots/ Same quantity.* | • Break Apart To Make Ten strategy<br>• Associative property of addition to make problems simpler (Ma)<br>• +/- fact fluency to 20<br>• Composer with tens and ones (C&S)<br>..............................................<br>• (Y1) Count collections to 100 by partitioning numbers using place value (ACMNA014)<br>• (Y1) Represent and solve simple addition and subtraction problems using a range of strategies including partitioning and rearranging parts (ACMNA015)<br>• (Y2) Solve simple addition and subtraction problems using a range of efficient mental strategies (ACMNA030)<br>• (Y7) Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151) |

2. The idea of the 'real name' and 'nickname' is from Liping Ma and Cathy Kessel, Knowing Mathematics, 2003. I have found this a very effective way to support students in processing numbers beyond nine. Asian languages have the math-to-brain 'advantage' in that their numerals are named in a strictly base ten fashion (for instance 'ten, one' is our 'eleven' (Dehaene, 1997)). This habit of discussing the real name and the nickname with students is an easy way to combat the relatively obscure language we use to describe numbers, particularly through the critical teen numbers.

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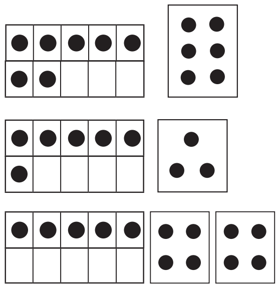
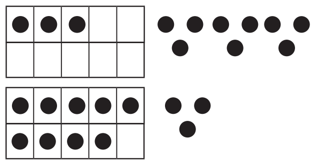
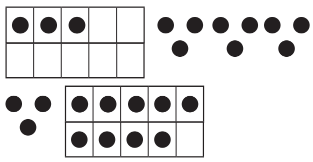
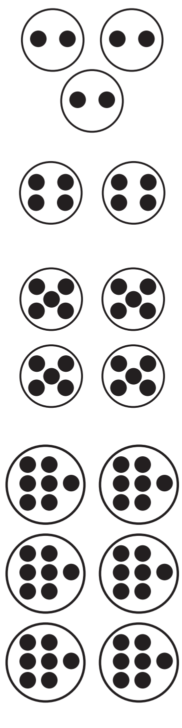
| **Conceptual 8 (C8)**<br>Break Apart to Make Ten (BAMT) with (7,6,5. | <br>7 + (8)<br>7 + (3 + 5)<br>(7 + 3) + 5<br>10 + 5<br>15 | As above in C6 with first addend of 7, 6, 5 plus a second addend. Also regularly point out that you are adding dots to dots (like units). So say things like "okay 6 what plus 3 what equals 9 what?"<br>*6 dots + 3 dots = 9 dots*<br>Continue making this point during the addition phase to support students in thinking in like units. | • Break Apart to Make Ten Strategy (C&S)<br>• Associative property of addition<br>• When we add we must add like units (Ma)<br>• +/-Fact fluency to 20<br>..........................................<br>• (Y1) Represent and solve simple addition and subtraction problems using a range of strategies including partitioning and rearranging parts (ACMNA015)<br>• (Y2) Solve simple addition and subtraction problems using a range of efficient mental strategies (ACMNA030) |
| **Conceptual 9 (C9)**<br>Break Apart to Make Ten (BAMT) with small first addend. | <br>3 + 9 = 3 + 9<br>Also,<br><br>3 + 9 = 3 + 9,<br>even when we put them in a different form.<br><br>Also we can still use the associative property:<br>3 + 9<br>(2 + 1) + 9<br>2 + (1 + 9) | What if I have a combination such as 3 + 9? How should we approach that?<br>*Break our 9 into 7 and 2 (so 3+ 7 = 10 plus 2 more)*<br>Yes we can do that. But we can also make this simpler by thinking of it as 9 and 3. Can we do that?<br>*We can use the commutative property.*<br>Press for reasoning: But doesn't that change the value? Why can you do that?<br>*It doesn't change the value when we add because it is still the same amount of dots. It is still the same value.*<br>Is it 'the same thing'?<br>*No it's not 'the same thing' because now it is 9 + 3 instead of 3 + 9. That's a different story. We changed the form, but the VALUE is the same!*<br>Why would you as a mathematician want to do that?<br>*To use our number sense to make it a simpler problem so we can do it in our heads.* | • Break Apart to Make Ten Strategy (C&S)<br>• Commutative property of addition<br>• Associative property of addition<br>• +/- Fact fluency to ten<br>..........................................<br>• (Y1) Represent and solve simple addition and subtraction problems using a range of strategies including partitioning and rearranging parts (ACMNA015)<br>• (Y2) Solve simple addition and subtraction problems using a range of efficient mental strategies (ACMNA030) |
| **Conceptual 10 (C10)**<br>Multiplication with whole numbers. |  | When we move from adding to multiplying we move 'up a level' in our thinking. Now we are adding repeated groups of the same number. So let's think of repeated equal-groups cards as groups of something.<br>Here we see 3 groups of 2 and that equals 6. We can say 3 times 2 rather than 2 + 2 + 2. So rather than dots + dots = dots, now we have groups times so many dots = a total amount of dots.<br>Do you see that? So what is this?<br>*3 groups of 2 dots!*<br>Is it 4 groups of 2 dots or 2 groups of 4 dots?<br>*2 groups of 4 dots!*<br>Can someone explain why it is 2 groups of 4 and not 4 groups of 2?<br>What are our 2 factors? What is our product?<br>Note: Also work on speed and accuracy of times tables facts through subitising. | • Fluency of mathematics multiplication facts<br>• Commutative property of multiplication<br>• Definition of multiplication<br>• When we multiply we move ' up a level' from unit + unit = unit to groups × unit = unit<br>..........................................<br>• (Y2) Recognise and represent multiplication as repeated addition (ACMNA031)<br>• (Y3) Represent and solve problems involving multiplication using efficient mental strategies (ACMNA057)<br>• (Y4) Develop efficient mental strategies for multiplication and division where there is no remainder (ACMNA076) |

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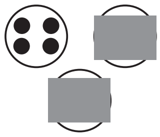
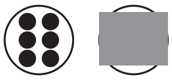
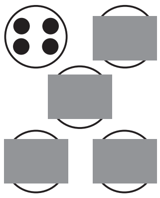
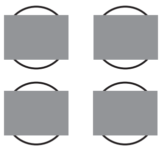
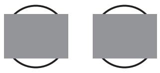
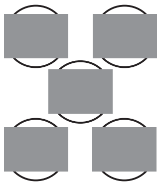
| **Conceptual 11 (C11)**<br>Multiplication with whole numbers—missing factor. | <br><br> | Look at this card. When we see a card like this we will assume that all of the circles have the same amount of dots — they are just covered up. So here the total product is hidden; Can you still tell me what the factors are?<br>*3 and 4*<br>How did you know that?<br>*Even though they are covered up I can see there are three groups so that is one factor and the other factor is 4 because there are 4 dots on the plate we can see and you said that all the plates have the same number of dots.*<br>So even without the dots, can you tell me the product?<br>*12!*<br>Excellent, so let's all get used to calling out the two factors: first the number of groups, then the number of dots in each group and then the product.<br>So what would we say for this one?<br>*2 groups of 6 dots = 12 dots*<br>(and eventually… 2 groups × 6 dots = 12 dots and then… 2 × 6 = 12) | • Fluency of multiplication facts<br>………………………………………<br>• (Y2) Recognise and represent multiplication as repeated addition (ACMNA031)<br>• (Y3) Represent and solve problems involving multiplication using efficient mental strategies (ACMNA057)<br>• (Y4) Develop efficient mental strategies for multiplication and division where there is no remainder (ACMNA076) |
| **Conceptual 12 (C12)**<br>Whole number division. | <br>12 total dots<br><br>8 total dots<br><br>30 total dots<br><br>21 total dots | Can you figure out one of the factors if I provide one factor and the product? Let's try: here I am telling you that there are the same amount of dots in each group, there are 4 groups, and a total of 12 dots. How many dots on each plate? *3.*<br>How do you know? …<br>*I just know that 4 × 3 = 12*<br>Or<br>*At first I thought 2 dots but then realised that wasn't enough so then I thought 3.*<br>So let's say this more mathematically, 8 total dots divided up into 2 groups = 4 dots per group.<br>30 dots divided into 5 groups:<br>*6 dots!* 6 total dots?<br>*No, 6 dots per group.*<br>21 total dots and 3 dots in each group?<br>*7 groups.* | • Deepen understanding of structure of multiplication and division through attention to units. (Ma)<br>• Foundation for partitive division (Ma)<br>• Foundation for measurement division. (Ma)<br>………………………………………<br>• (Y2) Recognise and represent division as grouping into equal sets (ACMNA032)<br>• (Y4) Develop efficient mental strategies for multiplication and division where there is no remainder (ACMNA076) |

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