Trigonometric Identities

Proving Trigonometric Identities

LI

- Prove equations involving sin x, cos x and tan x.

SC

- Pythagorean Identity.
- Link between sin x, cos x and tan x.

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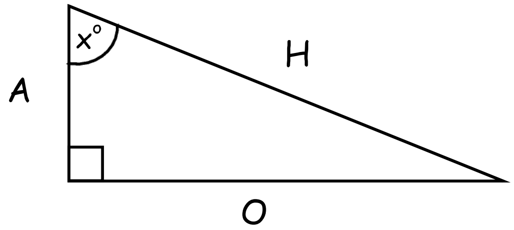

We use the abbreviation $\sin^2 x°$ as a shorthand for $(\sin x°)^2$ and similarly for cos x°

$$\sin x° = \frac{O}{H}$$

$$\cos x° = \frac{A}{H}$$

$$\tan x° = \frac{O}{A}$$

$$H^2 = A^2 + O^2$$

$$\sin^2 x° + \cos^2 x°$$

$$= \left(\frac{O}{H}\right)^2 + \left(\frac{A}{H}\right)^2$$

$$= \frac{O^2}{H^2} + \frac{A^2}{H^2}$$

$$= \frac{O^2 + A^2}{H^2}$$

$$= \frac{H^2}{H^2}$$

$$= \underline{1}$$

We thus have the Pythagorean Identity :

$$\boxed{\sin^2 x° + \cos^2 x° = 1}$$

$$\frac{\sin x°}{\cos x°} = \frac{O}{H} \div \frac{A}{H}$$

$$= \frac{O}{H} \times \frac{H}{A}$$

$$= \frac{O}{A}$$

$$= \underline{\tan x°}$$

Thus :

$$\boxed{\frac{\sin x°}{\cos x°} = \tan x°}$$

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A Trigonometric Identity is an equation involving trigonometric functions that is true for all allowable x - values

To show that an identity is true, normally start with the more complicated side and try to show that it equals the other side

The two sides of an equation have a standard shorthand :

LHS : Left-Hand Side

RHS : Right-Hand Side

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Example 1

Show that $\cos x^\circ \tan x^\circ = \sin x^\circ$.

$$\text{LHS} = \cos x^\circ \tan x^\circ$$

$$= \cos x^\circ \left(\frac{\sin x^\circ}{\cos x^\circ} \right)$$

$$= \frac{\cos x^\circ}{1} \times \frac{\sin x^\circ}{\cos x^\circ}$$

$$= \frac{\cos x^\circ \sin x^\circ}{\cos x^\circ}$$

$$= \sin x^\circ$$

$$= \text{RHS}$$

LHS = RHS; hence, $\cos x^\circ \tan x^\circ = \sin x^\circ$

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Example 2

Show that $7\cos^2 x^\circ + 7\sin^2 x^\circ = 7$.

$$\begin{aligned} \text{LHS} &= 7\cos^2 x^\circ + 7\sin^2 x^\circ \\ &= 7(\cos^2 x^\circ + \sin^2 x^\circ) \\ &= 7(1) \\ &= 7 \\ &= \text{RHS} \end{aligned}$$

LHS = RHS; hence, $7\cos^2 x^\circ + 7\sin^2 x^\circ = 7$

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Example 3

Show that $\dfrac{1 - \cos^2 x^\circ}{\cos^2 x^\circ} = \tan^2 x^\circ$.

$$\text{LHS} = \frac{1 - \cos^2 x^\circ}{\cos^2 x^\circ}$$

$$= \frac{\sin^2 x^\circ}{\cos^2 x^\circ}$$

$$= \left(\frac{\sin x^\circ}{\cos x^\circ}\right)^2$$

$$= (\tan x^\circ)^2$$

$$= \tan^2 x^\circ$$

$$= \text{RHS}$$

LHS = RHS; hence, $\dfrac{1 - \cos^2 x^\circ}{\cos^2 x^\circ} = \tan^2 x^\circ$

1 Show that $\sin x^\circ \tan x^\circ = \dfrac{\sin^2 x^\circ}{\cos x^\circ}$

2 Show that $\sin^3 x^\circ + \sin x^\circ \cos^2 x^\circ = \sin x^\circ$

3 Show that $3\sin^2 x^\circ + 3\cos^2 x^\circ = 3$

4 Show that $\dfrac{\sin x^\circ}{\tan x^\circ} = \cos x^\circ$

5 Show that $5 - 5\cos^2 x^\circ = 5\sin^2 x^\circ$

★ 6 Show that $\dfrac{\sin^2 x^\circ}{1 - \cos^2 x^\circ} = 1$

★ 7 Show that $\dfrac{\sin x^\circ \cos x^\circ}{\cos^2 x^\circ} = \tan x^\circ$