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Further Differentiation

AH Maths Exam Questions

Source: 2019 Specimen P2 Q2 AH Maths	
(1)	(a) Given $f(x) = \sin^{-1} 3x$, find $f'(x)$. (b) For $y\cos x + y^2 = 6x$, use implicit differentiation to find $\frac{dy}{dx}$.
Answers:	$(a)\ f'(x) = \frac{3}{\sqrt{1-9x^2}}$ $\qquad (b)\ \frac{dy}{dx} = \frac{6 + y\sin x}{\cos x + 2y}$

Source: 2019 Specimen P2 Q4 AH Maths – Same as 2018 Q6	
(2)	On a suitable domain, a curve is defined parametrically by $x = t^2 + 1$ and $y = \ln(3t+2)$. Find the equation of the tangent to the curve where $t = -\frac{1}{3}$.
Answers:	$y = -\frac{9}{2}x + 5$

Source: 2019 Specimen P2 Q10 AH Maths – Same as 2017 Q11	
(3)	Given $y = x^{2x^3+1}$ where $x > 0$, find $\frac{dy}{dx}$. Write your answer in terms of x.
Answer:	$\frac{dy}{dx} = x^{2x^3+1}\left(6x^2 \ln x + \frac{2x^3+1}{x}\right)$

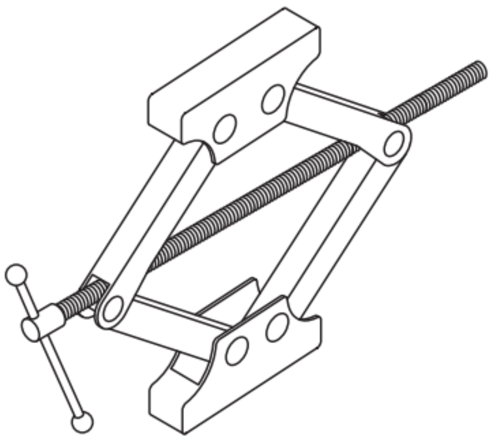
Source: 2019 Q5 AH Maths	
(4)	For $x = \ln(2t+7)$ and $y = t^2$, $t > 0$, find (a) $\frac{dy}{dx}$ (b) $\frac{d^2y}{dx^2}$.
Answers: $(a)\ 2t^2 + 7t \quad (b)\ \frac{1}{2}(2t+7)(4t+7)$	

Source: 2019 Q10 AH Maths	
(5)	A curve is defined implicitly by the equation $x^2 + y^2 = xy + 12$. (a) Find an expression for $\frac{dy}{dx}$ in terms of x and y. (b) There are two points where the tangent to the curve has equation $x = k$, $k \in \mathbb{R}$. Find the values of k.
Answers: $(a)\ \frac{dy}{dx} = \frac{y-2x}{2y-x} \quad (b)\ k = \pm 4$	

Source: 2016 Q11 AH Maths	
(6)	The height of a cube is increasing at the rate of $5\,\text{cm}\,\text{s}^{-1}$. Find the rate of increase of the volume when the height of the cube is 3 cm.
Answer: $\frac{dv}{dt} = 135cm^3s^{-1}$	

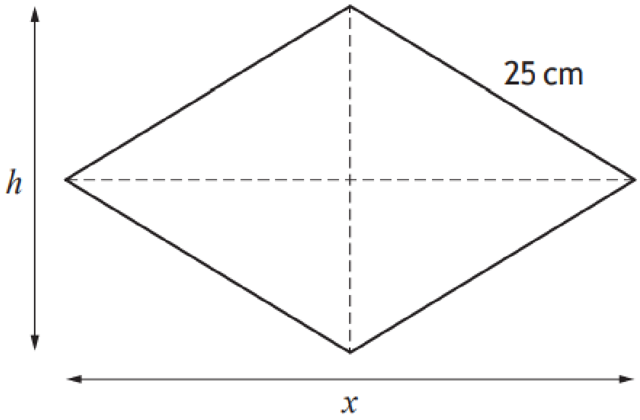
Source: 2018 Q13 AH Maths

(7) An engineer has designed a lifting device. The handle turns a screw which shortens the horizontal length and increases the vertical height.



The device is modelled by a rhombus, with each side 25 cm.

The horizontal length is x cm, and the vertical height is h cm as shown.



(a) Show that $h = \sqrt{2500 - x^2}$.

(b) The horizontal length decreases at a rate of 0·3 cm per second as the handle is turned.

Find the rate of change of the vertical height when $x = 30$.

Answers: $(a)\ Proof$ $\quad (b)\ \frac{dh}{dt} = \frac{9}{40} cms^{-1}$

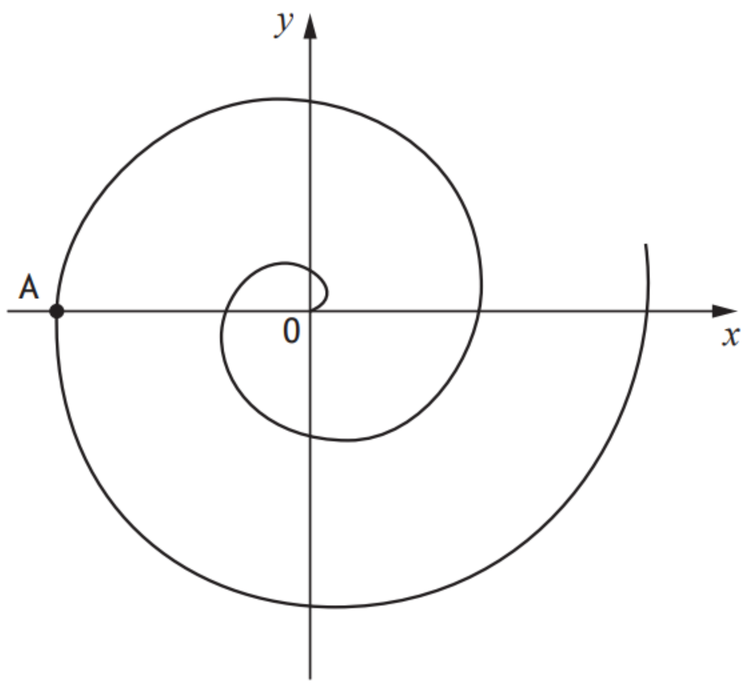
Source: 2017 Q18 AH Maths

(8) The position of a particle at time t is given by the parametric equations

$x = t\cos t, \quad y = t\sin t, \quad t \geq 0.$

(a) Find an expression for the instantaneous speed of the particle.

The diagram below shows the path that the particle takes.



(b) Calculate the instantaneous speed of the particle at point A.

Answers: $(a)\ \sqrt{(1+t^2}$ $\quad (b)\ Speed = \sqrt{(1+9\pi^2}$

Source: 2015 Q4 AH Maths

(9) The equation $x^4 + y^4 + 9x - 6y = 14$ defines a curve passing through the point A(1, 2).

Obtain the equation of the tangent to the curve at A.

Answer: $y = -\frac{1}{2}x + \frac{5}{2}$

Source: 2015 Q6 AH Maths	
(10)	For $y = 3^{x^2}$, obtain $\frac{dy}{dx}$.
Answer: $\frac{dy}{dx} = 2xln3.3^{x^2}$ or $2xln3.e^{x^2 ln3}$	

Source: 2015 Q8 AH Maths	
(11)	Given $x = \sqrt{t+1}$ and $y = \cot t,\ 0 < t < \pi$, obtain $\frac{dy}{dx}$ in terms of t.
Answer: $\frac{dy}{dx} = -2\sqrt{t+1}.cosec^2 t$	

Source: 2014 Q4 AH Maths	
(12)	Given $x = \ln(1 + t^2)$, $y = \ln(1 + 2t^2)$ use parametric differentiation to find $\frac{dy}{dx}$ in terms of t.
Answer: $\frac{dy}{dx} = \frac{2(1+t^2)}{1+2t^2}$	

Source: 2014 Q6 AH Maths	
(13)	Given $e^y = x^3\cos^2 x$, $x > 0$, show that $$\frac{dy}{dx} = \frac{a}{x} + b\tan x,$$ for some constants a and b. State the values of a and b.
Answers: $a = 3,\ b = -2$	

Source: 2013 Q11 AH Maths	
(14)	A curve has equation $$x^2 + 4xy + y^2 + 11 = 0.$$ Find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $(-2, 3)$.
Answer: $\frac{dy}{dx} = 4,\quad \frac{d^2y}{dx^2} = 33$	

Source: 2012 Q12 AH Maths	
(15)	The radius of a cylindrical column of liquid is decreasing at the rate of $0{\cdot}02\ \mathrm{m\,s^{-1}}$, while the height is increasing at the rate of $0{\cdot}01\ \mathrm{m\,s^{-1}}$. Find the rate of change of the volume when the radius is 0·6 metres and the height is 2 metres. [*Recall that the volume of a cylinder is given by* $V = \pi r^2 h$.]
Answer: $Rate\ of\ change\ of\ the\ volume = -0.0444\pi\ m^3 s^{-1}$	

Source: 2010 Q13 AH Maths	
(16)	Given $y = t^3 - \frac{5}{2}t^2$ and $x = \sqrt{t}$ for $t > 0$, use parametric differentiation to express $\frac{dy}{dx}$ in terms of t in simplified form. Show that $\frac{d^2y}{dx^2} = at^2 + bt$, determining the values of the constants a and b. Obtain an equation for the tangent to the curve which passes through the point of inflexion.
Answers: $\frac{dy}{dt} = 6t^{\frac{5}{2}} - 10t^{\frac{3}{2}}$ $a = 30, \qquad b = -30$ $2y + 8x = 5$	

Source: 2009 Q11 AH Maths	
(17)	The curve $y = x^{2x^2+1}$ is defined for $x > 0$. Obtain the values of y and $\frac{dy}{dx}$ at the point where $x = 1$.
Answer: $\frac{dy}{dx} = 3$	

Source: 2008 Q2 AH Maths	
(18)	(*a*) Differentiate $f(x) = \cos^{-1}(3x)$ where $-\frac{1}{3} < x < \frac{1}{3}$. (*b*) Given $x = 2\sec\theta$, $y = 3\sin\theta$, use parametric differentiation to find $\frac{dy}{dx}$ in terms of θ.
Answers: (a) $f'(x) = \frac{-3}{\sqrt{1-9x^2}}$ (b) $\frac{dy}{dx} = \frac{3cos^3\emptyset}{2sin\emptyset}$	

Source: 2008 Q5 AH Maths	
(19)	A curve is defined by the equation $xy^2 + 3x^2y = 4$ for $x > 0$ and $y > 0$. Use implicit differentiation to find $\frac{dy}{dx}$. Hence find an equation of the tangent to the curve where $x = 1$.
Answers: $\frac{dy}{dx} = \frac{-y^2 - 6xy}{2xy + 3x^2}$ $5y + 7x = 12$	

Source: 2007 Q13 AH Maths	
(20)	A curve is defined by the parametric equations $x = \cos 2t$, $y = \sin 2t$, $0 < t < \frac{\pi}{2}$. (*a*) Use parametric differentiation to find $\frac{dy}{dx}$. Hence find the equation of the tangent when $t = \frac{\pi}{8}$. (*b*) Obtain an expression for $\frac{d^2y}{dx^2}$ and hence show that $\sin 2t \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = k$, where k is an integer. State the value of k.

Answers:

(*a*) $\frac{dy}{dx} = \frac{2cos2t}{-2sin2t} = -cot2t$

Equation of tangent: $x + y = \sqrt{2}$

(*b*) $\frac{d^2y}{dx^2} = \frac{-1}{sin^3 2t}$, $k = -1$