advancedhighermaths.co.uk

Binomial Theorem

AH Maths Exam Questions

Source: 2019 Specimen P2 Q3 AH Maths (Same as 2016 Q3)	
(1)	Write down and simplify the general term in the binomial expansion of $\left(\frac{3}{x}-2x\right)^{13}$. Hence, or otherwise, find the term in x^9.
Answers: $^{13}C_r\,(3)^{13-r}\,(-2)^r\,x^{2r-13}$ $-1437696x^9\ (when\ r=11)$	

Source: 2019 Q9 AH Maths	
(2)	(a) Write down and simplify the general term in the binomial expansion of $\left(2x^2-\frac{d}{x^3}\right)^7$, where d is a constant. (b) Given that the coefficient of $\frac{1}{x}$ is $-70\,000$, find the value of d.
Answers: $(a)\ \binom{7}{2}.2^{7-r}.(-d)^r.x^{14-5r}$ $\quad(b)\ \ r=3,\ d=5$	

Source: 2018 Q3 AH Maths	
(3)	(a) Write down and simplify the general term in the binomial expansion of $\left(2x+\frac{5}{x^2}\right)^9$. (b) Hence, or otherwise, find the term independent of x.
Answers:	
$(a)\ \binom{9}{r}.2^{9-r}.5^r.x^{9-3r}$	$(b)\ 672000\ (when\ r=3)$

Source: 2017 Q1 AH Maths	
(4)	Write down the binomial expansion of $\left(\frac{2}{y^2}-5y\right)^3$ and simplify your answer.
Answer:	$\frac{8}{y^6}-\frac{60}{y^3}+150-123y^3$

Source: 2015 Q1 AH Maths	
(5)	Use the binomial theorem to expand and simplify $$\left(\frac{x^2}{3}-\frac{2}{x}\right)^5.$$
Answer:	$\frac{x^{10}}{243}-\frac{10x^7}{81}+\frac{40x^4}{27}-\frac{80x}{9}+\frac{80}{3x^2}-\frac{32}{x^5}$

Source: 2015 Q9 AH Maths	
(6)	Show that $$\binom{n+2}{3}-\binom{n}{3}=n^2,$$ for all integers, n, where $n \geq 3$.

Answer:

$\binom{n+2}{3}-\binom{n}{3}$	4	
$=\frac{(n+2)!}{(n+2-3)!\,3!}-\frac{n!}{(n-3)!\,3!}$		•[1] demonstrates understanding of factorial form *algebraically*[1].
$=\frac{(n+2)(n+1)n(n-1)!}{(n-1)!3!}-\frac{n(n-1)(n-2)(n-3)!}{(n-3)!\,3!}$		•[2] using property $n!=n(n-1)!$ [1]
$=\frac{(n+2)(n+1)n}{3!}-\frac{n(n-1)(n-2)}{3!}$		•[3] correctly expressing with common denominator **OR** as a single fraction[1,5].
$=\frac{n}{3!}\left[(n+2)(n+1)-(n-1)(n-2)\right]$*		
$=\frac{n}{6}\left(n^2+3n+2-n^2+3n-2\right)$		•[4] simplification to answer, line * (or equivalent) essential.
$=\frac{n}{6}(6n)$		
$=n^2$ as required		

Source: 2014 Q2 AH Maths	
(7)	Write down and simplify the general term in the expression $\left(\frac{2}{x}+\frac{1}{4x^2}\right)^{10}$. Hence, or otherwise, obtain the term in $\frac{1}{x^{13}}$.

Answers: $\binom{10}{r}2^{3r-20}x^{r-20}$ **OR** $\binom{10}{r}2^{10-3r}x^{-r-10}$ $\quad \frac{240}{x^{13}}$

Source: 2013 Q1 AH Maths	
(8)	Write down the binomial expansion of $\left(3x-\frac{2}{x^2}\right)^4$ and simplify your answer.
Answer: $81x^4 - 216x + \frac{216}{x^2} - \frac{96}{x^5} + \frac{16}{x^8}$	

Source: 2012 Q4 AH Maths	
(9)	Write down and simplify the general term in the expansion of $\left(2x-\frac{1}{x^2}\right)^9$. Hence, or otherwise, obtain the term independent of x.
Answer: -5376	

Source: 2011 Q2 AH Maths	
(10)	Use the binomial theorem to expand $\left(\frac{1}{2}x-3\right)^4$ and simplify your answer.
Answer: $\frac{x^4}{16} - \frac{3x^3}{2} + \frac{27x^2}{2} - 54x + 81.$	

Source: 2010 Q5 AH Maths

(11) Show that

$$\binom{n+1}{3} - \binom{n}{3} = \binom{n}{2}$$

where the integer n is greater than or equal to 3.

Answer:

$$\binom{n+1}{3} - \binom{n}{3} = \frac{(n+1)!}{3!(n-2)!} - \frac{n!}{3!(n-3)!}$$ **1** both terms correct

$$= \frac{(n+1)!}{3!(n-2)!} - \frac{n!(n-2)}{3!(n-2)!}$$ {alternative methods will appear}

$$= \frac{(n+1)! - n!(n-2)}{3!(n-2)!}$$

$$= \frac{n![(n+1)-(n-2)]}{3!(n-2)!}$$ **1** correct numerator **1** correct denominator

$$= \frac{n! \times 3}{3!(n-2)!} = \frac{n!}{2!(n-2)!}$$ **1**

$$= \binom{n}{2}$$

1 for knowing (anywhere) $(n-2)! = (n-2) \times (n-3)!$

Source: 2009 Q8 AH Maths

(12) (*a*) Write down the binomial expansion of $(1 + x)^5$.

(*b*) Hence show that $0{\cdot}9^5$ is $0{\cdot}59049$.

Answers

(a) $(1 + x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$

(b) Let $x = -0{\cdot}1$, then

$$0{\cdot}9^5 = (1 + (-0{\cdot}1))^5$$
$$= 1 - 0{\cdot}5 + 0{\cdot}1 - 0{\cdot}01 + 0{\cdot}0005 - 0{\cdot}00001$$
$$= 0{\cdot}5 + 0{\cdot}09 + 0{\cdot}00049$$
$$= 0{\cdot}59049$$

Source: 2008 Q8 AH Maths	
(13)	Write down and simplify the general term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{10}$. Hence, or otherwise, obtain the term in x^{14}.

Answers

$$\binom{10}{r}\left(x^2\right)^{10-r}\left(\frac{1}{x}\right)^r$$

$$= \binom{10}{r}x^{20-3r}$$

$$20 - 3r = 14 \Rightarrow r = 2$$

term is $45x^{14}$

$$\binom{10}{r}\left(x^2\right)^r\left(\frac{1}{x}\right)^{10-r} = \binom{10}{r}x^{3r-10}$$

$$3r - 10 = 14 \Rightarrow r = 8$$

term is $45x^{14}$

Source: 2007 Q1 AH Maths	
(14)	Express the binomial expansion of $\left(x - \frac{2}{x}\right)^4$ in the form $ax^4 + bx^2 + c + \frac{d}{x^2} + \frac{e}{x^4}$ for integers a, b, c, d and e.

Answers

$$\left(x - \frac{2}{x}\right)^4 = x^4 + 4x^3\left(-\frac{2}{x}\right) + 6x^2\left(-\frac{2}{x}\right)^2 + 4x\left(-\frac{2}{x}\right)^3 + \left(-\frac{2}{x}\right)^4$$

$$= x^4 - 8x^2 + 24 - \frac{32}{x^2} + \frac{16}{x^4}$$