advancedhighermaths.co.uk

Further Differential Equations

AH Maths Exam Questions

Source: 2019 Specimen P2 Q12 AH Maths

(1) Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 6e^{2x}$$

given $y = 4$ and $\frac{dy}{dx} = 7$ when $x = 0$.

Answer:

Generic scheme	Illustrative scheme	Max mark
•1 solve auxiliary equation	•1 $m = 2$ twice	10
•2 state complementary function	•2 $y = Ae^{2x} + Bxe^{2x}$	
•3 state form of particular integral	•3 $y = Cx^2e^{2x}$	
•4 find first derivative of particular integral	•4 $\frac{dy}{dx} = 2Cx^2e^{2x} + 2Cxe^{2x}$	
•5 find second derivative	•5 $\frac{d^2y}{dx^2} = 4Cx^2e^{2x} + 8Cxe^{2x} + 2Ce^{2x}$	
•6 determine coefficient of particular integral	•6 $C = 3$	
•7 state general solution	•7 $y = Ae^{2x} + Bxe^{2x} + 3x^2e^{2x}$	
•8 find derivative of general solution	•8 $\frac{dy}{dx} = 2Ae^{2x} + Be^{2x} + 2Bxe^{2x} + 6x^2e^{2x} + 6xe^{2x}$	
•9 find one constant	•9 $A = 4$ or $B = -1$	
•10 find second constant and state particular solution	•10 $y = 4e^{2x} - xe^{2x} + 3x^2e^{2x}$	

Source: 2019 Q8 AH Maths

(2) Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2}+11\frac{dy}{dx}+28y=0$$

given that $y=0$ and $\frac{dy}{dx}=9$, when $x=0$.

Answers:

Generic scheme	Illustrative scheme	Max mark
•[1] solve auxiliary equation	•[1] $m=-4,-7$	**5**
•[2] state general solution [1]	•[2] $y=Ae^{-4x}+Be^{-7x}$	
•[3] differentiate [2]	•[3] $\frac{dy}{dx}=-4Ae^{-4x}-7Be^{-7x}$ stated or implied at •[4]	
•[4] form equations and solve for a constant	•[4] $A=3$ or $B=-3$	
•[5] find second constant and state particular solution [3]	•[5] $y=3e^{-4x}-3e^{-7x}$	

Source: 2018 Q15b AH Maths

(3)

(a) Use integration by parts to find $\int x \sin 3x \, dx$.

(b) Hence find the particular solution of

$$\frac{dy}{dx} - \frac{2}{x} y = x^3 \sin 3x, \quad x \neq 0$$

given that $x = \pi$ when $y = 0$.

Express your answer in the form $y = f(x)$.

Answers:

(a)		•1 start integration by parts 1,2,3,4,6	•1 $-\frac{x}{3}\cos 3x - \ldots$	3
		•2 complete integration by parts 1,2,3,4,6	•2 $\ldots \int -\frac{1}{3}\cos 3x \, dx$	
		•3 complete integration 1,2,3,4,5,6	•3 $= -\frac{x}{3}\cos 3x + \frac{1}{9}\sin 3x + c$	

(b)		•4 identify integral form of integrating factor 1,2	•4 $e^{\int -\frac{2}{x} dx}$	7
		•5 determine integrating factor 3,6	•5 $\frac{1}{x^2}$	
		•6 begin solution	•6 $\frac{d}{dx}\left(\frac{1}{x^2} y\right) = \frac{1}{x^2}\left(x^3 \sin 3x\right)$ stated or implied at •7	
		•7 rewrite as integral equation	•7 $\frac{1}{x^2} y = \int x \sin 3x \, dx$	
		•8 integrate 4,5,6	•8 $\frac{1}{x^2} y = -\frac{x}{3}\cos 3x + \frac{1}{9}\sin 3x + c$	
		•9 evaluate constant 4,6,7,8	•9 $c = -\frac{\pi}{3}$	
		•10 form particular solution 4,6,7,8	•10 $y = -\frac{x^3}{3}\cos 3x + \frac{x^2}{9}\sin 3x - \frac{\pi x^2}{3}$	

Source: 2017 Q14 AH Maths

(4) Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=8\sin x+19\cos x$$

given that $y=7$ and $\frac{dy}{dx}=\frac{1}{2}$ when $x=0$.

Answer:

•[1] construct auxiliary equation [1,9]	•[1] $m^2-6m+9=0$	10
•[2] solve auxiliary equation and state CF [2,3,4,5,6,7,9]	•[2] $y=Ae^{3x}+Bxe^{3x}$	
•[3] state PI	•[3] $y=C\sin x+D\cos x$	
•[4] obtain first and second derivatives of PI	$\frac{dy}{dx}=C\cos x-D\sin x$ •[4] $\frac{d^2y}{dx^2}=-C\sin x-D\cos x$	
•[5] substitute	•[5] $-C\sin x-D\cos x$ $-6(C\cos x-D\sin x)$ $+9(C\sin x+D\cos x)=8\sin x+19\cos x$	
•[6] derive equations	•[6] $8C+6D=8$ $-6C+8D=19$	
•[7] obtain both constants of PI	•[7] $C=-\frac{1}{2},\ D=2$	
•[8] differentiate general solution [5,6,7,9,10]	•[8] $\frac{dy}{dx}=3Ae^{3x}+Be^{3x}+3Bxe^{3x}-\frac{1}{2}\cos x-2\sin x$	
•[9] determine first constant of general solution [7,8,9]	•[9] $A=5$ or $B=-14$	
•[10] determine second constant and state particular solution [3,7,9,10]	•[10] $y=5e^{3x}-14xe^{3x}-\frac{1}{2}\sin x+2\cos x$	

Source: 2016 Q15 AH Maths

(5) Solve the differential equation

$$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=12x^2+2x-5$$

given $y=-6$ and $\frac{dy}{dx}=3$, when $x=0$.

Answer:

•[1] state auxiliary equation [1]	•[1] $m^2+5m+6=0$ $m=-3,\ m=-2$	**10**
•[2] solve auxiliary equation and state complementary function [2,3]	•[2] $y=Ae^{-3x}+Be^{-2x}$	
•[3] construct particular integral	•[3] $y=Cx^2+Dx+E$	
•[4] differentiate particular integral	•[4] $\frac{dy}{dx}=2Cx+D$ and $\frac{d^2y}{dx^2}=2C$	
•[5] calculate one coefficient of the particular integral	•[5] $C=2$	
•[6] calculate remaining coefficients	•[6] $D=-3, E=1$ $y=Ae^{-3x}+Be^{-2x}+2x^2-3x+1$	
•[7] differentiate general solution [3]	•[7] $\frac{dy}{dx}=-3Ae^{-3x}-2Be^{-2x}+4x-3$	
•[8] construct equations using given conditions	•[8] $A+B=-7$ and $3A+2B=-6$ or equivalent	
•[9] Find one coefficient	•[9] $A=8$ or $B=-15$	
•[10] Find other coefficient and state particular solution	•[10] $y=8e^{-3x}-15e^{-2x}+2x^2-3x+1$	

Source: 2015 Q16 AH Maths

(6) Solve the second order differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 3e^{2x}$$

given that when $x = 0$, $y = 1$ and $\frac{dy}{dx} = 0$.

Answer:

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 3e^{2x}$ $m^2 + 2m + 10 = 0$	**10**	•[1] correct auxillary equation.
$m = \frac{-2 \pm \sqrt{4 - 4.1.10}}{2} = -1 \pm 3i$		•[2] solves correctly[1].
$y = e^{-x}(A\cos 3x + B\sin 3x)$ **OR** $y = ae^{(-1\pm 3i)x} + Be^{(-1-3i)x}$		•[3] appropriate complementary function.
try $y = Ce^{2x}$		•[4] particular integral
$\frac{dy}{dx} = 2Ce^{2x}$ $\frac{d^2y}{dx^2} = 4Ce^{2x}$		•[5] for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$
$4Ce^{2x} + 4Ce^{2x} + 10Ce^{2x} = 3e^{2x}$ $C = \frac{1}{6}$		•[6] for finding C
$y = Ae^{-x}\cos 3x + Be^{-x}\sin 3x + \frac{1}{6}e^{2x}$		•[7] combine CF and PI for general solution[3].
$1 = A + \frac{1}{6}$, $A = \frac{5}{6}$		•[8] value of A.
$\frac{dy}{dx} = -Ae^{-x}\cos 3x - 3Ae^{-x}\sin 3x - Be^{-x}\sin 3x + 3Be^{-x}\cos 3x + \frac{1}{3}e^{2x}$		•[9] for differentiating correctly[3].
$0 = -(A) + 3B + \frac{1}{3}$, $\frac{5}{6} - \frac{1}{3} = 3B$, $B = \frac{1}{6}$		
So particular solution is: $y = \frac{5}{6}e^{-x}\cos 3x + \frac{1}{6}e^{-x}\sin 3x + \frac{1}{6}e^{2x}$		•[10] value of B *and* statement of final answer[4].

Source: 2014 Q8 AH Maths

(7) Find the solution $y = f(x)$ to the differential equation

$$4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$$

given that $y = 4$ and $\frac{dy}{dx} = 3$ when $x = 0$.

Answer:

Expected Answer/s	Max Mark	Additional Guidance
$4m^2 - 4m + 1 = 0$ $(2m-1)^2 = 0$	6	•1 Correct auxiliary equation.[1]
$m = \frac{1}{2}$		•2 Correct solution of auxiliary equation.[4]
C.F./G.S. $y = Ae^{\frac{1}{2}x} + Bxe^{\frac{1}{2}x}$		•3 Statement of general solution/complementary function.[3,4,5,6]
$y = 4$ when $x = 0$ gives $4 = A.1+0$, so $A = 4$		•4 Correct evaluation of A.[2,4]
$\frac{dy}{dx} = \frac{1}{2}Ae^{\frac{1}{2}x} + Be^{\frac{1}{2}x} + \frac{1}{2}Bxe^{\frac{1}{2}x}$ $\frac{dy}{dx} = 3$ when $x = 0$ gives $3 = \frac{1}{2}Ae^0 + Be^0 + \frac{1}{2}B.0.e^0$ $3 = \frac{1}{2}.4 + B,$ so $B = 1$		•5 Correct differentiation of G.S.[4]
So P.S. is: $\boldsymbol{y = 4e^{\frac{1}{2}x} + xe^{\frac{1}{2}x}}$		•6 Substitution to obtain B *and* particular solution.[4]

Source: 2013 Q14 AH Maths	
(8)	Solve the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 4e^{3x}$, given that $y = 1$ and $\frac{dy}{dx} = -1$ when $x = 0$.

Answer:

$m^2 - 6m + 9 = 0$ $(m-3)^2 = 0$ $m = 3$		•[1] Correct auxiliary equation (or equivalent).[1]
C.F. $y = Ae^{3x} + Bxe^{3x}$		•[2] Correct solution of auxiliary equation ***and*** statement of complimentary function.
P.I. Try $y = Cx^2e^{3x}$		•[3] Correct form of particular integral.[1,7]
$\frac{dy}{dx} = 2Cxe^{3x} + 3Cx^2e^{3x}$		•[4] Correct first derivative of P.I.[2,3]
$\frac{d^2y}{dx^2} = 2Ce^{3x} + 6Cxe^{3x} + 6Cxe^{3x} + 9Cx^2e^{3x}$		•[5] Correct differentiation of first derivative.[4]
$2Ce^{3x} + 6Cxe^{3x} + 6Cxe^{3x} + 9Cx^2e^{3x}$ $-6(2Cxe^{3x} + 3Cx^2e^{3x}) + 9Cx^2e^{3x} = 4e^{3x}$		•[6] For correctly substituting expressions for both derivatives.
$2Ce^{3x} = 4e^{3x} \Rightarrow C = 2$		•[7] For correctly solving to obtain C.[5]
G.S. $y = Ae^{3x} + Bxe^{3x} + 2x^2e^{3x}$		•[8] Correct collation of above answers to obtain full General Solution.[6]
$\frac{dy}{dx} = 3Ae^{3x} + Be^{3x} + 3Bxe^{3x} + 4xe^{3x} + 6x^2e^{3x}$		•[9] Derivative of G.S.
When $x = 0, y = 1$ $A = 1$		•[10] Use of i.c.s to find first constant correctly.
$\frac{dy}{dx} = -1$ $-1 = 3 + B \Rightarrow B = -4$		•[11] Second constant.
P.S. $y = e^{3x} - 4xe^{3x} + 2x^2e^{3x}$		States solution.[6]

Source: 2012 Q15 AH Maths

(9)

(a) Express $\dfrac{1}{(x-1)(x+2)^2}$ in partial fractions.

(b) Obtain the general solution of the differential equation

$$(x-1)\frac{dy}{dx} - y = \frac{x-1}{(x+2)^2},$$

expressing your answer in the form $y = f(x)$.

Answers:

(a) $\dfrac{1}{(x-1)(x+2)^2} = \dfrac{A}{x-1} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$ **1M**

$1 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$

$x = 1 \Rightarrow A = \frac{1}{9}$ **1**

$x = -2 \Rightarrow C = -\frac{1}{3}$ **1**

$x = 0 \Rightarrow 1 = \frac{4}{9} - 2B + \frac{1}{3} \Rightarrow B = -\frac{1}{9}$ **1**

$\therefore \dfrac{1}{(x-1)(x+2)^2} = \dfrac{1}{9}\left(\dfrac{1}{x-1} - \dfrac{1}{x+2} - \dfrac{3}{(x+2)^2}\right)$

(b) $(x-1)\dfrac{dy}{dx} - y = \dfrac{x-1}{(x+2)^2}$

$\dfrac{dy}{dx} - \dfrac{1}{x-1}y = \dfrac{1}{(x+2)^2}$ **1M** for rearranging

Integrating factor: $\exp\left(\int -\frac{1}{x-1}dx\right)$ **1**

$= \exp(-\ln(x-1)) = (x-1)^{-1}$ **1**

$\dfrac{1}{(x-1)}\dfrac{dy}{dx} - \dfrac{1}{(x-1)^2}y = \dfrac{1}{(x-1)(x+2)^2}$

$\dfrac{d}{dx}\left(\dfrac{y}{x-1}\right) = \dfrac{1}{(x-1)(x+2)^2}$ **1**

$= \dfrac{1}{9}\left(\dfrac{1}{x-1} - \dfrac{1}{x+2} - \dfrac{3}{(x+2)^2}\right)$ **1**

$\dfrac{y}{x-1} = \dfrac{1}{9}\left(\ln|x-1| - \ln|x+2| + \dfrac{3}{x+2}\right) + c$ **1** constant of integration needed.

$y = \dfrac{x-1}{9}\left(\ln|x-1| - \ln|x+2| + \dfrac{3}{x+2}\right) + c(x-1)$ **1**

$= \dfrac{x-1}{9}\left(\ln\dfrac{|x-1|}{|x+2|} + \dfrac{3}{x+2}\right) + c(x-1)$

Source: 2011 Q14 AH Maths	
(10)	Find the general solution of the differential equation $$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x + 12.$$ Find the particular solution for which $y = -\frac{3}{2}$ and $\frac{dy}{dx} = \frac{1}{2}$ when $x = 0$.

Answer:

Auxiliary equation		
$m^2 - m - 2 = 0$	1	
$(m - 2)(m + 1) = 0$		
$m = -1$ or 2	1	
Complementary function is: $y = Ae^{-x} + Be^{2x}$	1	
The particular integral has the form $y = Ce^x + D$	1	
$y = Ce^x + D \Rightarrow \frac{dy}{dx} = Ce^x$		
$\Rightarrow \frac{d^2y}{dx^2} = Ce^x$	1	
Hence we need:		
$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x + 12$		
$[Ce^x] - [Ce^x] - 2[Ce^x + D] = e^x + 12$		
$-2Ce^x - 2D = e^x + 12$	1	
Hence $C = -\frac{1}{2}$ and $D = -6$.		
So the General Solution is		
$y = Ae^{-x} + Be^{2x} - \frac{1}{2}e^x - 6.$	1	
$x = 0$ and $y = -\frac{3}{2} \Rightarrow$		
$A + B - \frac{1}{2} - 6 = -\frac{3}{2}$		
$x = 0$ and $\frac{dy}{dx} = \frac{1}{2} \Rightarrow$	1	Setting up the equations
$-A + 2B - \frac{1}{2} = \frac{1}{2}$		
$3B - 7 = -1 \Rightarrow B = 2 \Rightarrow A = 3$	1	
So the particular solution is		
$y = 3e^{-x} + 2e^{2x} - \frac{1}{2}e^x - 6.$	1	

Source: 2010 Q11 AH Maths	
(11)	Obtain the general solution of the equation $$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0.$$ Hence obtain the solution for which $y = 3$ when $x = 0$ and $y = e^{-\pi}$ when $x = \frac{\pi}{2}$.

Answer:

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$		
$m^2 + 4m + 5 = 0$	**1**	
$(m + 2)^2 = -1$		
$m = -2 \pm i$	**1**	
The general solution is		
$y = e^{-2x}(A\cos x + B\sin x)$	**1M**	appropriate CF
	1	for accuracy
$x = 0, y = 3 \Rightarrow 3 = A$	**1**	
$x = \frac{\pi}{2}, y = e^{-\pi} \Rightarrow e^{-\pi} = e^{-\pi}(3\cos\frac{\pi}{2} + B\sin\frac{\pi}{2})$		
$\Rightarrow B = 1$	**1**	
The particular solution is:		
$y = e^{-2x}(3\cos x + \sin x).$	**1**	

Source: 2009 Q15 AH Maths

(12)

(*a*) Solve the differential equation

$$(x+1)\frac{dy}{dx}-3y=(x+1)^4$$

given that $y = 16$ when $x = 1$, expressing the answer in the form $y = f(x)$.

(*b*) Hence find the area enclosed by the graphs of $y = f(x)$, $y = (1-x)^4$ and the x-axis.

Answers:

(a)

$$(x+1)\frac{dy}{dx} - 3y = (x+1)^4$$

$$\frac{dy}{dx} - \frac{3}{x+1}y = (x+1)^3 \qquad \mathbf{1}$$

Integrating factor:

$$\text{since } \int \frac{-3}{x+1}dx = -3\ln(x+1). \qquad \mathbf{1}$$

Hence the integrating factor is $(x+1)^{-3}$. **1**

$$\frac{1}{(x+1)^3}\frac{dy}{dx} - \frac{3}{(x+1)^4}y = 1 \qquad \mathbf{1}$$

$$\frac{d}{dx}\left((x+1)^{-3}y\right) = 1$$

$$\frac{y}{(x+1)^3} = \int 1\,dx \qquad \mathbf{1}$$

$$= x + c$$

$y = 16$ when $x = 1$, so $2 = 1 + c \Rightarrow c = 1$. Hence

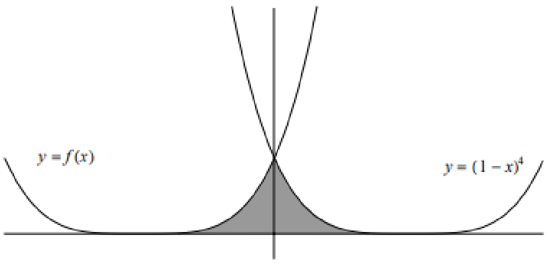
$$y = (x+1)^4 \qquad \mathbf{1}$$

(b)

$$(x+1)^4 = (1-x)^4$$

$$x + 1 = 1 - x \Rightarrow x = 0 \qquad \mathbf{1}$$

or $x + 1 = -1 + x$ which has no solutions.



$$\text{Area} = \int_{-1}^{0}(x+1)^4\,dx + \int_{0}^{1}(1-x)^4\,dx \qquad \mathbf{M1}$$

$$= 2\int_{-1}^{0}(x+1)^4\,dx \qquad \mathbf{1}$$

$$= \frac{2}{5}\left[(x+1)^5\right]_{-1}^{0} = \frac{2}{5} - 0 = \frac{2}{5} \qquad \mathbf{1}$$

Source: 2008 Q13 AH Maths

(13) Obtain the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2.$$

Given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = 1$, when $x = 0$, find the particular solution.

Answer:

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2$$

$$m^2 - 3m + 2 = 0 \qquad \mathbf{1}$$

$$(m - 1)(m - 2) = 0$$

$$m = 1 \text{ or } m = 2 \qquad \mathbf{1}$$

Complementary function: $y = Ae^x + Be^{2x}$ **1**

For particular integral try $y = ax^2 + bx + c$ **1M**

$$\Rightarrow \frac{dy}{dx} = 2ax + b; \frac{d^2y}{dx^2} = 2a$$

Hence require

$$2a - 3(2ax + b) + 2(ax^2 + bx + c) = 2x^2 \qquad \mathbf{1}$$

$$2ax^2 + (-6a + 2b)x + (2a - 3b + 2c) = 2x^2$$

$$\Rightarrow a = 1; b = 3; c = \frac{7}{2} \qquad \mathbf{1}$$

General solution is: $y = Ae^x + Be^{2x} + x^2 + 3x + \frac{7}{2}$ **1**

When $x = 0, y = \frac{1}{2}$ and $\frac{dy}{dx} = 1$.

$$\frac{1}{2} = A + B + \frac{7}{2} \Rightarrow A + B = -3 \qquad \mathbf{1}$$

$$\frac{dy}{dx} = Ae^x + 2Be^{2x} + 2x + 3 \Rightarrow 1 = A + 2B + 3 \Rightarrow A + 2B = -2 \qquad \mathbf{1}$$

$$B = 1 \qquad A = -4 \qquad \mathbf{1}$$

Particular solution is

$$y = -4e^x + e^{2x} + x^2 + 3x + \frac{7}{2}.$$

Source: 2007 Q8 AH Maths	
(14)	Obtain the general solution of the equation $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=e^{2x}$.

Answer:

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = e^{2x}$$

Auxiliary equation: $m^2 + 6m + 9 = 0$

So $(m + 3)^2 = 0$ giving $m = -3$.

Complementary function:

$$y = (A + Bx)e^{-3x}$$

For the Particular Integral try $y = ke^{2x}$

$$\Rightarrow \frac{dy}{dx} = 2ke^{2x};\ \frac{d^2y}{dx^2} = 4ke^{2x}$$

$$4ke^{2x} + 12ke^{2x} + 9ke^{2x} = e^{2x} \Rightarrow 25k = 1$$

Hence the General Solution is:

$$y = (A + Bx)e^{-3x} + \tfrac{1}{25}e^{2x}$$

Source: 2006 Q8 AH Maths

(15) Solve the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

given that when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 2$.

Answer:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

A.E. $m^2 + 2m + 2 = 0$ **1**

$$m = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm i$$ **1**

General solution is

$$y = e^{-x}(A\cos x + B\sin x)$$ **1**

$$y = 0 \text{ when } x = 0 \Rightarrow 0 = A$$ **1**

$$\frac{dy}{dx} = -e^{-x}B\sin x + e^{-x}B\cos x$$ **1**

$$2 = 0 + B$$ **1**

The solution is $y = 2e^{-x}\sin x$.

Source: 2005 Q14 AH Maths	
(16)	Obtain the general solution of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 20\sin x$ Hence find the particular solution for which $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$.

Answer:

Let $y = e^{mx}$, then the auxiliary equations is

$$m^2 - 3m + 2 = 0 \qquad 1$$

$$(m-1)(m-2) = 0$$

$$m = 1 \text{ or } m = 2 \qquad 1$$

The Complementary Function is $y = Ae^x + Be^{2x}$. 1

For the Particular Integral, try $y = a\sin x + b\cos x$. 1

$$\frac{dy}{dx} = a\cos x - b\sin x$$

$$\frac{d^2y}{dx^2} = -a\sin x - b\cos x$$

Substituting:

$$(-a\sin x - b\cos x) - 3(a\cos x - b\sin x) + 2(a\sin x + b\cos x) = 20\sin x \qquad 1$$

$$(-a + 3b + 2a)\sin x + (-b - 3a + 2b)\cos x = 20\sin x$$

$$a + 3b = 20; \qquad -3a + b = 0$$

$$a = 2;\ b = 6. \qquad 1$$

The general solution is

$$y = Ae^x + Be^{2x} + 2\sin x + 6\cos x \qquad 1$$

$$\frac{dy}{dx} = Ae^x + 2Be^{2x} + 2\cos x - 6\sin x$$

$y = 0$ when $x = 0$ so $A + B + 6 = 0$. 1

$\frac{dy}{dx} = 0$ when $x = 0$ so $A + 2B + 2 = 0$. 1

$$B = 4; \qquad A = -10 \qquad 1$$

The particular solution is

$$y = -10e^x + 4e^{2x} + 2\sin x + 6\cos x.$$