
EDUCATION REVIEW OFFICE
TE TARI AROTAKE MATAURANGA

# EARLY MATHEMATICS: A GUIDE FOR IMPROVING TEACHING AND LEARNING

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# Contents

**Introduction** **4**
Improving mathematics teaching and learning 4
**Background** **5**
The importance of early mathematics 5
Creating a disposition for mathematical learning 6
Mathematics in *Te Whāriki* 7
Summary of Effective Pedagogy in Mathematics/Pāngarau Best Evidence Synthesis Iteration (BES) 8
**A guide to children's early mathematics learning** **10**
A balance of deliberate teaching and spontaneous learning across the breadth of mathematics 11
Assessment for learning 24
Improving teaching of mathematics in the early years 28
Internal evaluation to improve practice 29
Resource One: A continuum for improving early mathematical learning for children 30
Resource Two: Investigative prompts and indicators of practice 31
**Conclusion** 35
What needs to happen to ensure that all children have rich mathematics learning? 35

# Introduction

## Improving mathematics teaching and learning

This guide describes how children can be supported to become confident and capable mathematical learners in the early years.

ERO has found two factors that are critical – pedagogical leadership and teacher knowledge. Effective pedagogical leaders understand how to integrate mathematics into the curriculum to best support children's learning. They support teachers to develop an in-depth and broad understanding of early mathematics, and associated practice, and promote a culture where teachers can reflect on their practice to continually make improvements. Where there is strong pedagogical leadership teachers are more likely to be up to date with current research and good practice about early mathematics. They are able to use this knowledge to be innovative and intentional in their planning and teaching. Such teachers confidently recognise and extend children's understanding of mathematical concepts in a range of contexts.

The curriculum in most early learning settings provides children with a wide range of opportunities to explore mathematical concepts. A balance of child-initiated learning experiences and deliberately planned activities provide a platform for teachers to extend children's developing mathematical understanding. Children can develop as confident and capable mathematical learners through learning opportunities that reflect the six strands of *Te Kākano*[1] – patterning, measuring, sorting, locating, counting and grouping, and shape. Skilled teachers intentionally plan experiences to extend children's developing mathematical understandings.

This guide includes examples where there are both deliberate and responsive mathematical learning opportunities for children, and where the children have opportunities for learning from across the six mathematics strands. The focus is on:

- how teachers in the early years continually reflect on and improve what they do
- the importance of leaders and teachers' understanding of how mathematics is implemented as part of their curriculum
- their professional knowledge, ongoing learning and deliberate teaching that supports mathematical learning
- their use of assessment information so that they can confidently notice, recognise, respond to, and extend children's understanding of mathematical concepts.

This guide also outlines aspects of practice for leaders to watch out for or be concerned about when thinking about mathematics within their service's curriculum. In particular, a 'hands off' philosophy, where teachers rely solely on children to take the lead in their own learning, can result in missed teaching and learning opportunities. This misinterpretation of the notion of a 'child-centred' curriculum fails to appreciate the critical role of the teacher in deliberately extending and scaffolding children's learning. Leaders wanting to provide an in-depth mathematical curriculum should focus on developing teachers' subject and pedagogical knowledge, so that they can confidently engage with mathematics in ways that support and extend children's learning.

1. Ministry of Education (2012). *Te Aho Tukutuku – Early Mathematics*. Wellington: Ministry of Education. This resourse has previously been distributed to all early childhood services and will be made available online later in 2016.

# Background

## The importance of early mathematics

Children's early experiences of mathematics form the foundation for their future mathematics learning and success. Mathematics enables children to think logically, strategically, creatively and critically. Mathematical knowledge and skills provide building blocks for success in many areas of life and work.[2]

New Zealand and international research on children's learning in the early years confirms the importance of early experiences in mathematics for future educational success.[3] New Zealand's Competent Children study has tracked the development of a group of learners from early childhood education, through school and into adulthood. It shows that at age 10, the quality of early childhood education still influences children's competencies that lead to a successful adulthood, and that mathematical ability is one of the most influential factors. The study shows that most children acquire basic mathematical knowledge and skills before the age of eight years. Duncan and Murnane suggest that early mathematics skills are highly predictive of later academic achievement.[4]

2. Ministry of Education. (2015). *Spotlight on Mathematics/Pāngarau*. Retrieved from www.educationcounts.govt.nz/topics/BES/spotlight-on/spotlight-on-mathematics-pangarau
3. Duncan, G. et al. (2007). School readiness and later achievement, Developmental Psychology, 43, 6: pp 1428-1446. Clements, D. et al (Eds). (2004). *Engaging Young Children in Mathematics*, New Jersey: Lawrence Erlbaum. Wylie, C. (2001, April). Competent children: Findings and issues from the first 7 years. Paper presented at the Ministry of Social Policy seminar, *The long road to knowledge: Longitudinal research and social policy*, Wellington: Ministry of Education.
4. Duncan, G. and Murnane, R. (2011). Introduction: the American dream, then and now, Duncan, G. and Murnane, R. (Eds) *Whither Opportunity? Rising Inequality, Schools, and Children's Life Chances*. New York. Russell Sage Foundation: Ministry of Education. (1996)

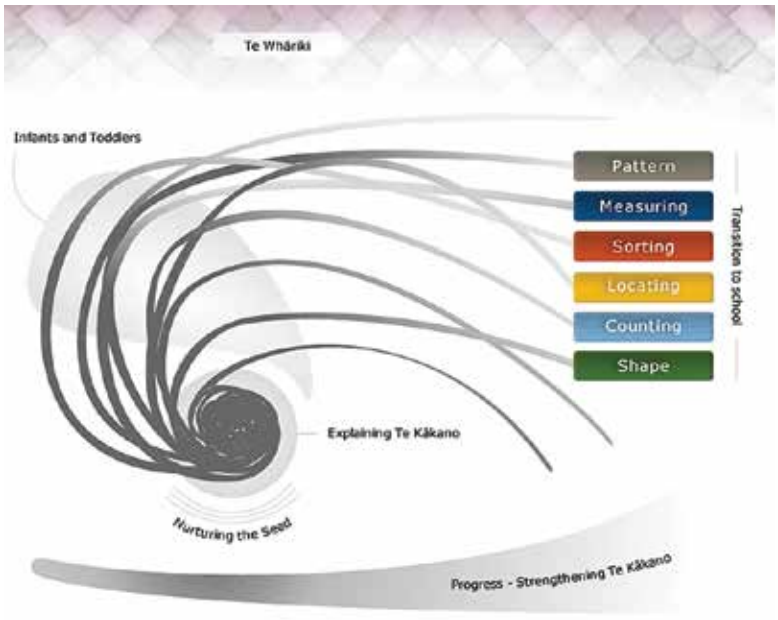
## Creating a disposition for mathematical learning

New Zealand's early childhood curriculum, *Te Whāriki*, focuses on supporting children's learning dispositions and broad competencies that can be readily transferred to new situations (such as starting school). The attitudes and expectations that are formed at an early age continue to influence a child's learning throughout life.[5] The ECE Taskforce report, *An Agenda for Amazing Children* in 2011 highlighted the importance of learning dispositions, such as curiosity and perseverance, to later educational success.[6] These are critical dispositions for children's early learning experiences of mathematics.

The concept of *Te Kākano* – the seed – has been used to describe teaching and learning of mathematics in early childhood. See Figure 1.

Peters and Rameka, in an article about *Te Kākano*, explain that dispositions influence how people tend to invest their capabilities. They state that motivation is crucial to the development of children's early mathematical concepts and it is important that the disposition to explore these concepts is not lost. "*Pedagogical approaches play a key role in ensuring that the inclination to use mathematics is fostered and teachers' own positive dispositions towards mathematics are likely to be central to this process.*"[7] Peters and Rameka argue that inappropriate teaching practice, such as rote learning in isolation of any meaningful contexts, may result in children developing negative attitudes to mathematics.

Figure 1: *Te Kākano*




Source: *Te Aho Tukutuku*

*Image credit: Ministry of Education © Crown 2016*

5. Ministry of Education (1996). *Te Whāriki, He Whāriki Mātauranga mō ngā Mokopuna o Aotearoa, Early Childhood Curriculum.* Retrieved from http://www.education.govt.nz/early-childhood/teaching-and-learning/ece-curriculum/te-whariki/
6. ECE Taskforce report, An Agenda for Amazing Children: p107. Retrieved from www.taskforce.ece.govt.nz/wp-content/uploads/2011/06/Final_Report_ECE_Taskforce.pdf
7. Peters, S. and Rameka, L. (2010). Te Kākano (the seed): growing rich mathematics in ECE settings. *Early Childhood Folio,* 14, 2. Pp8-14.

## Mathematics in *Te Whāriki*

Mathematics is woven throughout the strands of *Te Whāriki*, specifically in Mana Reo – Communication and Mana Aotūroa – Exploration. However, there is potential for mathematics in all strands.

### Mana Reo – Communication

Mathematical learning outcomes relating to the goals of communication include:

- familiarity with numbers and their uses
- skill in using the counting system and mathematical symbols and concepts such as numbers, length, weight, volume, shape and pattern.

### Mana Aotūroa – Exploration

Mathematical learning outcomes relating to the goals of exploration include:

- setting and solving problems
- looking for patterns
- classifying things for a purpose
- guessing
- using trial and error
- thinking logically and making comparisons.[8]

### Te Aho Tukutuku/Early Mathematics

*Te Aho Tukutuku* is a Ministry of Education (the Ministry) resource designed to support and strengthen the teaching and learning of mathematics in early childhood education. The Ministry intended the material to spark mathematical discussions and investigations, provide information and examples to support 'noticing, recognising and responding' to children's mathematical learning, and be a useful resource for teachers and leaders to explore.

The resource is grounded in *Te Whāriki* and the *Te Kākano* framework in *Kei Tua o te Pae/ Assessment for Learning: Early Childhood Exemplars Book 18: Mathematics Pāngarau*.[9] It includes short papers as well as information about mathematical learning and examples of young children putting mathematical ideas into action.

*Te Aho Tukutuku* uses the *Te Kākano* framework as a metaphor for growing rich mathematics. It recognises the movement and unfolding from "te kore, ki te pō, ki te ao mārama" (from nothingness, to the night, to the world of light) and the dynamic, integrated lifelong nature of mathematical learning.

8. Ministry of Education. (1996). *Te Whāriki, He Whāriki Mātauranga mō ngā Mokopuna o Aotearoa, Early Childhood Curriculum.* Retrieved from http://www.education.govt.nz/early-childhood/teaching-and-learning/ece-curriculum/te-whariki/
9. Ministry of Education (2009). *Kei Tua o te Pae. Assessment for Learning. Early Childhood Exemplars Mathematics (18).* Retrieved from www.education.govt.nz/assets/Documents/Early-Childhood/Kei-Tua-o-te-Pae/ECEBk18Full.pdf

## Nurturing the seed

Many things nurture the seed: teacher pedagogy, teacher content knowledge, family/whānau knowledge, and resources. The *Te Aho Tukutuku* resource provides information on the importance of these aspects, and offers a way to strengthen and enrich, foster and develop mathematics teaching and learning.

## Strands of early mathematics

*Te Aho Tukutuku* outlines six strands of early mathematics:

- Pattern - the process of exploring, making and using patterns.
- Measuring - answering the question "How big is it?"
- Sorting - separating objects into groups with similar characteristics.
- Locating - exploring space or finding or 'locating' something, such as a place (location), or an item in space.
- Counting and grouping - the process for working out the answer to a question about "How many?" Grouping involves putting things together.
- Shape - naming shapes and identifying the unique specific properties or features of shapes.

# Summary of Effective Pedagogy in Mathematics/Pāngarau Best Evidence Synthesis Iteration (BES)[10]

## Key findings: The early years

Young children are powerful mathematics learners. Research has consistently demonstrated how a wide range of children's everyday activities, play and interests can be used to engage, challenge and extend children's mathematical knowledge and skills. There is now strong evidence that the most effective settings for young learners provide a balance between opportunities for children to benefit from teacher-initiated group work and freely chosen, yet potentially instructive play activities. Teachers in early childhood settings need a sound understanding of mathematics to effectively capture the learning opportunities within the child's environment and make available a range of appropriate resources and purposeful and challenging activities. Using this knowledge, effective teachers provide scaffolding that extends the child's mathematical thinking while simultaneously valuing the child's contribution.

10. Anthony, G. and Walshaw, M. (2007). *Effective Pedagogy in Mathematics/Pāngarau Best Evidence Synthesis Iteration [BES].* Retrieved from www.educationcounts.govt.nz/publications/series/2515/5951

## Key ideas: The early years

- All children can be powerful mathematics learners.
- Children have their own purpose for activities.
- Children's involvement in mathematical learning experiences depends on interest.
- Mathematics learning experiences should be both planned and informal/spontaneous.
- Everyday activities and play situations provide a wealth of mathematical experiences.
- Teachers can extend the child-initiated activities by scaffolding, thematic instruction, or instruction.
- Teachers need to cater for children's interests and mathematical abilities and to engage children in challenging learning experiences.
- Content matter is important.

## Issues identified in early mathematics teaching

Low levels of content knowledge and the resulting lack of confidence limits the ability of teachers to engage children in the mathematics learning opportunities present in existing activities. This low-level knowledge also limits the ability of teachers to introduce more focused, interventional activities designed to cater for diverse learners. Other issues identified included teachers:

- being unaware of the need to cater for children's interests and mathematical abilities, and to engage children in challenging learning experiences
- missing many opportunities for sustained, shared cognitive engagement
- seeing mathematics as numeracy only
- being unaware of home mathematical experiences, expectations and aspirations
- not following their planning for mathematics teaching and learning.

# A guide to children's early mathematics learning

Two factors are critical in supporting children to be confident and capable mathematics learners – pedagogical leadership and teacher knowledge (subject and pedagogical).[11]

## Pedagogical leadership

Strong pedagogical leaders understand how to integrate mathematics throughout their service's curriculum to best support children's learning. They have extensive subject knowledge and use this to support teachers to improve their knowledge and practice and to teach mathematics more intentionally as part of the curriculum.

Leaders promote a culture where teachers reflect on their teaching practice to continually make improvements. In services with strong pedagogical leadership, teachers are more likely to be up to date with current research and good practice about mathematics in early childhood. They are able to use this knowledge and high quality resources to be innovative and intentional in their planning and teaching. As a result, teachers confidently recognise and extend children's understanding of mathematical concepts in a range of contexts.

## Teacher knowledge

Effective teachers successfully integrate the different strands of mathematics of *Te Kākano* in the curriculum and provide each child with rich learning experiences across the breadth of mathematics. The curriculum reflects a balance of child-initiated learning experiences and deliberately planned activities.

Teachers understand how cultural concepts such as kowhaiwhai, rangoli, and tapa designs, relate to mathematics and children's learning. They know how to integrate these into their teaching practice. They share children's progress and learning with parents and whānau in wall displays and in assessment information such as portfolios.

11. Pedagogical knowledge – knowledge, skills, attitudes and attributes needed to provide clear, purposeful, stimulating and effective learning experiences. Lee, S. (2010). Toddlers and maths: fact, fiction of just fun? *The First Years: Ngā Tau Tuatahi. New Zealand Journal of Infant and Toddler Education.* 12(2), pp38-43:p40.

## A balance of deliberate teaching and spontaneous learning across the breadth of mathematics

New Zealand and international research highlights the need for a balance between spontaneous child-initiated play and planned mathematical learning.[12] When supporting children's mathematical learning, teachers need to be clear about what they are trying to achieve and then deliberately reflect on the actions they can take to help the child achieve the intended outcome. Epstein points out the need for both approaches.

> *"Children need opportunities to initiate activities and follow their interests, but teachers are not passive during these [child] initiated and directed activities. Similarly, children should be actively engaged and responsive during teacher-initiated and directed activities. Good teachers help support the child's learning in both types of activities."*[13]

In this resource we have used the term deliberate teaching but acknowledge that sometimes the term *intentional teaching* is used. Esptein explains what intentional teaching is.

> *"Intentional teaching means teachers act with specific outcomes or goals in mind for children's development and learning. Teachers must know when to use a given strategy to accommodate the different ways that individual children learn and the specific content they are learning."*[14]

12. Anthony, G. and Walshaw, M. (2007). *Effective Pedagogy in Mathematics/Pāngarau Best Evidence Synthesis Iteration [BES].* Retrieved from www.educationcounts.govt.nz/publications/series/2515/5951

    O'Brien, N. (2015). Strategies for teachers developing strategies for mathematics in early childhood education. He Kupu 4(1), pp18-22. Retrieved from www.hekupu.ac.nz/index.php?type=journal&issue=21&journal=341.

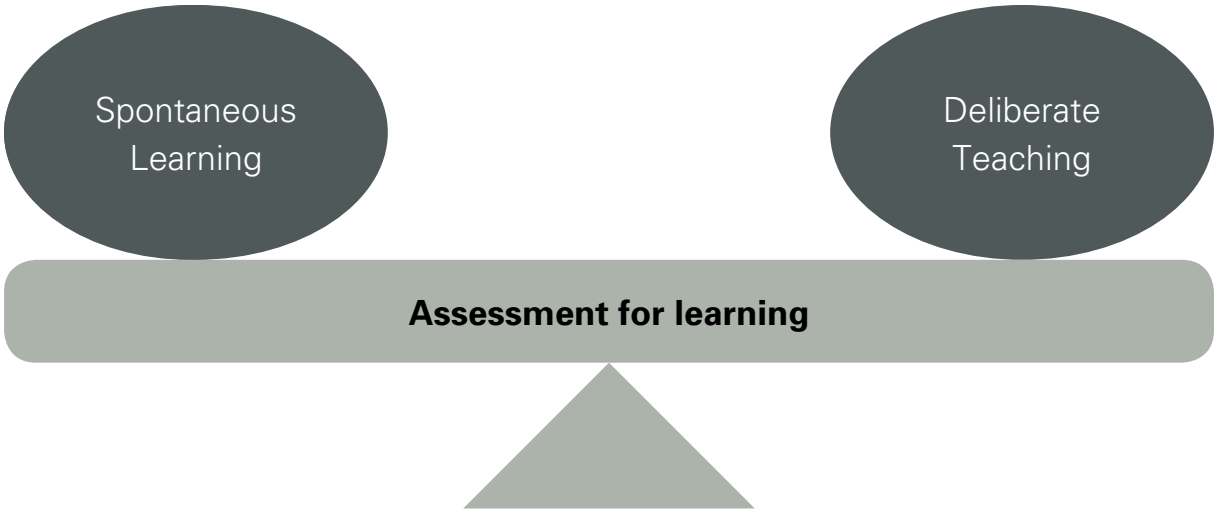
    Bobis, J., Clarke, B., Clarke, D., Thomas, G., Wright, R., Young-Loveridge, J., & Gould, P. (2005). Supporting teachers in the development of young children's mathematical thinking: Three large scale cases. *Mathematics Education Research Journal*, 16(3), pp27-57.

    Edens, K., M., & Potter, E. F. (2013). An exploratory look at the relationships among math skills, motivational factors and activity choice. *Early Childhood Education Journal*, 41, 235-243.

13. Epstein, A. (2007) *The intentional teacher: Choosing the best strategies for young children's learning.* Washington DC. National Association for the Education of Young Children, pp8-9

14. Epstein, A. (2007) *The intentional teacher: Choosing the best strategies for young children's learning.* Washington DC. National Association for the Education of Young Children, p1.

## Balancing spontaneous learning with deliberate teaching




In the Best Evidence Synthesis, Anthony and Walshaw state:

> *There is now strong evidence that the most effective settings for young learners provide a balance between opportunities for children to benefit from teacher-initiated group work and freely chosen, yet potentially instructive, play activities. Teachers in early childhood settings need a sound understanding of mathematics to effectively capture the learning opportunities within the child's environment and make available a range of appropriate resources and purposeful and challenging activities. Using this knowledge, effective teachers provide scaffolding that extends the child's mathematical thinking while simultaneously valuing the child's contribution.*[15]

### Deliberate teaching

Teachers' planning guides their deliberate use of teaching strategies as part of their broader curriculum. Teachers know how to provide a range of experiences for children to support and extend their learning. They can explain and discuss their pedagogy related to mathematics, how they integrate mathematics throughout the curriculum, and build on individual children's knowledge in both spontaneous and planned ways. Teaching of mathematics is intentional and embedded in a holistic curriculum.

Leaders play a significant role in promoting deliberate teaching. They access professional learning for themselves and teachers, encourage and lead reflective practice (such as peer observations, videoed practice to review, and collaborative planning), professional reading, action research and internal evaluation (self review). This leadership means teachers can keep up to date with current research and good practice and have the knowledge necessary to be innovative in their teaching of mathematics.

15. Anthony, G. and Walshaw, M. (2007). *Effective Pedagogy in Mathematics/Pāngarau Best Evidence Synthesis Iteration [BES], p2.* Retrieved from www.educationcounts.govt.nz/publications/series/2515/5951

In services with a balance of deliberate teaching and spontaneous learning, children have access to high quality mathematics resources. Teachers are skilled at using interesting and challenging resources to scaffold children's learning and extend mathematical concepts. Teachers consider:

- children's interests
- parent aspirations
- their service's priorities for children's learning
- current research about mathematics
- next steps identified in assessment information.

In the following example, ERO observed how a leader and teachers used their pedagogical and subject knowledge to deliberately plan for and extend children's mathematical learning. In this service, teachers empowered children to make choices about their learning. They also fostered children's exploration and experimentation by respecting and encouraging the children's thinking and ideas. With this support, children in this service developed elaborate, imaginative and deep interests that extended well beyond their initial ideas.

Curriculum and teaching practice

## Creating 3D structures

A four-year-old at this service had seen a *YouTube* video about constructing with plastic cups and he had shared his excitement about making some similar constructions with his teachers. A series of learning stories show the development of his ideas and the deliberate extension of his mathematical exploration by his teachers, which culminates in a long-term project not just for him but for many of the children in his group.

**Assessment**

The learning stories in his portfolio showed him making complex and accurate 2D drawings of the 3D structures he wanted to create. Teachers placed these drawings on display for him to refer to as he began to construct in 3D.

**Curriculum and teaching practice**

In the middle of his project, the teachers and children attended a light show at a local art gallery. The use of light fascinated him, and he then wanted to add light to his structures. Teachers provided torches and helped to make the room dark, and he added another dimension to his structures.

**Teacher knowledge**

By this stage, other children in the group had caught his enthusiasm for designing and creating 3D structures from the plastic cups and had started building their own – with and without light. The teachers' planning shows a clear focus on supporting this mathematical learning. They made sure that resources were available to prompt children's interests through construction sets, blocks of a variety of shapes, and a variety of games with a mathematical focus; and they intentionally used mathematical language with children. Children were urged to draw their ideas and then to transfer their 2D drawings into a 3D reality.

Assessment

The following comments show the children's development of and reflection on their mathematical understanding – not only about shape, but also pattern, locating and counting.

- "It's at the apex."
- "It's about making the cups balance."
- "Remember the one where it went along the floor for a long way, and then went up the wall? I had to angle the cups to start going up the wall to make it balance."
- "I had to use equal numbers of red and green cups to make the pattern. The yellow ones are the contrast, but they are all the same size. The yellow ones just look bigger because they are nearer when you stand in front."
- "Can you see how it alters the dimensions of my face when I stand behind the tower and you can see me in front?"
- "I had to work hard to get the balance right. Sometimes the cups are tricky, but I just had to concentrate to get it right. First I just made a triangle shape, but now I'm making more linear shapes that can go across the floor."

In this next example, teachers noticed that a trip to the library provoked one child's interest in large numbers. The teacher recognised opportunities for extending this interest when they returned to the centre. Open-ended questioning encouraged the children to explain and extend their thinking.

## Big numbers


**Curriculum and teaching practice**

The children went on a trip to the local library, where the children's librarian explained how the Dewey Decimal System used numbers to categorise books so people could find books they were interested in. They went for a number hunt around the library, and found each of the big sorting numbers 000, 100, 200, 300, 400, 500, 600, 700, 800 and 900.

**Teacher knowledge**

One child was especially excited by the big numbers and, back at the centre, shared that his favourite number was 100. He wrote the number on the whiteboard to show his friends, and also wrote the number 100 on a birthday card for his mum.

Teacher knowledge

He was having a conversation with his friend, and said he wanted to be 100 years old. His friend said he wanted to be 102. The teacher noticed the conversation, and asked why the second child chose 102 – he responded that it was bigger than 100.

Later, the children were sitting with the number puzzles, where each piece is a digit. The teacher asked them what big numbers they could make, suggesting 90 to start with. This was easily done, so the teacher suggested an even bigger number – 100. The children did this, and then continued changing only the first digit, so they made all the hundreds up to 900. One child was unsure about naming these numbers, saying "twenty hundred, thirty hundred". The teacher guided him to name them correctly, and he quickly caught on, confidently counting in hundreds up to 900.

In another service, teachers supported new learning about measurement by planning to extend this learning in several different and meaningful contexts.

### Measuring, estimating and comparing

**Curriculum and teaching practice**

The children experimented with multiple ways of measuring things, such as how much water their gumboots could hold, how long they could hang off a beam, and how far playdough could bounce. Teachers extended this interest by showing them different ways the measurement information could be presented.

They used the tape measure to measure each other's height and recorded these on a height chart. They counted how many segments were in their mandarins each day and recorded this data in a pictograph.

**Teacher knowledge**

One child took the tape measure and began stretching it alongside pieces of wood. He told the teacher that he wanted to make two pieces of wood that were the same length. The teacher helped him by explaining that he would need to make sure the number on the tape measure was the same for each piece. There were not any pieces already the same length, so he realised he would have to cut one to match the length of another. The teacher prompted him to find the same number on his measuring tape as before, and mark the wood with a pencil to show where he needed to cut it. The boy was able to do this, and learnt a practical application for his new measuring skills.

In the following service, teachers took a planned approach to support children's learning in a way that responded to the different interests, strengths and ages of children in the group.

## Comparing size

Curriculum and teaching practice

Children at the centre had been exploring size. Teachers provided three rectangular mats, labelled big, medium and small; and a collection of objects, with a big, medium and small one for each object type. The teacher demonstrated the activity, placing the biggest pompom on the mat labelled 'big', the next biggest pompom on the mat labelled 'medium', and the smallest pompom on the mat labelled 'small'.

A child quickly picked up on the task, and placed the appropriate lock, bell, square etc. on each mat, describing the biggest objects as the 'daddy' ones, the medium ones as the 'mummy' ones, and the smallest ones as the 'baby' ones. This tied in to the language and comparative sizing used earlier in the *Goldilocks and the Three Bears* story.

Curriculum and teaching practice

The teachers planned to extend that child's learning with activities that helped her to sort and classify based on more specific attributes, such as length, weight or height.

An older child was comparing her shoe size to that of her teacher. The teacher suggested measuring their feet as a way of describing the length more accurately. They got a ruler and measured the teacher's foot, followed by the girl's foot. They compared the numbers, and saw that the teacher had the bigger foot and the bigger number.


Teacher knowledge

The teacher then showed her how to measure the length of a line with her feet, carefully balancing and putting one foot exactly in front of the other. The teacher counted how many foot-lengths a line was, and then the girl counted how many of her foot-lengths the line was. The numbers were different, and the teacher asked why that might be. The child replied that the teacher used fewer foot-lengths because his feet were bigger.

Curriculum and teaching practice

The teacher planned to continue this child's interest in measuring by exploring other non-standard ways of measuring.

## Spontaneous learning

When spontaneous learning is happening well, children experience a curriculum in which mathematics is included in meaningful contexts that recognise their strengths and interests and build on these. Leaders and teachers understand how to integrate mathematics into the child's interests. Teachers may also integrate mathematics into their bicultural practice, for example, learning mathematics though waiata, creating and exploring kowhaiwhai patterns and Māori carvings, and using resources that reflect the cultures of children in the centre.[16]

The example below highlights how one child's learning was extended as a result of the collaborative culture evident in the service she attended. Teachers in the service worked collegially to reflect on their practices, children's interests and new learning. This collaboration meant that a child's success seen by one teacher was extended later by another teacher.

### Making it fit

**Curriculum and teaching practice**

The teacher noticed a child's interest in arranging blocks on a baseboard. After sitting together for a while making patterns, the teacher suggested she could try to cover the whole board with blocks. This was a challenge, as there were a number of nodules on the board and the blocks were all different sizes.

The child carefully started putting the blocks on from one side, making sure the blocks lined up. She then started adding more blocks from the opposite side of the board, and found herself stuck with a gap in the middle that was smaller than any of her blocks. She asked her teacher if there were any smaller blocks to fill the gap, and was prompted to look for some other holes in her block arrangement. She tried to put a block in one of these spaces, and commented that her block was too big.

**Teaching practice**

The teacher then helped her to count the number of nodules on the board in the space, and the number of nodules on her smallest block. They compared the numbers, and realised that they were different. The teacher suggested making a bigger hole might help, so after considering for a while, the child moved the pieces across to fill one hole, and enlarge another. She then counted the nodules in the new, bigger hole, and compared the number to the nodules on her brick. They were the same, so she was able to fill the board completely.

16. Waiata – song(s). Kowhaiwhai patterns – refers to a traditional red, white and black coloured pattern. These colours resemble the origin of the Māori tribes and were commonly found on rafters and meeting houses.

| | |
|---|---|
| Teacher knowledge | A few weeks later, this child was observed by another teacher repeating the activity by herself, and doing a great job of it. The teacher noticed this, and thought it was time to extend the girl's thinking, by introducing different shapes, as well as different sizes. The teacher got the number-teaching blocks for the girl, and straight away, she started trying to cover the board with them. She got nearly to the end, before realising that the blocks she had left would not fit the remaining spaces on the board. This time, she was able to use her problem-solving skills from the earlier activity. She moved some pieces, looked at the different sizes and shapes of the blocks and the holes, and tried different ways of putting in her remaining blocks, until she solved the puzzle. |

The following examples show the ways in which teachers were supporting and extending children's interests in mathematics.

| | |
|---|---|
| | **Sorting and patterning** |
| Child initiates activity | One child was playing with magnetic shapes. "I'm making shapes." She sorted the shapes into groups and then organised them into patterns. The teacher asked what she knew about shapes, and pointing to each group in turn, the child said, "Circles, squares, triangles and rectangles." |
| Teacher notices learning happening | As part of this the teacher recognised the child's learning and identified an opportunity to extend the mathematical learning by engaging the child further in sorting, ordering and creating with shapes. The teacher recognised shape and spatial awareness, patterns, and sorting and organising into categories as some of the mathematical concepts being explored by the child.<br><br>The teacher then supported the child to create artwork using a complex mosaic technique. |

The Playcentre, in the example below, is led by a group of Playcentre-qualified parents who meaningfully integrate mathematical concepts into children's play.

Noticing of mathematics occurring naturally

Questioning prompts transfer of concepts

## Measuring and growth

Parents talked about mathematics with the children as they played in the sandpit, mixed play-dough, played with water using measuring cups, read books, and shared finger plays, rhymes and favourite songs.

"Maths is a natural conversation that we have with children all the time... there are lots of opportunities for questioning." (Supervisor)

A group of three boys went to the children's growth chart. They began to measure themselves and compare their heights: "Where are you up to?" One child reads the ruler – "I am six-nine, see. That's taller then you, ay?" The boys continued to compare each other against the chart. Next to the growth chart is another type of chart, used to measure growth using photos of children over time. The children moved from the height chart to comment on the photo growth chart. "Look at how small I was, now I am six-nine and much bigger. I am bigger sideways too." A parent asked the boys what other things they could track the growth of. One child remembered that they had a metre ruler in the vegetable garden and they had been tracking the growth of the bean plants.

The children raced off to investigate and measure the beans.

In the third example, the teacher offers the child a pen and paper to record his thinking while at the same time giving him the opportunity to decide how he wants to record his findings.

Children initiate activity

Teacher notices and extends the learning

## Recording data

The child and his friend were working together to explore weight and counting. They had a set of scales and were watching the number change as they carefully added more objects. He used his finger to point to each of the numbers, counting as he went.

The teacher noticed his counting and pointed out the pens and paper, suggesting he might like to record his counting. He used his finger to count some rocks in a container he had been using on the scales. He got to 10, and then tried writing 10 lines on a piece of paper. This was not working as he wanted, so he changed his approach. He laid out the rocks in a line, and drew a line for each rock. He then counted the lines.

The child had problem solved independently, and had used his number skills in ways that were meaningful to him.

## What are the challenges for deliberate and spontaneous teaching?

Some leaders and teachers advocate a 'child-centred' curriculum and consider that because mathematics is everywhere children will engage spontaneously with mathematical ideas at their own pace and that there is little need to deliberately plan, instigate or extend specific experiences into deeper learning.

This can be a result of limited teacher knowledge, a lack of ongoing professional learning, or simply that the prevailing teaching culture discourages deliberate teacher intervention in children's play. Teachers may have good subject knowledge in mathematics, but over-reliance on children to manage their own curriculum means that teachers feel constrained from using their knowledge and teaching practice to extend children's learning. An absence of deliberate teaching means that opportunities for deeper engagement in or recognition of mathematical learning are missed.

Children have many and varied opportunities to explore the six strands of mathematics. Teachers provide lots of resources focused on mathematics, and use these and open-ended questioning to stimulate children's immediate interest and learning. They may be aware of home mathematical experiences, parents' expectations and aspirations, and incorporate these, particularly those relating to the child's cultural identity, into hands-on mathematical activities.

Nevertheless, despite the curriculum providing many opportunities to learn about mathematical concepts, children's engagement with these remains largely incidental and relies upon them taking an interest in the activities provided. Significant learning opportunities are frequently overlooked because mathematical experiences are often one-off events, rather than enabling children to explore concepts in more depth or take their new learning into other contexts.

Without a balance of deliberate teaching and spontaneous learning, a 'hands off' approach does not benefit children's learning. When teachers do not deliberately or intentionally extend children's interests and build on their learning over time, children are disadvantaged.

In reflecting on how well mathematics is implemented as part of their curriculum, leaders should consider not only how effectively teachers notice, recognise, and respond to children's immediate and obvious interests, but also how they deliberately extend and scaffold children's deeper learning and understandings. Teachers have a critical role to inspire and engage children in mathematics, rather than leave the child to initiate engagement independently and at random.

## Assessment for learning

Teachers document children's learning and progress in mathematics as part of their assessment practices. Rich assessment information shows the mathematical learning in the children's play and in conversations with their peers and teachers. In the best instances, assessment information can show the development of children's working theories about mathematical concepts. Assessment information includes next steps to build children's mathematical learning. Such assessment informs planning and highlights the child's achievement and progress to parents, whānau and the child.

The following examples show how two services recorded children's developing understanding of mathematics and used this documentation to extend their learning. The first example is of a child developing confidence with balancing and comparing weights. Carefully scaffolding his learning gave the child a sense of achievement and the confidence to teach his new skills to a friend.

## Making comparisons with weight

**Curriculum and teaching practice**

The child had devised his own see-saw with wooden blocks, and was balancing different combinations of items at each end, commenting that heavier things went down and lighter things went up. He compared the weight of different types of objects too – one cow was heavier than two blocks; plastic lids were lighter than anything else.

**Assessment**

The assessment information showed the teacher planned to extend this learning by helping him to make predictions about weight, then seeing if his predictions were accurate.

**Teacher knowledge**

The next learning story has the boy working with his teacher and a set of balance scales. The teacher prompted him to hold objects in his hands and asked which he thought was lighter. He looked at the objects, and felt their weight, before replying "the blue block is lighter... and it's smaller". He was then able to check his prediction by placing the objects in the cups on each end of the balance scale, and use his earlier observation of "heavier goes down, lighter goes up" to see which of all the objects was lightest.

**Curriculum and teaching practice**

Later on, a teacher noticed that the boy was sharing some pule (samoan shell beads), dividing them up in different ways, and showing another boy how the balance scales work. He prompted the other boy to predict which was heavier, then showed him how to check. The boys discussed which they thought would be heavier, then put the shells in the cups on the balance scale. He explained to his friend "This one is heavier. It's down low, so it's the heavy one."

**Learning partnerships**

The boy's mum commented in his portfolio that he shared his learning with his family when he got home, explaining about heavy and light, and how heavy things went down and light things went up. He was very proud of his learning!

The next example shows one child's mathematical progress in sorting and patterning and how she is working in increasingly complex ways. The assessment information documented the child's learning over almost a year.

Curriculum and teaching practice

Teacher knowledge

Assessment

## Complexity over time

The first record shows the child sorting beads into groups by their colour. She sang a counting song in Māori as she did this. The teacher noticed the girl's interest in patterning, and planned to include more opportunities for sorting and patterning in future.

A week later, the teacher noticed the child creating more complex groupings, by both colour and shape. The child arranged these groups all in a line, and stuck the pieces together with tape. The teacher extended the girl's understanding by exploring the environment with the child, looking for patterns.

The next assessment record shows the same child again grouping objects by their characteristics and explaining why each object belongs in the group: "these ones are all squares" and "these are the ones you can look through". The objects in each group had both similar and different characteristics, but she had decided what feature she was basing her grouping on, and did this accurately. The teacher thought that the child was ready to try adding to a pre-existing pattern.

Assessment

The child sat with cards that had patterns printed on them, with an empty spot. She looked at the card carefully, and talked about the different features. She decided the pattern was by colour, so pointed to each pattern and named the colour. When she got to the empty spot, she immediately knew what colour she would need to put there, and went looking for a pattern of the right colour.

A few months later, the girl was arranging driftwood, shells and stones into a pattern on the floor, when another child noticed what she was doing. She explained what she was doing, and asked the other child questions to guide her in creating her own pattern. She offered advice when needed, and commented on the other child's pattern. The complexity of her patterning has increased over time, and she understands the concept well, as shown by her willingness and ability to support others to create patterns of their own.

Such informative assessments highlight the child's development of understandings and dispositions as well as their strengths and interests. This type of information can be shared with the child's primary school teacher to support their continuity of learning.

Children's portfolios also provide a useful source of information for parents to see their child's development, enabling them to support and extend their child's learning in authentic ways at home. Leaders and teachers share their knowledge and resources with parents and whānau, including explaining the links between *Te Whāriki* and *The New Zealand Curriculum* so that parents can better understand and support their child's learning.

## What are the assessment for learning challenges?

Many services need to move from just having mathematics visible in children's assessment records, where it is not necessarily the focus of the assessment. The challenge for leaders in these services is to deepen teachers' assessments of children's strengths, capabilities and interests, and to highlight children's developing understanding of mathematics concepts in subsequent curriculum planning and assessment records.

## Improving teaching of mathematics in the early years

This guide provides examples of deliberate teaching, assessment that underpins deeper learning, and teacher response to spontaneous learning. Despite evidence of good practice in many settings, ERO has found that mathematical teaching practice in the early years remains variable.

In considering how to improve mathematics in the curriculum, leaders and teachers in the early years should be aware of the following issues which have the potential to limit or disengage children's propensity for learning mathematics:

- Leaders and teachers have some knowledge about mathematics but they are not using this to intentionally plan strategies that will enable children to explore the breadth of the curriculum. The curriculum lacks a focus on mathematics.
- Teachers wait for children to take an interest in mathematics rather than actively engaging and stimulating children's interest.
- There is limited explicit teaching of mathematical concepts, or, where there is, it is through overly teacher-directed activities and not in meaningful contexts. Often activities focus on counting and shape only.
- Assessment information varies in the extent to which it makes mathematics learning visible. Either no mathematics assessment information is collected or, when information does show children engaging in mathematic experiences, there is little about children's learning or progress.
- Children do not have opportunities to develop deeper understandings through their play or explore concepts in new contexts.

### Continuity of learning

In some services, overly teacher-directed activities are undertaken in response to parental pressures to introduce formal programmes perceived to prepare children for school. Such practices have good intentions but are misguided. In the worst cases, these activities are inappropriate and demotivating for children. They are not aligned to the objectives of *Te Whāriki* or *The New Zealand Curriculum* as they are not based on children's interests and inquiries, and therefore have the potential to lead to children developing negative attitudes to mathematics.

Leaders and teachers with the most deliberate and knowledgeable approach to mathematical learning for children avoid such misunderstandings about best practice. They do this by sharing their knowledge and resources with parents and whānau to support their child's mathematical learning in authentic ways. Assessment information, such as that documented in portfolios, empowers children, enhances their identities as learners, is a resource to connect knowledge at home with new learning, and fosters a sense of belonging and engagement.[17] Teachers provide parents and whānau with useful information to share, during their early years and as children make their next steps to school. They work with local schools to facilitate collaborative relationships and sharing of responsibility, becoming familiar with both curriculum documents, and making comparisons and links to better support children's learning.

Teachers in the early years have the potential to implement a curriculum that will better support children's life-long mathematical learning.

17. Peters S., Hartley C., Rogers P., Smith J. and Carr M. (2009). Supporting the transitions from early childhood education to school. Insights from one Centre of Innovation project. *Early Childhood Folio*, 13: 2-6.

## Internal evaluation to improve practice

This guide is intended to support leaders and teachers to undertake internal evaluation (self review) of their leadership and the curriculum, teaching practices and assessment processes that underpin mathematical learning in the early years.

As a leader, you can think about how you can encourage teachers to:

- be more intentional in their teaching to make sure children have opportunities for deep learning across the six mathematics strands (*Te Kākano*)
- make children's developing understanding of mathematics more visible in assessment information and use it to inform deliberate planning of mathematical learning
- use spontaneous learning experiences as an opportunity to sustain and extend children's mathematical understandings
- work collaboratively to ensure each child experiences seamless continuity of learning as they move from one learning setting to the next.

ERO has developed two resources that will help with your internal evaluation:

- The first resource is a continuum which can be used to identify your current practice and key next steps for improvement.
- The second resource is a set of prompts and indicators to guide your internal evaluation of mathematics in the curriculum.

# **Resource one:** A continuum for improving early mathematical learning for children

## Incidental mathematical learning for children

- Mathematics is not a deliberate focus for leaders. They believe mathematics is everywhere, and there is no need to plan specific experiences.
- Teaching practice is variable, with a reliance on children to independently engage with mathematics-related play.
- Some overly teacher-directed activities that lack any meaningful contexts.
- Learning opportunities have a strong focus on counting and shape. Some teachers are knowledgeable about mathematics.
- Potential to build on opportunities to extend mathematics learning through more deliberate planning. Professional learning and development (PLD) is required to improve some teachers' knowledge.
- Mathematics learning is generally not visible in assessment. When learning is visible, progress is not identified, and neither are the child's next steps.
- Partnerships not focused on mathematics learning. Very few shared mathematics activities with parents and whānau.

## Responsive mathematical learning for children

- Leaders initiate professional discussions about deliberate planning for mathematics.
- Change in leadership does not have an impact on the mathematics focus of the curriculum.
- Good use of resources to extend children's learning across the strands of mathematics.
- Teachers notice, recognise and respond when mathematics learning occurs spontaneously. The risk is that some children might not engage with mathematics learning if it is not planned or deliberate.
- Teachers have good general mathematics knowledge.
- Teachers need to deepen their knowledge to extend children's mathematical learning. PLD focus is on curriculum in general.
- Mathematics learning is visible in assessment information. Next steps and continuity of learning are not always clear.
- The six strands of mathematics are not recognised in assessments.
- Proactive partnerships about learning. However there is a lack of engagement with parents and whānau about their child's mathematics learning.

## Deliberate and spontaneous mathematical learning for children

- Leaders have clear expectations about how mathematics is integrated throughout the curriculum which is shared with teachers and parents and whānau.
- A balance of spontaneous and deliberately planned mathematics teaching and learning through play.
- Learning is based on children's strengths, interests and needs across the six strands of mathematics.
- Planning and practice is informed by assessment information.
- Teachers are skilled at scaffolding learning and using resources to provide challenges.
- Teachers have strong subject and pedagogical knowledge.
- Teachers are reflective and collaborative practitioners.
- Teachers are up to date with research and good practice about early mathematics.
- Teachers are supported to develop knowledge and teaching practice through well-planned professional learning and development.
- Rich assessment information that highlights children's progress and next learning steps.
- Clearly linking how mathematics is integrated throughout the curriculum.
- Leaders and teachers are responsive to parent and whānau aspirations, children's interests and home mathematics experiences.
- Teachers provide parents with resources and best practice in early mathematics to continue the learning at home.

# Resource two: Investigative prompts and indicators of practice

Leaders and teachers may like to use these prompts and indicators to guide an internal evaluation of mathematics in their curriculum.

## Leadership, teachers' subject and pedagogical knowledge and professional development

### Investigative prompts

- How do we:
  - promote quality learning and teaching relating to mathematics within our curriculum?
  - build and support professional practice relating to mathematics within our curriculum?
- What professional learning and development (PLD) have we participated in relating to mathematics within our curriculum? What do we know about the impact of this PLD?
- What reflective practice/inquiry have we been involved in that has led us to question and modify our approach regarding mathematics within our curriculum?
- In what ways are we intentional and deliberate in our teaching regarding mathematics within our curriculum?

### Indicators

- We use evidence to reflect on and improve our practice relating to mathematics within our curriculum.
- We reflect on our personal values, beliefs, attitudes and philosophy when we make decisions relating to mathematics within our curriculum.
- We are able to explain and discuss our pedagogy relating to mathematics within our curriculum.
- We are able to articulate and share the rationale for decisions relating to mathematics within our curriculum.
- We have a depth of mathematical subject knowledge that enables us to respond meaningfully to all children's mathematical interests and enquiries, or we access information with children.
- We are reflective practitioners who critique our own practices regarding mathematics within our curriculum and engage in discussion and debate that informs improvement of our practice.

## Assessment information

### Investigative prompts

> How do our assessment processes/practices show how we notice, recognise and respond to children's interests and strengths in mathematics?

> Does our information show continuity of mathematics learning for children and progression in terms of strengthening *Te Kākano?*

### Indicators

> Our assessment builds children's identity as successful mathematics learners.

> We value Māori children's identities as Māori as the foundation for a key to their success as Māori and this is reflected in our assessment information.

> Our assessment practices value and respond to Pacific cultures, knowledge and ways of learning.

> Our assessment information:

- shows continuity of learning and progression for children in terms of:
  - increasing the range of mathematical ideas
  - the use of mathematics in a wider range of contexts
  - increasing the complexity within the strands of mathematics
  - strengthening children's learning dispositions.
- reflects a credit-based approach that pays attention to children's strengths, interests and dispositions.

> Our assessment includes multiple perspectives that enhance the interpretation and analysis of learning.

## Curriculum design and implementation

### Investigative prompts

- What informs our decisions about mathematics within our curriculum?
- How does mathematics within our curriculum recognise and build on the knowledge and expertise children and their parents and whānau bring to our service?
- How does mathematics within our curriculum reflect the interests, professional knowledge, curriculum and subject knowledge of us, as teachers?

### Indicators

- We can explain how mathematics within our curriculum aligns to the principles and strands of *Te Whāriki*.
- We extend children's understanding of mathematical concepts from a te ao Māori perspective.
- We have an understanding of each child as a unique mathematical learner including their working theories, and encourage their development.
- Children contribute to the development of mathematics within our curriculum so it is responsive to their interests, culture, language and identities.
- We recognise and value the importance of children learning through play.
- We use the languages and symbols of Māori children and Pacific children to contribute to the richness of a vibrant mathematical learning environment.
- Children's mathematical learning is enhanced through meaningful and interesting opportunities to:
  - use mathematics in everyday life
  - use open-ended resources for mathematical exploration
  - engage in games in which children can vary the level of challenge.
- We provide resources and environments that encourage choice, a level of challenge, and mathematical exploration that is meaningful and enjoyable for children.
- Children and we, as teachers, have fun as part of the mathematical learning process.

## Internal evaluation and inquiry

### Investigative prompts

> What do we know about the learning outcomes for children with regard to mathematics?
> What do we know about the effectiveness of mathematics within our curriculum to support children?
> How does our internal evaluation guide decision making and lead to improved outcomes for children?

### Indicators

> There is compelling evidence that our internal evaluation leads to improved outcomes for children as mathematical learners.
> We consult parents and whānau and give them opportunities to contribute to internal evaluation about mathematics within our curriculum.
> We:
  - question whose mathematical knowledge is valued and reflected in our curriculum as part of internal evaluation
  - evaluate the impact of decisions about mathematics within our curriculum for all children
  - use research and relevant resources, including *Te Whāriki*, as a basis for evaluating mathematics within our curriculum.

# Conclusion

## What needs to happen to ensure that all children have rich mathematics learning?

This guide acknowledges and describes effective practice in services where the curriculum integrates the breadth and depth of mathematics. Children in these services experience many and varied mathematical learning experiences that respond to and extend their strengths and interests. These children are establishing sound foundations for understanding mathematical concepts that will be critical for their future learning success.

Pedagogical leadership and teacher knowledge underpin this effective practice. Effective pedagogical leaders understand how to integrate mathematics into the curriculum to best support children's learning. Skilled teachers intentionally plan experiences to extend children's developing mathematical understandings. ERO encourages leaders and teachers to be increasingly intentional and deliberate in the way mathematics teaching builds on spontaneous child-initiated opportunities for early learning.

In supporting children to be confident and capable mathematical learners in their early years the following aspects are critical:

- Leaders placing an emphasis on mathematics as a foundation for children's learning.
- Leaders having a good understanding of how mathematics is implemented as part of their curriculum, including what this means for planning, assessment and internal evaluation.
- Mathematics teaching and learning occurring in meaningful contexts as part of a holistic curriculum that covers all strands of mathematics learning appropriate to early years.
- Teachers having sufficient in-depth subject and pedagogical knowledge to notice, recognise and respond to children's mathematical learning; making this learning visible to children, parents and whānau; responding appropriately to parental expectations and providing useful information as children transition to school.

# Notes

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New Zealand Government