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Further Number Theory

AH Maths Exam Questions

Source: 2019 Q12 AH Maths

(1) Express 231_{11} in base 7.

Answer:

•1 convert to base 10 [1]	•1 276
•2 method leading to a quotient of 0 or equivalent	•2 eg $276 = 7\times39+3$ $39 = 7\times5+4$ $5 = 7\times0+5$
•3 express in base 7 [1,2,3]	•3 543_7

Source: 2018 Q5 AH Maths

(2) Use the Euclidean algorithm to find integers a and b such that $306a+119b=17$.

Answers:

•1 start process	•1 $306 = 2\times119 + 68$ $(119 = 1\times68 + 51)$
•2 obtain remainder of 17 [1]	•2 $68 = 1\times51 + 17$ $(51 = 3\times17)$
•3 express gcd in terms of 306 and 119	•3 $17 = -1\times119 + 2\ (306 - 2\times119)$
•4 obtain a and b [2,3]	•4 $a = 2,\ b = -5$

Source: 2017 Q8 AH Maths	
(3)	Use the Euclidean algorithm to find integers a and b such that $1595a+1218b=29$.

Answers:

•[1] start process	•[1] $1595=1\times1218+377$
•[2] obtain remainder of 29 [1]	$1218=3\times377+87$ •[2] $377=4\times87+29$ $87=3\times29+0$
•[3] express gcd in terms of 377 and 1218	•[3] $29=377-4(1218-3\times377)$
•[4] state values of a and b [2]	•[4] $a=13, b=-17$

Source: 2015 Q7 AH Maths	
(4)	Use the Euclidean algorithm to find integers p and q such that $3066p + 713q = 1.$

Answer:

$3066=713\times4+214$ $713=214\times3+71$ $214=71\times3+1$ $1=214-71\times3$ $=214-3(713-214\times3)$ $=214\times10-713\times3$ $=(3066-713\times4)\times10-713\times3$ $=3066\times10-713\times43$ $p=10$ $q=-43$	**4**	•[1] starting correctly •[2] reach GCD •[3] equates GCD from •[2] *and* evidence of substitution •[4] obtains values of p and q.

Source: 2013 Q5 AH Maths

(5) Use the Euclidean algorithm to obtain the greatest common divisor of 1204 and 833, expressing it in the form $1204a + 833b$, where a and b are integers.

Answer:

$1204 = 1 \times 833 + 371$	•[1] Starting correctly.
$833 = 2 \times 371 + 91$ $371 = 4 \times 91 + 7$ $91 = 13 \times 7$ so gcd is 7	•[2] Obtains GCD. Accept $(833, 1204) = 7$
$7 = 371 - 4 \times 91$ $= 371 - 4(833 - 2 \times 371)$	•[3] Equates GCD from •[2] and evidence of correct back substitution.[1,4]
$= 9 \times 371 - 4 \times 833$ $= 9(1204 - 1 \times 833) - 4 \times 833$ $= 9 \times 1204 - 13 \times 833$ $(a = 9, b = -13)$	•[4] Correct form of final answer.[5]

Source: 2012 Q10 AH Maths

(6) Use the division algorithm to express 1234_{10} in base 7.

Answer:

Method 1

$$1234 = 7 \times 176 + 2$$
$$176 = 7 \times 25 + 1$$
$$25 = 7 \times 3 + 4$$

Hence

$$1234_{10} = 3412_7$$

Method 2

$1234 = 7 \times 176 + 2$

$= 7 \times (7 \times 25 + 1) + 2$

$= 7 \times (7 \times (7 \times 3 + 4) + 1) + 2$

$= 3 \times 7^3 + 4 \times 7^2 + 1 \times 7 + 2$

Hence

$$1234_{10} = 3412_7$$

Source: 2009 Q10 AH Maths	
(7)	Use the Euclidean algorithm to obtain the greatest common divisor of 1326 and 14654, expressing it in the form $1326a + 14654b$, where a and b are integers.

Answer:

$$\begin{aligned} 14654 &= 11 \times 1326 + 68 \\ 1326 &= 19 \times 68 + 34 \\ 68 &= 2 \times 34 \\ 34 &= 1326 - 19 \times 68 \\ &= 1326 - 19(14654 - 11 \times 1326) \\ &= 210 \times 1326 - 19 \times 14654 \end{aligned}$$

Source: 2007 Q7 AH Maths	
(8)	Use the Euclidean algorithm to find integers p and q such that $599p + 53q = 1$.

Answer:

$$\begin{aligned} 599 &= 53 \times 11 + 16 \\ 53 &= 16 \times 3 + 5 \\ 16 &= 5 \times 3 + 1 \end{aligned}$$

$$\begin{aligned} 1 &= 16 - 5 \times 3 \\ &= 16 - (53 - 16 \times 3) \times 3 \\ &= 16 \times 10 - 53 \times 3 \\ &= (599 - 53 \times 11) \times 10 - 53 \times 3 \\ &= 599 \times 10 - 53 \times 113 \end{aligned}$$

Hence $599p + 53q = 1$ when $p = 10$ and $q = -113$.

Source: 2004 Q8 AH Maths	
(9)	Use the Euclidean algorithm to show that $(231, 17) = 1$ where (a, b) denotes the highest common factor of a and b. Hence find integers x and y such that $231x + 17y = 1$.

Answers:

$$231 = 13 \times 17 + 10$$ **1 for method**

$$17 = 1 \times 10 + 7$$

$$10 = 1 \times 7 + 3$$

$$7 = 2 \times 3 + 1$$ **1**

Thus the highest common factor is 1.

$$1 = 7 - 2 \times 3$$

$$= 7 - 2 \times (10 - 7) = 3 \times 7 - 2 \times 10$$ **1 for method**

$$= 3 \times (17 - 10) - 2 \times 10 = 3 \times 17 - 5 \times 10$$

$$= 3 \times 17 - 5 \times (231 - 13 \times 17) = 68 \times 17 - 5 \times 231.$$ **1**

So $x = -5$ and $y = 68$.