

Simultaneous Equations - Lesson 5

Simultaneous Equations 5 (Intersections of Straight Lines)

LI

- Find the coordinates where straight lines cross using simultaneous equations.

SC

- Elimination.

Example 1

Find the coordinates of the point where the straight lines
 $4y = -5x - 2$ and $7y = 4x - 29$ intersect.

$$4y = -5x - 2$$

$$\underline{5x + 4y = -2}$$

$$7y = 4x - 29$$

$$\underline{4x - 7y = 29}$$

$$5x + 4y = -2 \quad (1) \quad \times 4$$

$$4x - 7y = 29 \quad (2) \quad \times 5$$

$$20x + 16y = -8 \quad (3)$$

$$20x - 35y = 145 \quad (4)$$

Signs of x are the same (+), so subtract: $(3) - (4)$

$$51y = -153$$

$$\Rightarrow \underline{y = -3}$$

Substitute $y = -3$ into (1) :

$$5x + 4y = -2$$

$$\therefore 5x + 4(-3) = -2$$

$$\Rightarrow 5x - 12 = -2$$

$$\Rightarrow 5x = 10$$

$$\Rightarrow \underline{x = 2}$$

$$\boxed{(2, -3)}$$

Example 2

Find the coordinates of the point where the straight lines
 $3x + 2y = 15$ and $21x - 3y = -14$ intersect.

$$3x + 2y = 15 \quad (1) \quad \times 3$$

$$21x - 3y = -14 \quad (2) \quad \times 2$$

$$9x + 6y = 45 \quad (3)$$

$$42x - 6y = -28 \quad (4)$$

Signs of y are different, so add: $(3) + (4)$

$$51x = 17$$

$$\Rightarrow \underline{x = 1/3}$$

Substitute $x = 1/3$ into (1) :

$$3x + 2y = 15$$

$$\therefore 3(1/3) + 2y = 15$$

$$\Rightarrow 1 + 2y = 15$$

$$\Rightarrow 2y = 14$$

$$\Rightarrow \underline{y = 7}$$

$$\boxed{(1/3, 7)}$$

When the straight line equations are given in the form $y = m x + c$, a simpler technique can be used (put the y 's equal to each other)

Example 3

Find the coordinates of the point where the straight lines $y = -x + 8$ and $y = 2x + 11$ intersect.

$$y = -x + 8 \quad \textcircled{1}$$

$$y = 2x + 11 \quad \textcircled{2}$$

$$\therefore 2x + 11 = -x + 8$$

$$\Rightarrow 3x + 11 = 8$$

$$\Rightarrow 3x = -3$$

$$\Rightarrow \underline{x = -1}$$

Substitute $x = -1$ into $\textcircled{1}$:

$$y = -x + 8$$

$$\therefore y = -(-1) + 8$$

$$\Rightarrow y = 1 + 8$$

$$\Rightarrow \underline{y = 9}$$

$$\boxed{(-1, 9)}$$

Questions

Find the coordinates of the point where these lines meet :

$$\begin{aligned} 1) \quad & 2x + 3y = 7 \\ & 3x + 2y = 8 \end{aligned}$$

$$\begin{aligned} 2) \quad & 5x - y = 1 \\ & 2x + y = 6 \end{aligned}$$

$$\begin{aligned} 3) \quad & 5x - 4y = 3 \\ & 2x - 5y = 8 \end{aligned}$$

$$\begin{aligned} 4) \quad & 10x + 3y = 4 \\ & 8x - 9y = -31 \end{aligned}$$

$$\begin{aligned} 5) \quad & y = 4x - 3 \\ & y = -2x + 9 \end{aligned}$$

$$\begin{aligned} 6) \quad & y = 3x + 11 \\ & y = -3x + 9 \end{aligned}$$

$$\begin{aligned} 7) \quad & y = 18x + 1 \\ & y = -x + 20 \end{aligned}$$

$$\begin{aligned} 8) \quad & y = 2000x + 53 \\ & y = -1000x + 3053 \end{aligned}$$

<i>Answers</i>	
1) $2x + 3y = 7$ $3x + 2y = 8$ (2, 1)	5) $y = 4x - 3$ $y = -2x + 9$ (2, 5)
2) $5x - y = 1$ $2x + y = 6$ (1, 4)	6) $y = 3x + 11$ $y = -3x + 9$ (-1/3, 10)
3) $5x - 4y = 3$ $2x - 5y = 8$ (-1, -2)	7) $y = 18x + 1$ $y = -x + 20$ (1, 19)
4) $10x + 3y = 4$ $8x - 9y = -31$ (-1/2, 3)	8) $y = 2\,000x + 53$ (1, 2\,053) $y = -1\,000x + 3\,053$