Differentiation

AH Maths Exam Questions

Source: 2019 Specimen P1 Q2 AH Maths	
(1)	Given $f(x) = 2x\tan x$, where $0 < x < \frac{\pi}{2}$, obtain $f'\left(\frac{\pi}{4}\right)$.
Answer: $f'\left(\frac{\pi}{4}\right) = 2 + \pi$	

Source: 2019 Q1a,b AH Maths	
(2)	(a) Differentiate $f(x) = x^6 \cot 5x$. (b) Given $y = \frac{2x^3 + 1}{x^3 - 4}$, find $\frac{dy}{dx}$. Simplify your answer. (c) For $f(x) = \cos^{-1} 2x$ evaluate $f'\left(\frac{\sqrt{3}}{4}\right)$.
Answers: $(a)\ 6x^5 cot 5x - 5x^6 cosec^2 5x$ $(b)\ \frac{-27x^2}{(x^3-4)^2}$ $(c)\ -4$	

Source: 2019 Q6 AH Maths

(3) A spherical balloon of radius r cm, $r > 0$, deflates at a constant rate of $60\,\text{cm}^3\,\text{s}^{-1}$.

Calculate the rate of change of the radius with respect to time when $r = 3$.

[The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.]

Answer: $Rate\ of\ change = -\frac{5}{3\pi}\ cms^{-1}$

Source: 2018 Q1b AH Maths

(4) (a) Given $f(x) = \sin^{-1} 3x$, find $f'(x)$.

(b) Differentiate $y = \dfrac{e^{5x}}{7x+1}$.

(c) For $y\cos x + y^2 = 6x$, use implicit differentiation to find $\dfrac{dy}{dx}$.

Answers: $(a)\ \dfrac{3}{\sqrt{1-9x^2}}$ $(b)\ \dfrac{5(7x+1)e^{5x} - 7e^{5x}}{(7x+1)^2}$ $(c)\ \dfrac{6 + ysinx}{cosx + 2y}$

Source: 2017 Q3 AH Maths

(5) On a suitable domain, a function is defined by $f(x) = \dfrac{e^{x^2-1}}{x^2-1}$.

Find $f'(x)$, simplifying your answer.

Answer: $f'(x) = \dfrac{2xe^{x^2-1}(x^2-2)}{(x^2-1)^2}$

Source: 2016 Q1a,b AH Maths

(6)

(a) Differentiate $y = x\tan^{-1}2x$.

(b) Given $f(x) = \dfrac{1-x^2}{1+4x^2}$, find $f'(x)$, simplifying your answer.

(c) A curve is given by the parametric equations

$$x = 6t \text{ and } y = 1-\cos t.$$

Find $\dfrac{dy}{dx}$ in terms of t.

Answers: $(a)\ tan^{-1}2x + \frac{2x}{1+4x^2}$ $(b)\ \frac{-10x}{(1+4x^2)^2}$ $(c)\ \frac{1}{6}sint$

Source: 2015 Q2 AH Maths

(7)

(a) For $y = \dfrac{5x+1}{x^2+2}$, find $\dfrac{dy}{dx}$. Express your answer as a single, simplified fraction.

(b) Given $f(x) = e^{2x}\sin^2 3x$, obtain $f'(x)$.

Answers:

$(a)\ \frac{dy}{dx} = \frac{-5x^2-2x+10}{(x^2+2)^2}$ $(b)\ f'(x) = e^{2x}(2sin^2 3x + 3sin6x)$

Source: 2014 Q1 AH Maths

(8) (a) Given

$$f(x) = \frac{x^2 - 1}{x^2 + 1},$$

obtain $f'(x)$ and simplify your answer.

(b) Differentiate $y = \tan^{-1}(3x^2)$.

Answers: $(a)\ f'(x) = \frac{4x}{(x^2+1)^2}$ $(b)\ f'(x) = \frac{6x}{1+9x^4}$

Source: 2014 Q13 AH Maths

(9) The fuel efficiency, F, in km per litre, of a vehicle varies with its speed, s km per hour, and for a particular vehicle the relationship is thought to be

$$F = 15 + e^x(\sin x - \cos x - \sqrt{2}), \quad \text{where } x = \frac{\pi(s-40)}{80},$$

for speeds in the range $40 \leq s \leq 120$ km per hour.

What is the greatest and least efficiency over the range and at what speeds do they occur?

Answers:

Greatest Efficiency is 15 *km/litre at* 100 *km/h*

Least Efficiency is 5.4 *km/litre at* 120 *km/h*

Source: 2013 Q2 AH Maths

(10) Differentiate $f(x) = e^{\cos x}\sin^2 x$.

Answer: $e^{cosx}sinx(2cosx - sin^2x)$

Source: 2012 Q1 AH Maths

(11) (a) Given $f(x) = \frac{3x+1}{x^2+1}$, obtain $f'(x)$.

(b) Let $g(x) = \cos^2 x \exp(\tan x)$. Obtain an expression for $g'(x)$ and simplify your answer.

Answers:

(a) $f'(x) = \frac{-3x^2-2x+3}{(x^2+1)^2}$

(b) $g'(x) = (1 - sin2x)tanx$

Source: 2011 Q7 AH Maths

(12) A curve is defined by the equation $y = \frac{e^{\sin x}(2+x)^3}{\sqrt{1-x}}$ for $x < 1$.

Calculate the gradient of the curve when $x = 0$.

Answer: $Gradient = 24$

Source: 2010 Q1 AH Maths

(13) Differentiate the following functions.

(a) $f(x) = e^x \sin x^2$.

(b) $g(x) = \frac{x^3}{(1+\tan x)}$.

Answers:

(a) $f'(x) = e^x sinx^2 + e^x(2xcosx^2)$

(b) $g'(x) = \frac{3x^2(1+tanx) - x^3sec^2x}{(1+tanx)^2}$

Source: 2009 Q1a AH Maths

(14)

(*a*) Given $f(x) = (x + 1)(x - 2)^3$, obtain the values of x for which $f'(x) = 0$.

(*b*) Calculate the gradient of the curve defined by $\frac{x^2}{y} + x = y - 5$ at the point $(3, -1)$.

Answers:

(*a*) $y = \ln\left(2 - \frac{1}{x}\right)$

(*b*) $m = \frac{dy}{dx} = -\frac{1}{2}$

Source: 2008 Q10 AH Maths

(15)

A body moves along a straight line with velocity $v = t^3 - 12t^2 + 32t$ at time t.

(*a*) Obtain the value of its acceleration when $t = 0$.

(*b*) At time $t = 0$, the body is at the origin O. Obtain a formula for the displacement of the body at time t.

Show that the body returns to O, and obtain the time, T, when this happens.

Answers:

(*a*) *Acceleration* $= 32$ *when* $t = 0$

(*b*) *Displacement of body at time* t: $x(t) = \frac{t^4}{4} - 4^3 + 16t^2$

The body returns to O when $t = 8$

Source: 2008 Q15 AH Maths

(16) Let $f(x) = \frac{x}{\ln x}$ for $x > 1$.

(a) Derive expressions for $f'(x)$ and $f''(x)$, simplifying your answers.

(b) Obtain the coordinates and nature of the stationary point of the curve $y = f(x)$.

(c) Obtain the coordinates of the point of inflexion.

Answers:

$(a)\ f'(x) = \frac{lnx - 1}{(lnx)^2}, \quad f''(x) = \frac{2 - lnx}{x(lnx)^3}$

(b) *Minimum stationaty point at coordinates* (e, e)

$(c)\ Coordinates = \left(e^2, \frac{e^2}{2}\right)$

Source: 2007 Q2 AH Maths

(17) Obtain the derivative of each of the following functions:

(a) $f(x) = \exp(\sin 2x)$;

(b) $y = 4^{(x^2 + 1)}$.

Answers:

$(a)\ f'(x) = 2cos2x\ exp(sin2x)$

$(b)\ \frac{dy}{dx} = 2x\ ln4.4^{(x^2+1)}$