

Advanced Trigonometry - Lesson 6

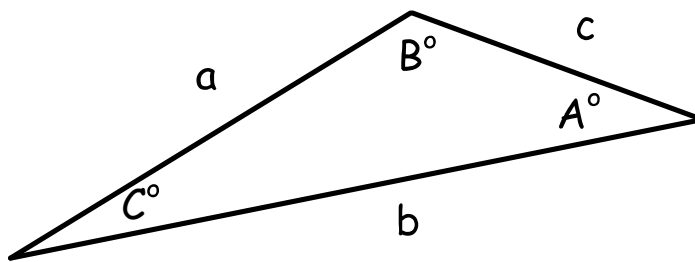
Area of a Triangle

LI

- Calculate the area of a triangle using trigonometry.

SC

- Use a calculator properly.

Area of a Triangle

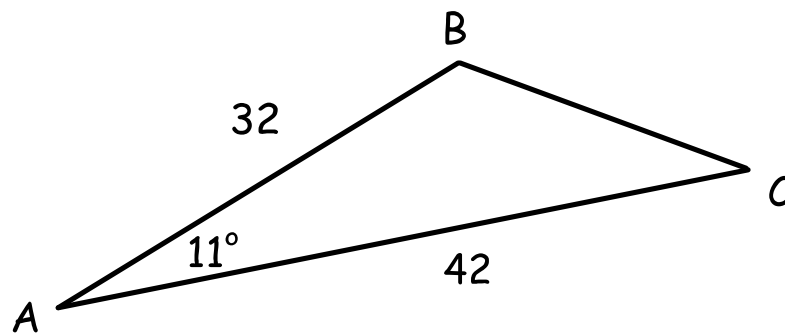
$$\text{AREA} = \frac{1}{2} a b \sin C^\circ$$

$$\text{AREA} = \frac{1}{2} b c \sin A^\circ$$

$$\text{AREA} = \frac{1}{2} c a \sin B^\circ$$

Example 1

Calculate the area of this triangle to 1 d.p. :



$$\begin{array}{l} A^\circ = 11^\circ, \quad a = \\ B^\circ = \quad, \quad b = 42 \\ C^\circ = \quad, \quad c = 32 \end{array}$$

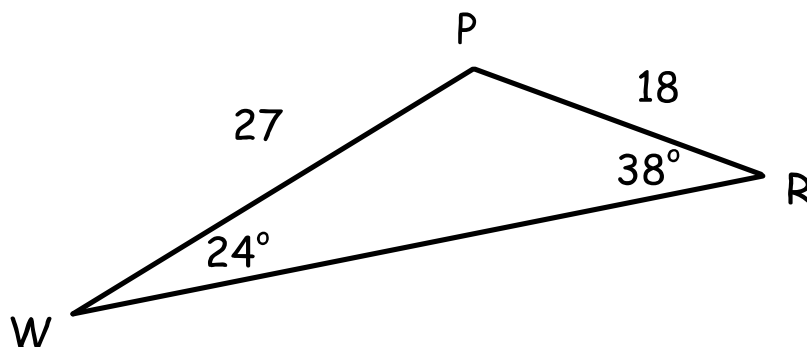
$$\text{AREA} = \frac{1}{2} b c \sin A^\circ$$

$$\text{AREA} = \frac{1}{2} \times 42 \times 32 \times \sin 11^\circ$$

$$\text{AREA} = 128.2 \text{ units}^2$$

Example 2

Calculate the area of this triangle to 1 d.p. :



$$W^\circ = 24^\circ, w = 18$$

$$R^\circ = 38^\circ, r = 27$$

$$P^\circ = 118^\circ, p =$$

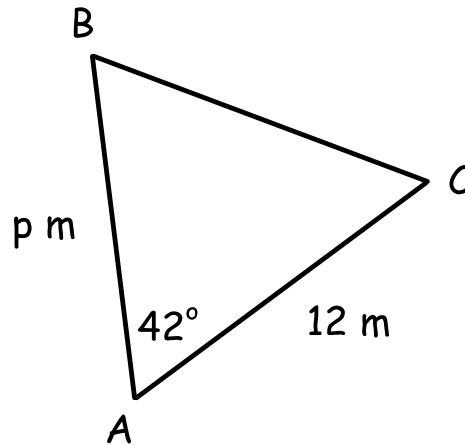
$$\text{AREA} = \frac{1}{2} r w \sin P^\circ$$

$$\text{AREA} = \frac{1}{2} \times 27 \times 18 \times \sin 118^\circ$$

$$\text{AREA} = 214.6 \text{ units}^2$$

Example 3

If the following triangle has an area of 52.2 m^2 , find the length AB (to the nearest metre):



$A^\circ = 42^\circ$,	$a =$
$B^\circ =$,	$b = 12$
$C^\circ =$,	$c = p$

$$\text{AREA} = \frac{1}{2} b c \sin A^\circ$$

$$52.2 = \frac{1}{2} \times 12 \times p \times \sin 42^\circ$$

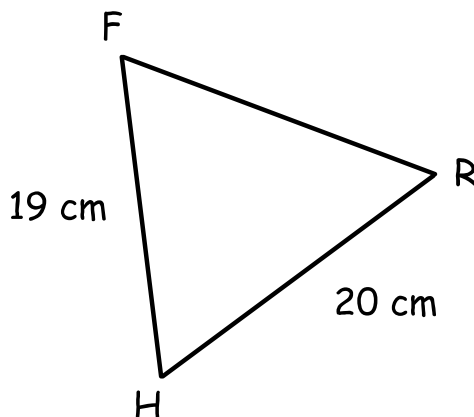
$$52.2 = (6 \sin 42^\circ) p$$

$$p = 52.2 / (6 \sin 42^\circ)$$

$$p = 13 \text{ m}$$

Example 4

If the following triangle has an area of 47 cm^2 , find the angle at H (to 1 d.p.):



$R^\circ =$,	$r = 19$
$F^\circ =$,	$f = 20$
$H^\circ =$,	$p =$

$$\text{AREA} = \frac{1}{2} f r \sin H^\circ$$

$$47 = \frac{1}{2} \times 20 \times 19 \times \sin H^\circ$$

$$47 = 190 \sin H^\circ$$

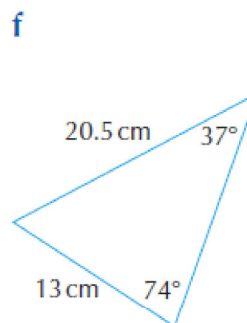
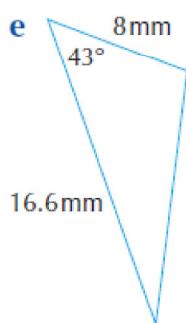
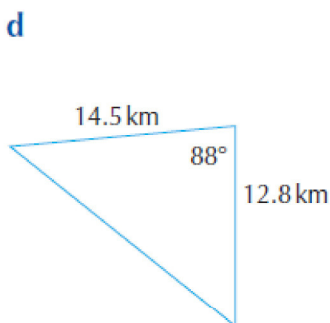
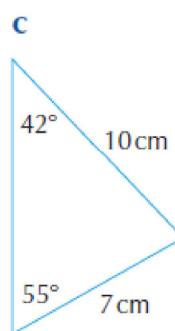
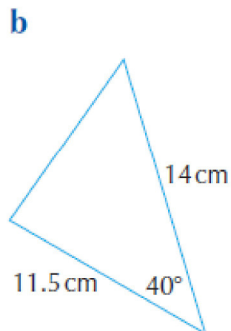
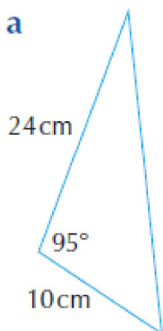
$$\sin H^\circ = 47/190$$

$$H^\circ = \sin^{-1}(47 \div 190)$$

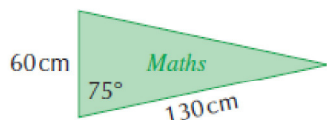
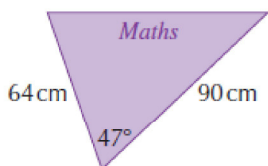
$$H^\circ = 14.3^\circ$$

Questions

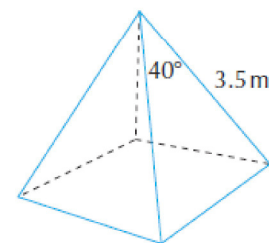
1 Use the area formula to calculate the area of each triangle giving your answers to 1 decimal place.



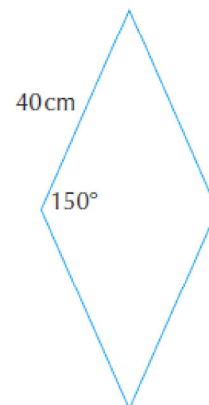
2 A banner for a maths club is being designed in the shape of a triangle. The cost of material for the banner is £3.50 per m^2 . Which of the two designs shown would be cheaper and by how much?



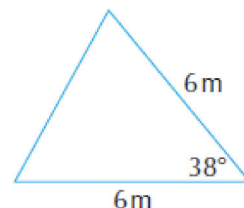
3 An artist has created a pyramid as part of an installation and has to paint all the triangular faces. Each tin of paint he buys will cover $8m^2$. How many tins of paint will he require to complete the job?



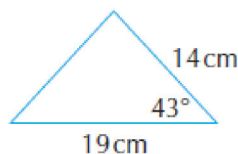
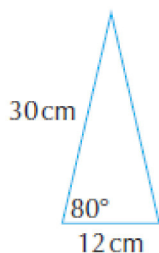
4 Find the area of the rhombus shown giving your answer to 3 significant figures.



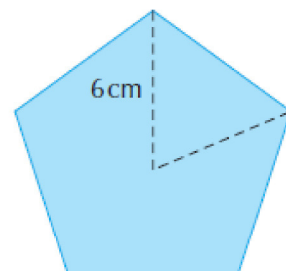
- 6 The sail for a boat is to be designed as shown.
Calculate the area of the sail to 3 significant figures.



- 7 A manufacturing company needs to make triangular tiles for a bathroom design. Two possible samples are shown below. Which one would be cheaper assuming both are made using the same material?



- 8 Calculate the area of the regular pentagon shown.



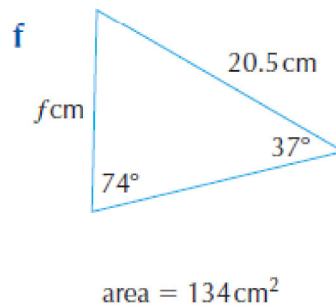
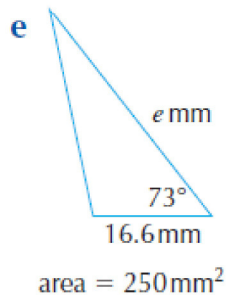
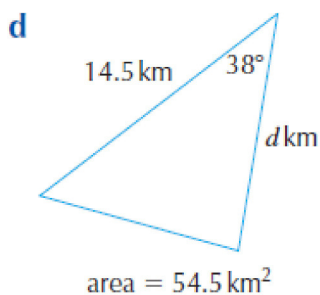
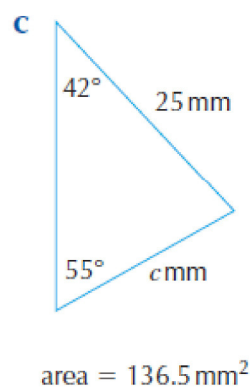
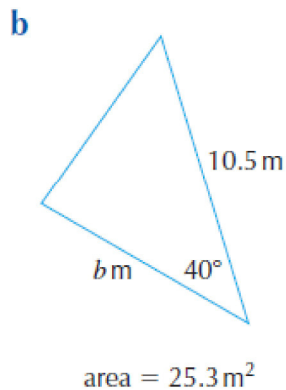
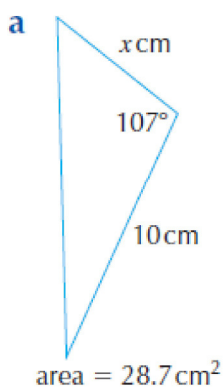
- 9 Using the same measurements from the centre to a vertex as in Question 8, calculate the area of:
- a a regular hexagon
 - b a regular octagon
 - c a regular decagon

Answers

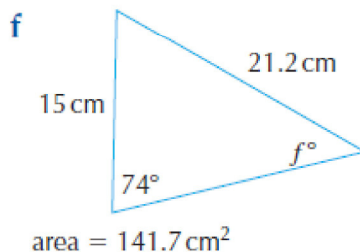
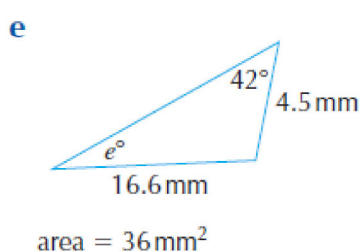
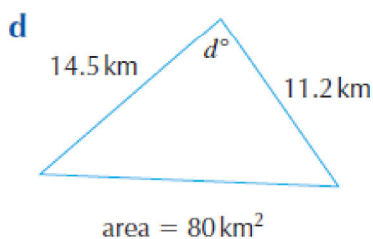
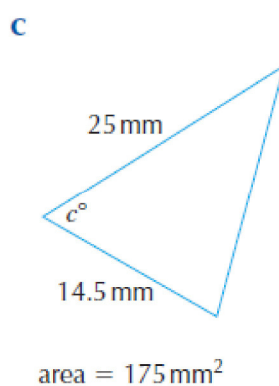
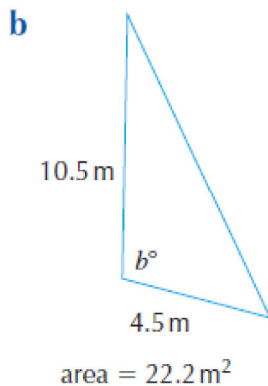
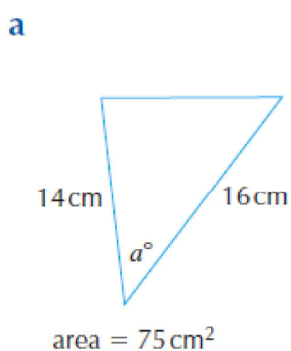
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|----------|----------------------------|--------------------|----------|----------------------------|
| 1 | a | 119.5cm^2 | 5 | 6.7km^2 |
| | b | 51.7cm^2 | 6 | 11.1m^2 |
| | c | 34.7cm^2 | 7 | The triangle on the right |
| | d | 92.7km^2 | 8 | 17.1cm^2 |
| | e | 45.3mm^2 | 9 | a 15.6cm^2 |
| | f | 124.4cm^2 | | b 12.7cm^2 |
| 2 | Purple flag cheaper by 58p | | | c 10.6cm^2 |
| 3 | 2 | | | |
| 4 | 800cm^2 | | | |

Questions

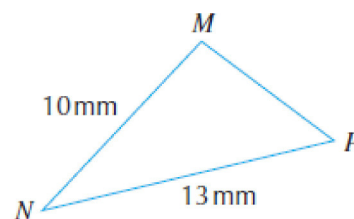
1 Calculate the missing side in each example shown giving your answers to 2 decimal places.



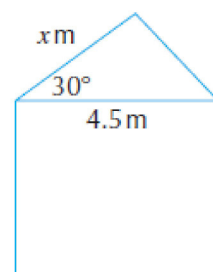
2 Calculate the missing angle in each triangle giving your answers to 1 decimal place.



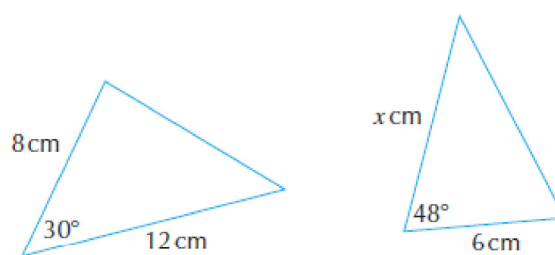
- 3 A badge is made in the shape of a triangle and has area 46 mm^2 . Calculate the size of angle MNP to the nearest degree.



- 4 The roof of a barn is made of a triangle as shown. Calculate the size of the missing side of the roof x if the cross-sectional area of the roof is 4.3 m^2 . Give your answer to 1 decimal place.



- 5 The two triangles have the same area. Calculate the length of the missing side shown giving your answer to 2 significant figures.



Answers

- | | | |
|------------------|----------------|------------|
| 1) (a) 6 . 00 cm | 2) (a) 42 . 0° | 3) 45° |
| (b) 7 . 50 cm | (b) 110 . 0° | 4) 3 . 8 m |
| (c) 11 . 00 cm | (c) 74 . 9° | 5) 11 cm |
| (d) 12 . 21 km | (d) 80 . 1° | |
| (e) 31 . 50 mm | (e) 63 . 5° | |
| (f) 14 . 00 cm | (f) 43 . 0° | |