

Algebraic Fractions - Lesson 2

Simplifying Algebraic Fractions by Factorisation

LI

- Simplify algebraic fractions using factorisation.

SC

- Factorise expressions.

Important Reminders and Ideas

To simplify a fraction, **divide top and bottom** by the **same** quantity

$$\frac{a}{p} + \frac{b}{p} = \frac{a + b}{p} = (a + b) \div p$$

If possible, **factorise numerator and/or denominator fully** before dividing

Example 1

Simplify fully,

$$\begin{aligned} & \frac{3x - 6}{3} \\ = & \frac{(3x - 6)}{3} \quad \div 3 \\ & \quad \div 3 \\ = & \frac{(x - 2)}{1} \\ = & \boxed{x - 2} \end{aligned}$$

Example 2

Simplify fully,

$$\begin{aligned} & \frac{3x + 9}{4x^2 + 12x} \\ = & \frac{3(x + 3)}{4x(x + 3)} \quad \div (x + 3) \\ & \quad \quad \quad \div (x + 3) \\ = & \boxed{\frac{3}{4x}} \end{aligned}$$

Example 3

Simplify fully,

$$\begin{aligned} & \frac{7x}{14x^2 - 7x} \\ = & \frac{7x}{7x(2x - 1)} \quad \div 7x \\ = & \boxed{\frac{1}{2x - 1}} \end{aligned}$$

Example 4

Simplify fully,

$$\begin{aligned} & \frac{x + 1}{x^2 - 1} \\ = & \frac{(x + 1)}{(x - 1)(x + 1)} \quad \div (x + 1) \\ = & \boxed{\frac{1}{x - 1}} \quad \div (x + 1) \end{aligned}$$

Example 5

Simplify fully,

$$\frac{x^2 + 5x + 6}{x^2 - 2x - 8}$$
$$= \frac{(x + 2)(x + 3) \div (x + 2)}{(x + 2)(x - 4) \div (x + 2)}$$
$$= \frac{x + 3}{x - 4}$$

Example 6

Simplify fully,

$$\frac{13x + \frac{1}{5}}{x - \frac{1}{5}}$$
$$= \frac{\left(13x + \frac{1}{5}\right) \times 5}{\left(x - \frac{1}{5}\right) \times 5}$$
$$= \frac{5\left(13x + \frac{1}{5}\right)}{5\left(x - \frac{1}{5}\right)}$$
$$= \frac{65x + 1}{5x - 1}$$

Example 7

Simplify fully,

$$\frac{\frac{D}{x} + \frac{D}{3p}}{\frac{D}{3x} - \frac{D}{p}}$$

$$= \frac{\left(\frac{D(3p + x)}{3px} \right)}{\left(\frac{D(p - 3x)}{3px} \right)} \quad \begin{array}{l} \times 3px \\ \times 3px \end{array}$$

$$= \frac{D(3p + x)}{D(p - 3x)} \quad \begin{array}{l} \div D \\ \div D \end{array}$$

$$= \boxed{\frac{3p + x}{p - 3x}}$$

1) Simplify fully :

| | | | |
|-----------------------------|-------------------------------|---|--------------------------------|
| (a) $\frac{4d + 6}{2}$ | (b) $\frac{9x + 6}{3}$ | (c) $\frac{8a + 10b}{2}$ | (d) $\frac{15m - 10n}{5}$ |
| (e) $\frac{8x - 4y}{2}$ | (f) $\frac{ax + bx}{x}$ | (g) $\frac{x^2 - x}{x}$ | (h) $\frac{2x^2 - 4}{2}$ |
| (i) $\frac{12}{3x - 9}$ | (j) $\frac{5x + 10}{15}$ | (k) $\frac{2x + 4y + 6z}{4x - 6y + 2z}$ | (l) $\frac{-2x - 4}{-6x - 4}$ |
| (m) $\frac{2x}{6x - 4}$ | (n) $\frac{3x}{6x^2 - 3}$ | (o) $\frac{3x}{6x^2 - 3x}$ | (p) $\frac{2x - 1}{6x^2 - 3x}$ |
| (q) $\frac{3m - 6}{2m - 4}$ | (r) $\frac{m^2 + 3m}{3m + 9}$ | (s) $\frac{x^2 - 3x}{x^2 + 2x}$ | (t) $\frac{5x - 10}{6 - 3x}$ |

2) Simplify fully :

| | | |
|--|---|---------------------------------------|
| (a) $\frac{x^2 + 3x}{x^2 + 4x + 3}$ | (b) $\frac{x^2 + 3x + 2}{x^2 + 4x + 3}$ | (c) $\frac{x^2 - 2x}{x^2 + x - 6}$ |
| (d) $\frac{x^2 - x - 20}{x^2 + 7x + 12}$ | (e) $\frac{x^2 - 4x + 4}{x^2 - 5x + 6}$ | (f) $\frac{x^2 - 5x}{x^2 + 4x}$ |
| (g) $\frac{x^2 - 1}{x + 1}$ | (h) $\frac{x - 2}{x^2 - 4}$ | (i) $\frac{x^2 - 2x - 3}{2x - 6}$ |
| (j) $\frac{2x^2 - 2x - 12}{2x^2 - 18}$ | (k) $\frac{2x^2 - 7x - 15}{2x^2 - 5x - 12}$ | (l) $\frac{6x^2 - 13x - 5}{9x^2 - 1}$ |

Answers

1)

| | | |
|----------------------------|---------------------------|---------------------------------------|
| (a) $2d + 3$ | (b) $3x + 2$ | (c) $4a + 5b$ |
| (d) $3m - 2n$ | (e) $4x - 2y$ | (f) $a + b$ |
| (g) $x - 1$ | (h) $x^2 - 2$ | |
| (i) $\frac{4}{x - 3}$ | (j) $\frac{x + 2}{3}$ | (k) $\frac{x + 2y + 3z}{2x - 3y + z}$ |
| (l) $\frac{x + 2}{3x + 2}$ | (m) $\frac{x}{3x - 2}$ | (n) $\frac{x}{2x^2 - 1}$ |
| (o) $\frac{1}{2x - 1}$ | (p) $\frac{1}{3x}$ | (q) $\frac{3}{2}$ |
| (r) $\frac{m}{3}$ | (s) $\frac{x - 3}{x + 2}$ | (t) $-\frac{5}{3}$ |

2)

| | | |
|---------------------------|---------------------------|-----------------------------|
| (a) $\frac{x}{x + 1}$ | (b) $\frac{x + 2}{x + 3}$ | (c) $\frac{x}{x + 3}$ |
| (d) $\frac{x - 5}{x + 3}$ | (e) $\frac{x - 2}{x - 3}$ | (f) $\frac{x - 5}{x + 4}$ |
| (g) $x - 1$ | (h) $\frac{1}{x + 2}$ | (i) $\frac{x + 1}{2}$ |
| (j) $\frac{x + 2}{x + 3}$ | (k) $\frac{x - 5}{x - 4}$ | (l) $\frac{2x - 5}{3x - 1}$ |