

Gradients - Lesson 2

Gradients of Straight Lines (Non-Calculator)

LI

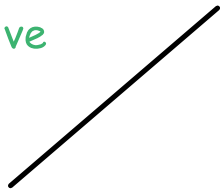
- Know the meaning of Gradient of a Straight Line.
- Know the 4 types of gradients.
- Calculate the gradient of a straight line when told 2 points on it.

SC

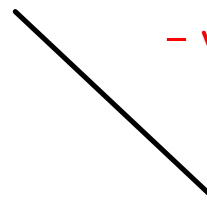
- - integers.
- Simplify fractions.

Gradient is a measure of steepness
from left to right

+ve



-ve



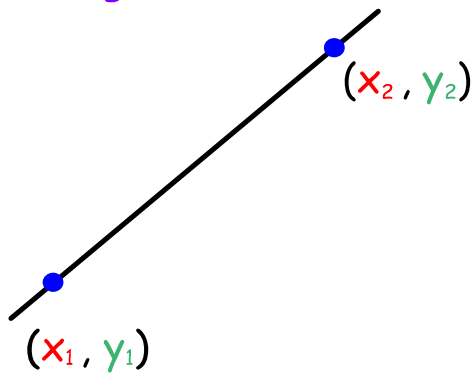
0



∞ (aka undefined)



Gradient Formula

 $m = \text{gradient}$ 

$$m = \frac{\text{vertical distance}}{\text{horizontal distance}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- If $y_2 = y_1$, $m = 0$.
- If $x_2 = x_1$, m is undefined.

Example 1

Calculate the gradient of the line joining the points (5, 7) and (8, 9).

$$\begin{array}{l} x_1 \ y_1 \\ (5, 7) \end{array}$$

$$\begin{array}{l} (8, 9) \\ x_2 \ y_2 \end{array}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{9 - 7}{8 - 5}$$

$$m = \frac{2}{3}$$

Example 2

Calculate the gradient of the line joining the points
(6, - 6) and (2, 12).

$$\begin{matrix} x_1 & y_1 \\ (6, & -6) \end{matrix}$$

$$\begin{matrix} (2, & 12) \\ x_2 & y_2 \end{matrix}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{12 - (-6)}{2 - 6}$$

$$m = \frac{18}{-4}$$

$$m = -\frac{9}{2}$$

Example 3

Calculate the gradient of the line joining the points (9, 7) and (6, 7).

$$\begin{array}{l} x_1 \quad y_1 \\ (9, 7) \end{array}$$

$$\begin{array}{l} (6, 7) \\ x_2 \quad y_2 \end{array}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7 - 7}{6 - 9}$$

$$m = \frac{0}{-3}$$

$$m = 0$$

Example 4

Calculate the gradient of the line joining the points
(- 2, 4) and (- 2, 5).

$$\begin{array}{l} x_1 \quad y_1 \\ (- 2, 4) \end{array}$$

$$\begin{array}{l} (- 2, 5) \\ x_2 \quad y_2 \end{array}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - 4}{- 2 - (- 2)}$$

$$m = \frac{1}{0}$$

$$m = \text{undefined}$$

Calculate the gradients of the straight lines which have these coordinates on them (simplified where possible) :

1) $(2, 5)$ and $(4, 11)$

2) $(0, 15)$ and $(4, 19)$

3) $(0, 15)$ and $(4, 30)$

4) $(2, 5)$ and $(6, 3)$

5) $(5, -4)$ and $(9, -4)$

6) $(3, 65)$ and $(3, 109)$

7) $(2, 2)$ and $(10, -4)$

8) $(7, 14)$ and $(-14, 21)$

9) $(9, 64)$ and $(18, 1)$

10) $(0, -11)$ and $(1, 0)$

11) $(3, -35)$ and $(21, 1)$

12) $(40, 100)$ and $(20, 20)$

13) $(600, 50)$ and $(4, 20)$

14) $(-40, 5)$ and $(80, 10)$

Answers1) (2, 5) and (4, 11) **3**2) (0, 15) and (4, 19) **1**3) (0, 15) and (4, 30) **15/4**4) (2, 5) and (6, 3) **- 1/2**5) (5, - 4) and (9, - 4) **0**6) (3, 65) and (3, 109) **∞** 7) (2, 2) and (10, - 4) **- 3/4**8) (7, 14) and (- 14, 21) **- 1/3**9) (9, 64) and (18, 1) **- 7**10) (0, - 11) and (1, 0) **11**11) (3, - 35) and (21, 1) **2**12) (40, 100) and (20, 20) **4**13) (600, 50) and (4, 20) **15/298**14) (- 40, 5) and (80, 10) **1/24**

- 1 Calculate the gradient of the straight line joining the following pairs of points.
 - a $A(2, 1)$ and $B(3, 4)$
 - b $C(-1, 6)$ and $D(0, 4)$
 - c $E(-1, -3)$ and $F(1, 5)$
 - d $G(-7, 2)$ and $H(-4, -1)$
 - e $J(-2, 5)$ and $K(1, 7)$
 - f $M(-7, 3)$ and $N(-3, -2)$
 - g $P(-7, -8)$ and $Q(-3, -2)$
 - h $R(-11, 4)$ and $S(-2, -8)$
 - i $T(-9, -10)$ and $U(-3, 5)$
- 2
 - a Calculate the gradient of the line joining $A(-3, 5)$ and $B(7, 5)$.
 - b The line in part a is horizontal. Which axis is it parallel to?
 - c How could you have known from the coordinates that the line in a is horizontal?
- 3
 - a Calculate the gradient of the line joining $C(5, 4)$ and $D(5, -7)$.
 - b The line in part a is vertical, its gradient is undefined. Which axis is it parallel to?
 - c How could you have known from the coordinates that the line is a is vertical?
- 4 Calculate the gradient of the straight line joining the following pairs of points.
 - a $T(-\frac{1}{2}, -\frac{3}{2})$ and $U(-\frac{9}{4}, \frac{15}{4})$
 - b $V(\frac{1}{3}, \frac{1}{5})$ and $W(-\frac{16}{9}, \frac{41}{15})$
- 5 A is the point $(5, 1)$ and B is the point $(8, y)$. Find y if $m_{AB} = 2$.
- 6 C is the point $(-1, -7)$ and D is the point $(x, -5)$. Find x if $m_{CD} = -\frac{2}{5}$.
- 7 E is the point $(\frac{3}{2}, \frac{5}{3})$ and F is the point $(\frac{11}{2}, y)$. Find y if $m_{EF} = -\frac{3}{4}$.
- 8 Find and simplify an expression for the gradient of the line joining $G(a, a^2)$ and $H(-2, 4)$.

Answers

<p>1 a $m_{AB} = 3$</p> <p>b $m_{CD} = -2$</p> <p>c $m_{EF} = 4$</p> <p>d $m_{GH} = -1$</p> <p>e $m_{JK} = \frac{2}{3}$</p> <p>f $m_{MN} = -\frac{5}{4}$</p> <p>g $m_{PQ} = \frac{3}{2}$</p> <p>h $m_{RS} = -\frac{4}{3}$</p> <p>i $m_{TU} = \frac{5}{2}$</p>	<p>3 a The gradient is undefined.</p> <p>b The line is parallel to the y-axis.</p> <p>c The line is vertical because the horizontal change is zero.</p>
<p>2 a $m_{AB} = 0$</p> <p>b The line is parallel to the x-axis.</p> <p>c The line is horizontal, because the vertical change is zero.</p>	<p>4 a $m_{TU} = -3$</p> <p>b $m_{VW} = -\frac{6}{5}$</p> <p>5 $y = 7$</p> <p>6 $x = -6$</p> <p>7 $y = -\frac{4}{3}$</p> <p>8 $m_{GH} = a - 2$</p>