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Further Sequences & Series (Maclaurin Expansion)

AH Maths Exam Questions

Source: 2018 Q17 AH Maths

(1)

(a) Given $f(x) = e^{2x}$, obtain the Maclaurin expansion for $f(x)$ up to, and including, the term in x^3.

(b) On a suitable domain, let $g(x) = \tan x$.

(i) Show that the third derivative of $g(x)$ is given by $g'''(x) = 2\sec^4 x + 4\tan^2 x \sec^2 x$.

(ii) Hence obtain the Maclaurin expansion for $g(x)$ up to and including the term in x^3.

(c) Hence, or otherwise, obtain the Maclaurin expansion for $e^{2x}\tan x$ up to, and including, the term in x^3.

(d) Write down the first three non-zero terms in the Maclaurin expansion for $2e^{2x}\tan x + e^{2x}\sec^2 x$.

Answers:

$(a)\ f(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 \ldots$

$(b)\ (i)\ \ g'''(x) = 2sec^2x(sec^2x) + (4sec^2xtanx)tanx$

$(ii)\ g(x) = x + \frac{1}{3}x^3 \ldots$

$(c)\ \ x + 2x^2 + \frac{7}{3}x^3 \ldots$

$(d)\ 1 + 4x + 7x^2$

Source: 2016 Q6 AH Maths	
(2)	Find Maclaurin expansions for $\sin 3x$ and e^{4x} up to and including the term in x^3. Hence obtain an expansion for $e^{4x}\sin 3x$ up to and including the term in x^3.
Answers:	$f(x) = 3x - \frac{9}{2}x^3$, $f(x) = 1 + 4x + 8x^2 + \frac{32}{3}x^3$ $3x + 12x^2 + \frac{39}{2}x^3 + \cdots$

Source: 2014 Q9 AH Maths	
(3)	Give the first three non-zero terms of the Maclaurin series for $\cos 3x$. Write down the first four terms of the Maclaurin series for e^{2x}. Hence, or otherwise, determine the Maclaurin series for $e^{2x}\cos 3x$ up to, and including, the term in x^3.
Answers:	$cos3x = 1 - \frac{9x^2}{2} + \frac{27x^4}{8} \ldots$ $e^{2x}cos3x = 1 + 2x - \frac{5x^2}{2} - \frac{23x^3}{3}$

Source: 2012 Q6 AH Maths	
(4)	Write down the Maclaurin expansion of e^x as far as the term in x^3. Hence, or otherwise, obtain the Maclaurin expansion of $(1 + e^x)^2$ as far as the term in x^3.
Answers:	$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$ $(1 + e^x)^2 = 4 + 4x + 3x^2 + \frac{5}{3}x^3 + \ldots$

Source: 2010 Q9 AH Maths	
(5)	Obtain the first three non-zero terms in the Maclaurin expansion of $(1 + \sin^2 x)$.
Answer: $f(x) = 1 + x^2 - \frac{x^4}{3} + \cdots$	

Source: 2009 Q14 AH Maths	
(6)	Express $\frac{x^2 + 6x - 4}{(x+2)^2(x-4)}$ in partial fractions. Hence, or otherwise, obtain the first three non-zero terms in the Maclaurin expansion of $\frac{x^2 + 6x - 4}{(x+2)^2(x-4)}$.
Answers: $\frac{2}{(x+2)^2} + \frac{1}{x-4}$ $\frac{1}{4} - \frac{9x}{16} + \frac{23x^2}{64} + \cdots$	

Source: 2011 Q5 AH Maths

(7)	Obtain the first four terms in the Maclaurin series of $\sqrt{1+x}$, and hence write down the first four terms in the Maclaurin series of $\sqrt{1+x^2}$. Hence obtain the first four terms in the Maclaurin series of $\sqrt{(1+x)(1+x^2)}$.

Answers:

Let $f(x) = (1+x)^{\frac{1}{2}}$, then

$$f(x) = (1+x)^{\frac{1}{2}} \Rightarrow f(0) = 1$$

$$f'(x) = \tfrac{1}{2}(1+x)^{-\frac{1}{2}} \Rightarrow f'(0) = \tfrac{1}{2}$$

$$f''(x) = -\tfrac{1}{4}(1+x)^{-\frac{3}{2}} \Rightarrow f''(0) = -\tfrac{1}{4}$$

$$f'''(x) = \tfrac{3}{8}(1+x)^{-\frac{5}{2}} \Rightarrow f'''(0) = \tfrac{3}{8}$$

1 for derivatives

1 for values

Hence

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{4}\times\frac{x^2}{2} + \frac{3}{8}\times\frac{x^3}{6} - \ldots$$ **1**

$$= 1 + \frac{1}{2}x - \frac{x^2}{8} + \frac{x^3}{16} - \ldots$$

and replacing x by x^2 gives

$$\sqrt{1+x^2} = 1 + \frac{1}{2}x^2 - \frac{x^4}{8} + \frac{x^6}{16} - \ldots$$ **1**

Thus

$$\sqrt{(1+x)(1+x^2)} =$$

$$\left(1 + \frac{1}{2}x - \frac{x^2}{8} + \frac{x^3}{16} - \ldots\right)\left(1 + \frac{1}{2}x^2 - \frac{x^4}{8} + \frac{x^6}{16} - \ldots\right)$$ **1M** for multiplying

$$= 1 + \frac{1}{2}x + \frac{1}{2}x^2 - \frac{1}{8}x^2 + \frac{1}{4}x^3 + \frac{1}{16}x^3 + \ldots$$

$$= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \ldots$$ **1**

Source: 2008 Q12 AH Maths	
(8)	Obtain the first three non-zero terms in the Maclaurin expansion of $x\ln(2+x)$. Hence, or otherwise, deduce the first three non-zero terms in the Maclaurin expansion of $x\ln(2-x)$. Hence obtain the first **two** non-zero terms in the Maclaurin expansion of $x\ln(4-x^2)$. [*Throughout this question, it can be assumed that* $-2 < x < 2$.]

Answers:

$$First\ three\ non-zero\ terms:\ f(x) = \ln(2)\,x + \frac{x^2}{2} - \frac{x^3}{8} + \cdots$$

$$x\ln(2-x) = (\ln 2)\,x - \frac{x^2}{2} - \frac{x^3}{3} + \cdots$$

$$x\ln(4-x^2) = (2\ln 2)x - \frac{x^3}{4} + \cdots$$

Source: 2007 Q6 AH Maths	
(9)	Find the Maclaurin series for $\cos x$ as far as the term in x^4. Deduce the Maclaurin series for $f(x) = \frac{1}{2}\cos 2x$ as far as the term in x^4. Hence write down the first three non-zero terms of the series for $f(3x)$.

Answers:

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots$$

$$f(x) = \frac{1}{2} - x^2 + \frac{x^4}{3} - \cdots$$

$$f(3x) = \frac{1}{2} - 9x^2 + 24x^4 - \cdots$$