Integration

AH Maths Exam Questions

Source: 2019 Specimen P1 Q4 AH Maths (Same as 2013 Q4b)	
(1)	The velocity, v, of a particle P at time t is given by $$v = e^{3t} + 2e^t.$$ Find the distance covered by P between $t = 0$ and $t = \ln 3$.
Answer:	$Distance\ covered = 12\frac{2}{3}\ units$

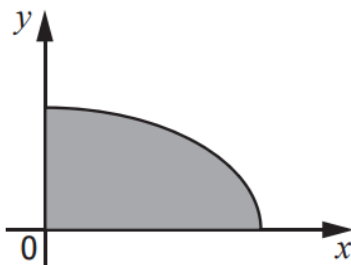| Source: 2018 Q8 AH Maths	
(2)	Using the substitution $u = \sin\theta$, or otherwise, evaluate $$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\sin^4\theta\cos\theta\,d\theta.$$
Answer:	$\dfrac{31}{80}$ or 0.3875

(3) On a suitable domain, a curve is defined by the equation $4x^2 + 9y^2 = 36$.

A section of the curve in the first quadrant, illustrated in the diagram below, is rotated 360° about the **y-axis**.

Calculate the exact value of the volume generated.

Answer: $Volume = 12\pi\ units^3$

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(4) Express $\dfrac{3x+32}{(x+4)(6-x)}$ in partial fractions and hence evaluate

$$\int_{3}^{4} \frac{3x+32}{(x+4)(6-x)}\,dx.$$

Give your answer in the form $\ln\left(\dfrac{p}{q}\right)$.

Answer: $\ln\left(\dfrac{486}{49}\right)$

(5) Find $\displaystyle\int \frac{2x^3 - x - 1}{(x-3)(x^2+1)}\, dx, \ x > 3.$

Answer:
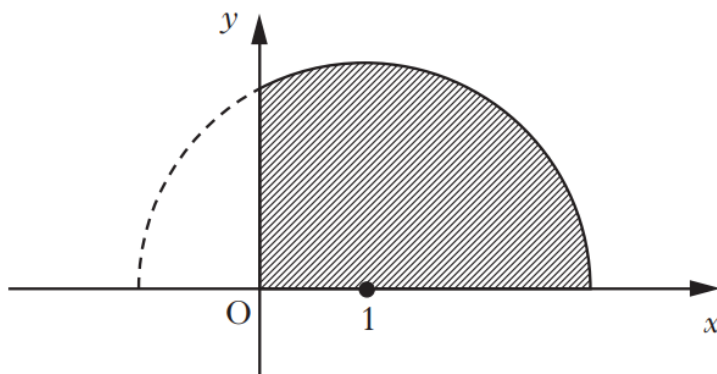
$$2x + 5\ln|x-3| + \frac{1}{2}\ln(x^2+1) + k$$

(6) A semi-circle with centre $(1, 0)$ and radius 2, lies on the x-axis as shown.

Find the volume of the solid of revolution formed when the shaded region is rotated completely about the x-axis.

Answer: $Volume = 9\pi\ units^3$

(7) Use the substitution $x = \tan\theta$ to determine the exact value of

$$\int_0^1 \frac{dx}{\left(1+x^2\right)^{\frac{3}{2}}}.$$

Answer: $\dfrac{1}{\sqrt{2}}$

(8) The velocity, v, of a particle P at time t is given by

$$v = e^{3t} + 2e^t.$$

(a) Find the acceleration of P at time t.

(b) Find the distance covered by P between $t = 0$ and $t = \ln 3$.

Answers: $(a)\ a = 3e^{3t} + 2e^t$ $\qquad (b)\ \dfrac{38}{3}\ $ or $\ 12\dfrac{2}{3}$

(9) Integrate $\dfrac{\sec^2 3x}{1 + \tan 3x}$ with respect to x.

Answer: $\dfrac{1}{3}\ln|1 + \tan 3x| + c$

(10) Use the substitution $x = 4 \sin \theta$ to evaluate $\int_0^2 \sqrt{16 - x^2} \; dx$.

Answer: $\frac{4\pi}{3} + 2\sqrt{3} = 7.65 \; (2 \; decimal \; places)$

(11) A new board game has been invented and the symmetrical design on the board is made from four identical "petal" shapes. One of these petals is the region enclosed between the curves $y = x^2$ and $y^2 = 8x$ as shown shaded in diagram 1 below.

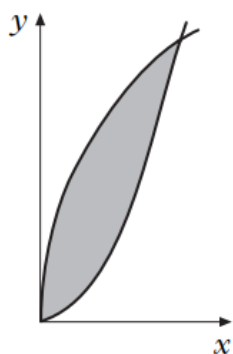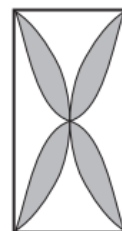Calculate the area of the complete design, as shown in diagram 2.

Diagram 1

Diagram 2

The counter used in the game is formed by rotating the shaded area shown in diagram 1 above, through $360°$ about the y-axis. Find the volume of plastic required to make one counter.

Answer: $A = \frac{32}{3} \; units^2 \qquad V = \frac{24\pi}{5} \; units^3$

(12) Show that

$$\int_{\ln \frac{3}{2}}^{\ln 2} \frac{e^x + e^{-x}}{e^x - e^{-x}} \, dx = \ln \frac{9}{5}.$$

Answer: *Prove LHS = RHS*

(13) Use the substitution $x = 2 \sin \theta$ to obtain the exact value of $\displaystyle\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4 - x^2}}\, dx.$

(Note that $\cos 2A = 1 - 2 \sin^2 A$.)

Answer: $Exact\ Value = \dfrac{\pi}{2} - 1$

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(14) Express $\dfrac{12x^2 + 20}{x(x^2 + 5)}$ in partial fractions.

Hence evaluate

$$\int_1^2 \frac{12x^2 + 20}{x(x^2 + 5)}\, dx.$$

Answers: $\dfrac{4}{x} + \dfrac{8x}{x^2 + 5}$ $\qquad 4ln3 \cong 4.39$

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(15) Write down the derivative of $\tan x$.

Show that $1 + \tan^2 x = \sec^2 x$.

Hence obtain $\int \tan^2 x\, dx$.

Answers:

$$\frac{d}{dx}(\tan x) = \sec^2 x.$$

$$1 + \tan^2 x = 1 + \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \sec^2 x.$$

$$\int \tan^2 x\, dx = \int (\sec^2 x - 1)\, dx$$

$$= \tan x - x + c$$

(16) A body moves along a straight line with velocity $v = t^3 - 12t^2 + 32t$ at time t.

(a) Obtain the value of its acceleration when $t = 0$.

(b) At time $t = 0$, the body is at the origin O. Obtain a formula for the displacement of the body at time t.

Show that the body returns to O, and obtain the time, T, when this happens.

Answers:

(a) *Acceleration* $= 32$ *when* $t = 0$

(b) *Displacement of body at time* t: $x(t) = \dfrac{t^4}{4} - 4^3 + 16t^2$

The body returns to O *when* $t = 8$

(17) Use the substitution $u = 1 + x^2$ to obtain $\displaystyle\int_0^1 \frac{x^5}{(1+x^2)^4}\,dx.$

A solid is formed by rotating the curve $y = \dfrac{x^{3/2}}{(1+x^2)^2}$ between $x = 0$ and $x = 1$ through $360°$ about the x-axis. Write down the volume of this solid.

Answers: $\dfrac{1}{24}, \quad Volume = \dfrac{\pi}{24}$