



Further Integration

AH Maths Exam Questions

Source: 2019 Specimen P2 Q6 AH Maths (Same as 2010 Q3a)

(1)

Use the substitution $t = x^4$ to obtain $\int \frac{x^3}{1+x^8} dx$.

Answer: $\frac{1}{4} \tan^{-1} x^4 + c$

Source: 2019 Q16a AH Maths

(2)

(a) Use integration by parts to find the exact value of $\int_0^1 (x^2 - 2x + 1)e^{4x} dx$.

(b) A solid is formed by rotating the curve with equation $y = 4(x-1)e^{2x}$ between $x = 0$ and $x = 1$ through 2π radians about the x -axis.

Find the exact value of the volume of this solid.

Answers: (a) $\frac{1}{32} (e^4 - 13)$ (b) $\frac{\pi}{2} (e^4 - 13)$

Source: 2018 Q15a AH Maths

(3) (a) Use integration by parts to find $\int x \sin 3x \, dx$.

(b) Hence find the particular solution of

$$\frac{dy}{dx} - \frac{2}{x}y = x^3 \sin 3x, \quad x \neq 0$$

given that $x = \pi$ when $y = 0$.

Express your answer in the form $y = f(x)$.

Answers:

$$(a) \quad -\frac{x}{3} \cos 3x + \frac{1}{9} \sin 3x + c \qquad (b) \quad y = -\frac{x^3}{3} \cos 3x + \frac{x^2}{9} \sin 3x - \frac{\pi x^2}{3}$$

Source: 2017 Q6 AH Maths

(4) Use the substitution $u = 5x^2$ to find the exact value of $\int_0^{\frac{1}{\sqrt{10}}} \frac{x}{\sqrt{1-25x^4}} \, dx$.

Answer: *Exact value* = $\frac{\pi}{60}$

Source: 2016 Q9 AH Maths

(5) Obtain $\int x^7 (\ln x)^2 \, dx$.

Answer:

$$\frac{1}{8}x^8 (\ln x)^2 - \frac{1}{32}x^8 (\ln x) + \frac{1}{256}x^8 + c$$

Source: 2015 Q10 AH Maths

(6)

Obtain the exact value of $\int_0^2 x^2 e^{4x} dx$.

Answer: $\frac{25}{32} e^8 - \frac{1}{32} = \frac{1}{32} (25e^8 - 1)$

Source: 2014 Q15 AH Maths

(7)

(a) Use integration by parts to obtain an expression for

$$\int e^x \cos x dx.$$

(b) Similarly, given $I_n = \int e^x \cos nx dx$ where $n \neq 0$,

obtain an expression for I_n .

(c) Hence evaluate $\int_0^{\frac{\pi}{2}} e^x \cos 8x dx$.

Answers:

(a) $\frac{1}{2} e^x (\sin x + \cos x) + c$

(b) $I_n = \left(\frac{e^x}{1+n^2} \right) (n \sin nx + \cos nx) + c$

(c) $\frac{1}{65} (e^{\frac{\pi}{2}} - 1)$

Source: 2013 Q8 AH Maths

(8) Use integration by parts to obtain $\int x^2 \cos 3x dx$.

Answer:

$$\frac{1}{3}x^2 \sin 3x + \frac{2}{9}x \cos 3x - \frac{2}{27} \sin 3x + c$$

Source: 2012 Q11 AH Maths

(9) (a) Write down the derivative of $\sin^{-1}x$.

(b) Use integration by parts to obtain $\int \sin^{-1}x \cdot \frac{x}{\sqrt{1-x^2}} dx$.

Answers:

$$(a) \frac{1}{\sqrt{1-x^2}}$$

$$(b) x - \sin^{-1}x \cdot \sqrt{1-x^2} + c$$

Source: 2011 Q11b AH Maths

(10) (a) Obtain the exact value of $\int_0^{\pi/4} (\sec x - x)(\sec x + x) dx$.

(b) Find $\int \frac{x}{\sqrt{1-49x^4}} dx$.

Answers:

$$(a) 1 - \frac{\pi^3}{192}$$

$$(b) \frac{1}{14} \sin^{-1} 7x^2 + c$$

Source: 2010 Q3 AH Maths

- (11) (a) Use the substitution $t = x^4$ to obtain $\int \frac{x^3}{1+x^8} dx$.
- (b) Integrate $x^2 \ln x$ with respect to x .

Answers:

(a) $\frac{1}{4} \tan^{-1} x^4 + c$ (b) $\frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + c$

Source: 2010 Q7 AH Maths

- (12) Evaluate

$$\int_1^2 \frac{3x+5}{(x+1)(x+2)(x+3)} dx$$

expressing your answer in the form $\ln \frac{a}{b}$, where a and b are integers.

Answer: $\ln \frac{32}{25}$

Source: 2009 Q9 AH Maths

- (13) Use integration by parts to obtain the exact value of $\int_0^1 x \tan^{-1} x^2 dx$.

Answer: $\frac{\pi}{8} - \frac{\ln 2}{4}$

Source: 2008 Q7 AH Maths

(14) Use integration by parts to obtain $\int 8x^2 \sin 4x \, dx$.

Answer:

$$-2x^2 \cos 4x + x \sin 4x + \frac{1}{4} \cos 4x + c$$

Source: 2007 Q4 AH Maths

(15) Express $\frac{2x^2 - 9x - 6}{x(x^2 - x - 6)}$ in partial fractions.

Given that

$$\int_4^6 \frac{2x^2 - 9x - 6}{x(x^2 - x - 6)} \, dx = \ln \frac{m}{n},$$

determine values for the integers m and n .

Answers:

$$\frac{1}{x} + \frac{2}{x+2} - \frac{1}{x-3}$$

$$m = 8, \quad n = 9$$