

Surds - Lesson 5

Rationalising the Denominator 2

LI

- Rationalise more complicated denominators.

SC

- Conjugate Surds.
- Expanding brackets.

Rationalising the Denominator means to
write a fraction with the denominator
not involving a root

How to rationalise a complicated denominator

Multiply top and bottom of the original fraction
by the **conjugate surd of the denominator**
of the original fraction

Conjugate surd is obtained by changing
the sign of the root term

Surd	Conjugate Surd
$1 + \sqrt{5}$	$1 - \sqrt{5}$
$3 - \sqrt{7}$	$3 + \sqrt{7}$
$\sqrt{2} - 7$	$-\sqrt{2} - 7$

Difference of 2 Squares Applied to Conjugate Surds

$$(a - \sqrt{b})(a + \sqrt{b})$$

$$= a^2 - b$$

$$(3 - \sqrt{7})(3 + \sqrt{7})$$

$$= 9 - 7$$

$$= \underline{2}$$

Example 1

Rationalise :

$$\begin{aligned} & \frac{1}{2 - \sqrt{5}} \times (2 + \sqrt{5}) \\ & \frac{1}{2 - \sqrt{5}} \times (2 + \sqrt{5}) \\ = & \frac{2 + \sqrt{5}}{(2 - \sqrt{5})(2 + \sqrt{5})} \\ = & \frac{2 + \sqrt{5}}{4 - 5} \\ = & \frac{2 + \sqrt{5}}{-1} \\ = & \boxed{-2 - \sqrt{5}} \end{aligned}$$

Example 2

Rationalise :

$$\begin{aligned} & \frac{1}{4 + \sqrt{7}} \times (4 - \sqrt{7}) \\ & \frac{4 - \sqrt{7}}{(4 + \sqrt{7})(4 - \sqrt{7})} \\ & = \frac{4 - \sqrt{7}}{16 - 7} \\ & = \frac{4 - \sqrt{7}}{9} \end{aligned}$$

Example 3

Rationalise :

$$\frac{4}{6 + \sqrt{3}} \quad \begin{array}{l} \times (6 - \sqrt{3}) \\ \times (6 - \sqrt{3}) \end{array}$$

$$= \frac{4(6 - \sqrt{3})}{(6 + \sqrt{3})(6 - \sqrt{3})}$$

$$= \frac{4(6 - \sqrt{3})}{36 - 3}$$

$$= \frac{4(6 - \sqrt{3})}{33}$$

Example 4

Rationalise :

$$\frac{1 + \sqrt{5}}{2 - \sqrt{3}} \times (2 + \sqrt{3})$$

$$= \frac{(1 + \sqrt{5})(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}$$

$$= \frac{2 + \sqrt{3} + 2\sqrt{5} + \sqrt{5}\sqrt{3}}{4 - 3}$$

$$= 2 + \sqrt{3} + 2\sqrt{5} + \sqrt{15}$$

Rationalise the denominator.

a $\frac{1}{1 - \sqrt{6}}$

b $\frac{1}{1 + \sqrt{2}}$

c $\frac{1}{2 - \sqrt{3}}$

d $\frac{3}{1 - \sqrt{2}}$

e $\frac{7}{4 + \sqrt{3}}$

f $\frac{8}{\sqrt{6} - \sqrt{2}}$

g $\frac{\sqrt{3}}{\sqrt{3} - 6}$

h $\frac{1 - \sqrt{2}}{1 - \sqrt{3}}$

i $\frac{1 + \sqrt{3}}{6 + \sqrt{2}}$

j $\frac{5 - \sqrt{2}}{\sqrt{3} - 1}$

k $\frac{7}{2\sqrt{3} - 1}$

l $\frac{1}{3\sqrt{7} + 2}$

m $\frac{2\sqrt{5}}{2\sqrt{5} + 1}$

Answers

a $\frac{-(1 + \sqrt{6})}{5}$

b $-(1 - \sqrt{2})$

c $2 + \sqrt{3}$

d $-(3 + 3\sqrt{2})$

e $\frac{28 - 7\sqrt{3}}{13}$

f $2\sqrt{6} + 2\sqrt{2}$

g $\frac{-(1 + 2\sqrt{3})}{11}$

h $\frac{\sqrt{2} + \sqrt{6} - \sqrt{3} - 1}{2}$

i $\frac{6 - \sqrt{2} + 6\sqrt{3} - \sqrt{6}}{34}$

j $\frac{5\sqrt{3} + 5 - \sqrt{6} - \sqrt{2}}{2}$

k $\frac{7(2\sqrt{3} + 1)}{11}$

l $\frac{3\sqrt{7} - 2}{59}$

m $\frac{20 - 2\sqrt{5}}{19}$