

Indices - Lesson 1

Indices - Multiplying and Dividing Rules

LI

- Know what an index (pl. indices) is.
- Know and use the Rules of Indices for \times and \div .

SC

- + and - numbers.

An **index** is a **power** (aka **exponent**)

$$2^3 = 2 \times 2 \times 2$$

$$3^4 = 3 \times 3 \times 3 \times 3$$

Rules of Indices

$$\begin{aligned}
 10^3 \times 10^2 &= (10 \times 10 \times 10) \times (10 \times 10) \\
 &= 1\,000 \times 100 \\
 &= 100\,000
 \end{aligned}$$

$$\therefore 10^3 \times 10^2 = 10^5$$

We thus have the 1st Rule of Indices :

$$a^m \times a^n = a^{m+n}$$

(m, n are any numbers)

Example 1

Simplify fully :

$$\begin{aligned}
 \text{(a)} \quad &3^4 \times 3^7 \\
 &= 3^{4+7} \\
 &= 3^{11}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad &w^{17} \times w^{-2} \\
 &= w^{17+(-2)} \\
 &= w^{15}
 \end{aligned}$$

Example 2

Simplify fully :

$$\begin{aligned}
 \text{(a)} \quad &5m^3 \times 3m^6 \\
 &= 15m^{3+6} \\
 &= 15m^9
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad &6w^{-3} \times 8w^{-9} \\
 &= 48w^{-3+(-9)} \\
 &= 48w^{-12}
 \end{aligned}$$

$$\begin{aligned}
 10^6 \div 10^2 &= (10 \times 10 \times 10 \times 10 \times 10 \times 10) \div (10 \times 10) \\
 &= 1\,000\,000 \div 100 \\
 &= 10\,000 \\
 \therefore 10^6 \div 10^2 &= 10^4
 \end{aligned}$$

We thus have the 2nd Rule of Indices :

$$a^m \div a^n = a^{m-n}$$

(m, n are any numbers)

Example 3

Simplify fully :

$$\begin{aligned}
 \text{(a)} \quad 5^9 \div 5^7 \\
 &= 5^{9-7} \\
 &= 5^2
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad f^{11} \div f^{-3} \\
 &= f^{11 - (-3)} \\
 &= f^{14}
 \end{aligned}$$

Example 4

Simplify fully :

$$\begin{aligned}
 \text{(a)} \quad 20 s^{30} \div 10 s^{20} \\
 &= 2 s^{30-20} \\
 &= 2 s^{10}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad 98 x^{14} \div 4 x^7 \\
 &= (98/4) x^{14-7} \\
 &= (49/2) x^7
 \end{aligned}$$

$$\begin{aligned}10^3 \div 10^3 &= (10 \times 10 \times 10) \div (10 \times 10 \times 10) \\ &= 1000 \div 1000 \\ &= 1\end{aligned}$$

But $10^3 \div 10^3 = 10^{3-3} = 10^0$ (using Rule 2)

We thus have the 3rd Rule of Indices :

$$a^0 = 1$$

Some Notation

$$a^{-m} = \frac{1}{a^m}$$

This is shorthand for this

Example 5

Simplify fully, expressing the answers with positive indices :

(a) $3D^3y^{-2} \times 4D^{-7}y^5$

$$= 12D^{-4}y^3$$

$$= 12 \times D^{-4} \times y^3$$

$$= 12 \times \frac{1}{D^4} \times y^3$$

$$= \frac{12y^3}{D^4}$$

(b) $\frac{8m^{13}a^6 \times 3m^{-9}a^2}{48m^4a^{87}}$

$$= \frac{24m^4a^8}{48m^4a^{87}}$$

$$= \frac{1}{2a^{79}}$$

1 Simplify these expressions. Write your answer in index form with a positive exponent.

a $4^5 \times 4^3$

b $7^4 \times 7$

c $x^{10} \times x^2$

d $t^2 \times t^3 \times t^4$

e $3^2 \times 3^{-7}$

f $c^3 \times c^{-9}$

g $a^8 \times a^{-8}$

h $4y^3 \times 5y^6$

i $c \times 4c^2 \times 2c^3$

j $8c^2 \times 3c^{-7}$

k $10a^7 \times 3a^{-20}$

l $4t^3 \times 3t^{-8} \times 2t^2$

2 Simplify these expressions leaving your answer in index form.

a $3^7 \div 3^2$

b $6 \div 6^3$

c $x^8 \div x^5$

d $t^3 \div t$

e $p^3 \div p^{-2}$

f $y^{-3} \div y^{-3}$

g $12y^{10} \div 3y^3$

h $24y^3 \div 12y^8$

i $15x^2 \div 3x^{-4}$

j $42p^6 \div (-7p)^{-2}$

k $\frac{4t^5 \times -7t^3}{14t^{-4}}$

l $\frac{5y^2 \times 4y^{-6}}{2y^3}$

3 Simplify these expressions.

a $3x^2y \times 5x^3y^2$

b $3a^2b^3 \times 7ab^4$

c $30x^3y \div 6x^2y^4$

Answers

1	a	4^8	2	a	3^5	3	a	$15x^5y^3$
	b	7^5		b	6^{-2}		b	$21a^3b^7$
	c	x^{12}		c	x^3		c	$5xy^{-3}$
	d	t^9		d	t^2			
	e	$\frac{1}{3^5}$		e	p^5			
	f	$\frac{1}{c^6}$		f	$y^0 = 1$			
	g	$a^0 = 1$		g	$4y^7$			
	h	$20y^9$		h	$2y^{-5}$			
	i	$8c^6$		i	$5x^6$			
	j	$\frac{24}{c^5}$		j	$2058p^8$			
	k	$\frac{30}{a^{13}}$		k	$-2t^{12}$			
	l	$\frac{24}{r^3}$		l	$10y^{-7}$			