*Indices - Lesson 3*
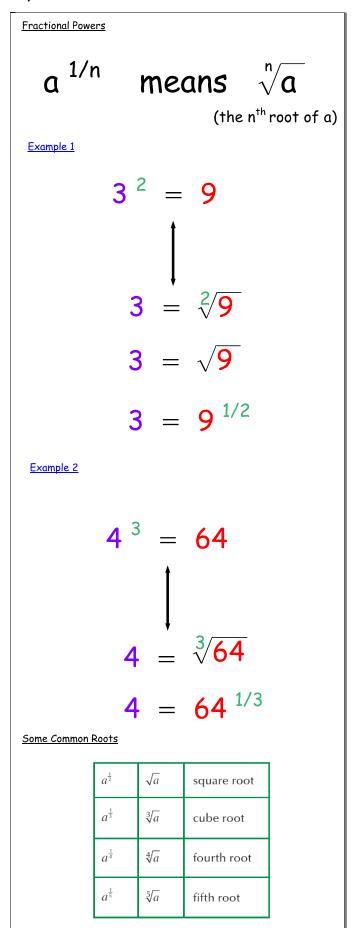
# Indices - Fractional Powers

## LI
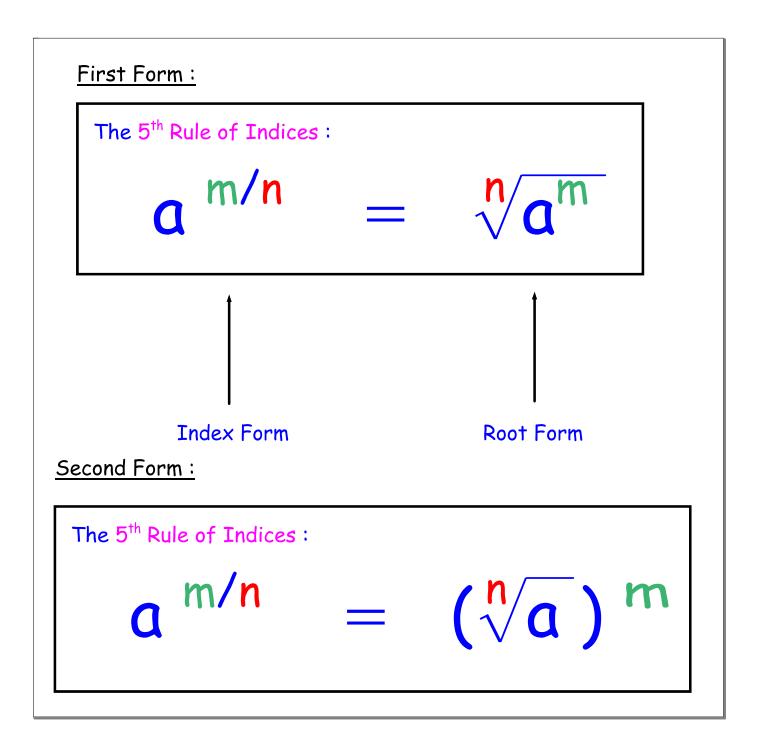
- Know how to work out fractional powers.

- Simplify expressions using fractional powers.

## SC

- Notation.

Fractional Powers

$$a^{1/n} \quad \text{means} \quad \sqrt[n]{a}$$

(the $n^{th}$ root of a)

Example 1

$$3^2 = 9$$

$$3 = \sqrt[2]{9}$$

$$3 = \sqrt{9}$$

$$3 = 9^{1/2}$$

Example 2

$$4^3 = 64$$

$$4 = \sqrt[3]{64}$$

$$4 = 64^{1/3}$$

Some Common Roots

| | | |
|---|---|---|
| $a^{\frac{1}{2}}$ | $\sqrt{a}$ | square root |
| $a^{\frac{1}{3}}$ | $\sqrt[3]{a}$ | cube root |
| $a^{\frac{1}{4}}$ | $\sqrt[4]{a}$ | fourth root |
| $a^{\frac{1}{5}}$ | $\sqrt[5]{a}$ | fifth root |

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First Form :

The 5<sup>th</sup> Rule of Indices :

$$a^{m/n} = \sqrt[n]{a^m}$$

Index Form            Root Form

Second Form :

The 5<sup>th</sup> Rule of Indices :

$$a^{m/n} = \left(\sqrt[n]{a}\right)^m$$

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## Example 3

Write these in root form :

(a)    $x^{3/5}$

$$= \boxed{\sqrt[5]{x^3}}$$

(b)    $p^{2/7}$

$$= \boxed{\sqrt[7]{p^2}}$$

(c)    $N^{13/11}$

$$= \boxed{\sqrt[11]{N^{13}}}$$

(d)    $f^{-7/9}$

$$= \frac{1}{f^{7/9}}$$

$$= \boxed{\frac{1}{\sqrt[9]{f^7}}}$$

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<u>Example 4</u>

Write these in index form :

(a)        $\sqrt[3]{b^4}$

    $=$    $\boxed{b^{4/3}}$

(b)        $\sqrt[8]{M^6}$

    $=$    $M^{6/8}$

    $=$    $\boxed{M^{3/4}}$

(c)        $\sqrt[6]{v^6}$

    $=$    $v^{6/6}$

    $=$    $v^1$

    $=$    $\boxed{v}$

(d)        $\dfrac{1}{\sqrt[17]{x^{15}}}$

    $=$    $\dfrac{1}{x^{15/17}}$

    $=$    $\boxed{x^{-15/17}}$

<u>Example 5</u>

Evaluate :

(a)     $49^{1/2}$

$= \sqrt{49}$

$= \boxed{7}$

(b)     $125^{-1/3}$

$= \dfrac{1}{125^{1/3}}$

$= \dfrac{1}{\sqrt[3]{125}}$

$= \boxed{\dfrac{1}{5}}$

(c)     $125^{2/3}$

$= \left(\sqrt[3]{125}\right)^2$

$= 5^2$

$= \boxed{25}$

(d)     $64^{-3/2}$

$= \dfrac{1}{64^{3/2}}$

$= \dfrac{1}{\left(\sqrt{64}\right)^3}$

$= \dfrac{1}{8^3}$

$= \boxed{\dfrac{1}{512}}$

1  Use the rules to express the following with root signs of the form $\sqrt[n]{a^m}$.

   a  $a^{\frac{1}{3}}$     b  $a^{\frac{1}{5}}$     c  $t^{\frac{1}{2}}$     d  $a^{\frac{2}{3}}$     e  $a^{\frac{3}{5}}$

   f  $t^{\frac{5}{2}}$     g  $x^{\frac{4}{3}}$     h  $y^{\frac{2}{5}}$     i  $p^{\frac{1}{4}}$     j  $m^{\frac{3}{4}}$

2  Write in index form.

   a  $\sqrt{t^5}$     b  $\sqrt[4]{a^3}$     c  $\sqrt[5]{x^3}$     d  $\sqrt[7]{m^4}$     e  $\sqrt[3]{a^{12}}$

3  Evaluate.

   a  $9^{\frac{1}{2}}$     b  $16^{\frac{1}{4}}$     c  $8^{\frac{2}{3}}$     d  $49^{\frac{3}{2}}$     e  $25^{-\frac{1}{2}}$

   f  $81^{-\frac{3}{4}}$     g  $100^{-\frac{3}{2}}$     h  $\left(\dfrac{1}{27}\right)^{\frac{2}{3}}$     i  $\left(\dfrac{49}{81}\right)^{\frac{1}{2}}$     j  $\left(\dfrac{16}{25}\right)^{\frac{3}{2}}$

# Answers

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **1** | **a** | $\sqrt[3]{a}$ | **2** | **a** | $t^{\frac{5}{2}}$ | **3** | **a** | 3 |
| | **b** | $\sqrt[5]{a}$ | | **b** | $a^{\frac{3}{4}}$ | | **b** | 2 |
| | **c** | $\sqrt{t}$ | | **c** | $x^{\frac{3}{5}}$ | | **c** | 4 |
| | **d** | $\sqrt[3]{a^2}$ | | **d** | $m^{\frac{4}{7}}$ | | **d** | 343 |
| | **e** | $\sqrt[5]{a^3}$ | | **e** | $a^{\frac{12}{3}} = a^4$ | | **e** | $\frac{1}{5}$ |
| | **f** | $\sqrt{t^5}$ | | | | | **f** | $\frac{1}{27}$ |
| | **g** | $\sqrt[3]{x^4}$ | | | | | **g** | $\frac{1}{1,000}$ |
| | **h** | $\sqrt[5]{y^2}$ | | | | | **h** | $\frac{1}{9}$ |
| | **i** | $\sqrt[4]{p}$ | | | | | **i** | $\frac{7}{9}$ |
| | **j** | $\sqrt[4]{m^3}$ | | | | | **j** | $\frac{64}{125}$ |