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Partial Fractions

AH Maths Exam Questions

Source: 2019 Specimen P2 Q1 AH Maths (same Question as 2017 Q2)	
(1)	Express $\dfrac{x^2-6x+20}{(x+1)(x-2)^2}$ in partial fractions.
Answer: $\dfrac{3}{(x+1)} - \dfrac{2}{(x-2)} + \dfrac{4}{(x-2)^2}$	

Source: 2019 Q4 AH Maths	
(2)	(a) Express $\dfrac{3x^2+x-17}{x^2-x-12}$ in the form $p+\dfrac{qx+r}{x^2-x-12}$, where p, q and r are integers. (b) Hence express $\dfrac{3x^2+x-17}{x^2-x-12}$ with partial fractions.
Answers: $(a)\ 3+\frac{4x+19}{x^2-x-12}$ $(b)\ 3-\frac{1}{x+3}+\frac{5}{x-4}$	

Source: 2018 Q2 AH Maths	
(3)	Use partial fractions to find $\displaystyle\int \frac{3x-7}{x^2-2x-15}\,dx$.
Answer: $2ln\|x+3\| + ln\|x-5\| + c$	

Source: 2016 Q13 AH Maths	
(4)	Express $\frac{3x+32}{(x+4)(6-x)}$ in partial fractions and hence evaluate $$\int_3^4 \frac{3x+32}{(x+4)(6-x)}\,dx.$$ Give your answer in the form $\ln\left(\frac{p}{q}\right)$.
Answer: $ln\,\frac{486}{49}$	

Source: 2012 Q15a AH Maths	
(5)	Express $\frac{1}{(x-1)(x+2)^2}$ in partial fractions.
Answer: $$\frac{1}{(x-1)(x+2)^2} = \frac{1}{9}\left(\frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2}\right)$$	

Source: 2014 Q14b AH Maths

(6)

(*a*) Given the series $1 + r + r^2 + r^3 + \ldots$, write down the sum to infinity when $|r| < 1$.

Hence obtain an infinite geometric series for $\frac{1}{2-3r}$.

For what values of r is this series valid?

(*b*) Express $\frac{1}{3r^2 - 5r + 2}$ in partial fractions.

Hence, or otherwise, determine the first three terms of an infinite series

for $\frac{1}{3r^2 - 5r + 2}$.

For what values of r does the series converge?

Answers

(*a*)

$$1 + r + r^2 + r^3 + \ldots = \frac{1}{1-r}$$

$$\frac{1}{2-3r} = \frac{1}{2\left(1-\frac{3r}{2}\right)} \textbf{ OR } \frac{1}{1-(3r-1)} \textbf{ OR } \frac{\frac{1}{2}}{1-\frac{3}{2}r}$$

$$= \frac{1}{2}\left(\frac{1}{1-\frac{3r}{2}}\right) = \frac{1}{2}\left(1 + \frac{3r}{2} + \left(\frac{3r}{2}\right)^2 + \ldots\right)$$

$$= \frac{1}{2}\left(1 + \frac{3r}{2} + \frac{9r^2}{4} + ..\right)$$

$$\left|\frac{3r}{2}\right| < 1, \quad \therefore |r| < \frac{2}{3}$$

(*b*)

$$\frac{1}{3r^2 - 5r + 2} = \frac{A}{(3r-2)} + \frac{B}{(r-1)}$$

$$\therefore A(r-1) + B(3r-2) \equiv 1; \qquad B = 1$$

$$A = -3$$

$$\frac{1}{3r^2 - 5r + 2} = \frac{-3}{(3r-2)} + \frac{1}{(r-1)}$$

$$= \frac{3}{(2-3r)} - \frac{1}{(1-r)}$$

$$= 3\left(\frac{1}{2}\left(1 + \frac{3r}{2} + \frac{9r^2}{4} + \ldots\right)\right) - \left(1 + r + r^2 + \ldots\right)$$

$$= \frac{1}{2} + \frac{5r}{4} + \frac{19r^2}{8} \ldots$$

$$\left|\frac{3r}{2}\right| < 1 \text{ and } |r| < 1, \text{ so } |r| < \frac{2}{3}$$

OR

$$f(x) = (3r^2 - 5r + 2)^{-1} \qquad f(0) = \frac{1}{2}$$

$$f'(x) = -(3r^2 - 5r + 2)^{-2}(6r-5) \qquad f'(0) = \frac{5}{4}$$

$$f''(x) = -6(3r^2 - 5r + 2)^{-2} + 2(3r^2 - 5r + 2)^{-3}(6r-5)^2$$

$$f''(0) = \frac{19}{4}$$

$$\therefore f(x) = \frac{1}{2} + \frac{5r}{4} + \frac{19r^2}{8} \ldots$$

Source: 2011 Q1 AH Maths	
(7)	Express $\dfrac{13-x}{x^2+4x-5}$ in partial fractions and hence obtain $$\int \frac{13-x}{x^2+4x-5}dx.$$
Answers: $\dfrac{2}{x-1}-\dfrac{3}{x+5}$	$2ln\lvert x-1\rvert - 3ln\lvert x+5\rvert + c$

Source: 2009 Q14 AH Maths	
(8)	Express $\dfrac{x^2+6x-4}{(x+2)^2(x-4)}$ in partial fractions. Hence, or otherwise, obtain the first three non-zero terms in the Maclaurin expansion of $\dfrac{x^2+6x-4}{(x+2)^2(x-4)}$.
Answers: $\dfrac{2}{(x+2)^2}+\dfrac{1}{x+4}$	$\dfrac{1}{4}-\dfrac{9x}{16}+\dfrac{23x^2}{64}+\cdots$

Source: 2008 Q4 AH Maths	
(9)	Express $\dfrac{12x^2+20}{x(x^2+5)}$ in partial fractions. Hence evaluate $$\int_1^2 \frac{12x^2+20}{x(x^2+5)}\,dx.$$
Answers: $\dfrac{4}{x}+\dfrac{8x}{x^2+5}$	$4ln3 \cong 4.39$

Source: 2007 Q4 AH Maths	
(10)	Express $\dfrac{2x^2-9x-6}{x(x^2-x-6)}$ in partial fractions. Given that $$\int_4^6 \frac{2x^2-9x-6}{x(x^2-x-6)}\,dx = \ln\frac{m}{n},$$ determine values for the integers m and n.
Answers: $\frac{1}{x}+\frac{2}{x+2}-\frac{3}{x-3}$ $\quad m=8,\ n=9$	