Systems of Equations

AH Maths Exam Questions

Source: 2019 Specimen P1 Q3 AH Maths	
(1)	Use Gaussian elimination to solve the following system of equations. $x + y + 3z = 2$ $2x + y + z = 2$ $3x + 2y + 5z = 5$
Answers: $x = 2, \quad y = -3, \quad z = 1$	

Source: 2017 Q5 AH Maths	
(2)	(a) (i) Use Gaussian elimination on the system of equations below to give an expression for z in terms of λ. $x + 2y - z = -3$ $4x - 2y + 3z = 11$ $3x + y + 2\lambda z = 8$ (ii) For what value of λ is this system of equations inconsistent? (b) Determine the solution of this system when $\lambda = -2{\cdot}5$.
Answers:	(a) $(i) \quad z = \frac{11}{4\lambda - 1}$ $\qquad (ii) \quad \lambda = \frac{1}{4}$ (b) $z = -1, \quad y = -3, \quad x = 2$

Source: 2016 Q4 AH Maths

(3) Below is a system of equations:

$$x+2y+3z=3$$
$$2x-y+4z=5$$
$$x-3y+2\lambda z=2$$

Use Gaussian elimination to find the value of λ which leads to redundancy.

Answer: $\lambda = \frac{1}{2}$

Source: 2014 Q3 AH Maths

(4) Use Gaussian elimination on the system of equations below to give an expression for z in terms of λ.

$$\begin{aligned} x+y+z &= 2 \\ 4x+3y-\lambda z &= 4 \\ 5x+6y+8z &= 11 \end{aligned}$$

For what values of λ does this system have a solution?

Determine the solution to this system of equations when $\lambda = 2$.

Answers: $\lambda \neq -1, \quad z=1, \quad y=-2, \quad x=3$

Source: 2012 Q14 AH Maths

(5)

(*a*) Use Gaussian elimination to obtain the solution of the following system of equations in terms of the parameter λ.

$$4x + 6z = 1$$
$$2x - 2y + 4z = -1$$
$$-x + y + \lambda z = 2$$

(*b*) Describe what happens when $\lambda = -2$.

(*c*) When $\lambda = -1{\cdot}9$ the solution is $x = -22{\cdot}25$, $y = 8{\cdot}25$, $z = 15$.

Find the solution when $\lambda = -2{\cdot}1$.

Comment on these solutions.

Answers:

(a) $x = \dfrac{\lambda - 7}{4(2+\lambda)}$

(b) When $\lambda = -2$, the final row gives $0 = 6$ which is inconsistent.
There are no solutions.

(c) $\lambda = -2.1$; $x = 22.75$; $y = -6.75$; $z = -15$
Although the values of λ are close, the values of x, y and z are quite different. The system is **ill-conditioned** near $\lambda = -2$.

Source: 2009 Q16a AH Maths

(6)

(*a*) Use Gaussian elimination to solve the following system of equations

$$x + y - z = 6$$
$$2x - 3y + 2z = 2$$
$$-5x + 2y - 4z = 1.$$

(*b*) Show that the line of intersection, L, of the planes $x + y - z = 6$ and $2x - 3y + 2z = 2$ has parametric equations

$$x = \lambda$$
$$y = 4\lambda - 14$$
$$z = 5\lambda - 20.$$

(*c*) Find the acute angle between line L and the plane $-5x + 2y - 4z = 1$.

Answers: $(a)\ x = 3,\ y = -2,\ z = -5$ $(b)\ Proof$ $(c)\ Acute\ angle = 23.0°$

Source: 2006 Q1 AH Maths

(7) Use Gaussian elimination to obtain solutions of the equations

$$2x - y + 2z = 1$$
$$x + y - 2z = 2$$
$$x - 2y + 4z = -1$$

Answers: $z = t, \quad y = 1 + 2t, \quad x = 1$

Source: 2005 Q6 AH Maths

(8) Use Gaussian Elimination to solve the system of equations below when $\lambda \neq 2$.

$$x + y + 2z = 1$$
$$2x + \lambda y + z = 0$$
$$3x + 3y + 9z = 5$$

Explain what happens when $\lambda = 2$.

Answers:

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 2 & \lambda & 1 & 0 \\ 3 & 3 & 9 & 5 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & \lambda - 2 & -3 & -2 \\ 0 & 0 & 3 & 2 \end{array}\right) \qquad \textbf{2E1}$$

$z = \frac{2}{3}$; **1**

$(\lambda - 2)y - 2 = -2 \Rightarrow y = 0; x = 1 - 0 - \frac{4}{3} = -\frac{1}{3}$. **1**

When $\lambda = 2$, the second and third rows of the second matrix are the same, **1(†)**
so there is an infinite number of solutions. **1**

(†) Use of 'redundant' is worth a mark.
Interpretation in geometrical terms can be given both the marks.

Source: 2003 Q6 AH Maths

(9) Use elementary row operations to reduce the following system of equations to upper triangular form.

$$x+y+3z=1$$
$$3x+ay+z=1$$
$$x+y+z=-1$$

Hence express x, y and z in terms of the parameter a.

Explain what happens when $a=3$.

Answers:

$$\begin{aligned} x + y + 3z &= 1 \\ 3x + ay + z &= 1 \\ x + y + z &= -1. \end{aligned}$$

Hence

$$\begin{aligned} x + y + 3z &= 1 \\ (a - 3)y - 8z &= -2 \\ -2z &= -2 \end{aligned}$$

When $a \neq 3$, we can solve to give a unique solution.

$$z = 1; \qquad y = \frac{6}{a-3}; \qquad x = -2 + \frac{6}{3-a}.$$

When $a = 3$, we get $z = \frac{1}{4}$ from the second equation but $z = 1$‡ from the third, i.e. inconsistent§ .

Source: 2002 Q1 AH Maths

(10) Use Gaussian Elimination to solve the following system of equations.

$$x+y+3z=2$$
$$2x+y+z=2$$
$$3x+2y+5z=5$$

Answers:

$$z=1, \qquad y=-3, \qquad x=2$$