Solving Quadratic Equations - Lesson 6

The Discriminant

LI

- Use the Discriminant to determine the number and nature of the roots.

SC

- Substitution.
- Solving simple equations.

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The Discriminant is a quantity that allows us to decide how many roots a quadratic has (and their nature)

Nature of Roots :

- Real
- Not real

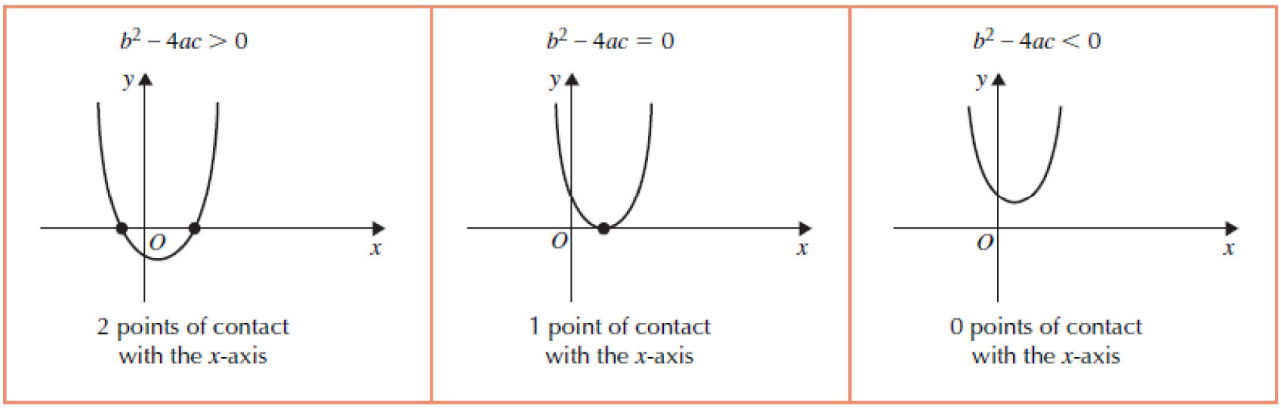
For the quadratic equation $a x^2 + b x + c = 0$, the Discriminant is the quantity :

$$D = b^2 - 4 a c$$

If $D = b^2 - 4 a c$ is :

- > 0 - 2 distinct, real roots
- $= 0$ - 1 (repeated) real root/equal roots
- < 0 - no real roots

Consider the graph $y = ax^2 + bx + c$, $a > 0$:



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Example 1

Find the number and nature of the roots of $3x^2 - 12x + 9 = 0$.

$$\boxed{\begin{aligned} a &= 3 \\ b &= -12 \\ c &= 9 \end{aligned}}$$

$$D = b^2 - 4ac$$

$$D = (-12)^2 - 4(3)(9)$$

$$D = 144 - 108$$

$$D = 36$$

As $D > 0$, there exist 2 distinct real roots

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Example 2

Show that the equation $-5x^2 + x - 2 = 0$ has no real roots.

$$\boxed{\begin{aligned} a &= -5 \\ b &= 1 \\ c &= -2 \end{aligned}}$$

$$D = b^2 - 4ac$$

$$D = (1)^2 - 4(-5)(-2)$$

$$D = 1 - 40$$

$$D = -39$$

As $D < 0$, there are no real roots

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Example 3

Find p so that the equation $-2x^2 - 6x + p = 0$ has equal roots.

$$\boxed{\begin{aligned} a &= -2 \\ b &= -6 \\ c &= p \end{aligned}}$$

$$D = b^2 - 4ac$$

$$D = (-6)^2 - 4(-2)(p)$$

$$D = 36 + 8p$$

For equal roots, $D = 0$:

$$36 + 8p = 0$$

$$8p = -36$$

$$p = -36/8$$

$$\boxed{p = -9/2}$$

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Example 4

Find k so that the equation $3x^2 + 6x + k = 0$ has real roots.

$$\boxed{\begin{aligned} a &= 3 \\ b &= 6 \\ c &= k \end{aligned}}$$

$$D = b^2 - 4ac$$

$$D = 6^2 - 4(3)(k)$$

$$D = 36 - 12k$$

For real roots, $D \geq 0$:

$$36 - 12k \geq 0$$

$$-12k \geq -36$$

$$\boxed{k \leq 3}$$

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Questions

1 By calculating the value of the discriminant, determine the nature of the roots of the following quadratic equations.

a $x^2 + 6x + 9 = 0$ **b** $3x^2 - 4x + 2 = 0$ **c** $x^2 + 5x - 1 = 0$

d $5x^2 + 4x + 9 = 0$ **e** $4 - 2x - x^2 = 0$ **f** $9x^2 - 6x + 1 = 0$

2 How many points of contact do graphs of the following quadratic functions have with the x-axis?

a $y = 3x^2 + 4x$ **b** $y = x^2 + \frac{1}{2}x + \frac{1}{4}$ **c** $y = 2x^2 - 9$

d $y = x^2 + 4x + 6$ **e** $y = 3x - x^2$ **f** $y = 5x^2 - 4x - 3$

3 By first rearranging each quadratic equation to its standard form, determine the nature of the roots of the following.

a $x(x + 3) = 2x - 3$ **b** $(x - 2)^2 - 3 = 0$ **c** $4(2x - 4) = x^2$

d $(2x - 1)^2 - 3(x + 1) = 0$ **e** $x(4x + 3) + 6 = 5 - x$ **f** $(x + 3)^2 - 4x - 8 = 0$

4 Show that the equation $x^2 + 3x + 5 = 0$ has no real roots.

5 Show that the equation $x(x - 2) = 7 + 2x$ has real roots.

6 Find the range of values of k for which the equation $2x^2 + 4x + k = 0$ has real roots.

7 Find the range of values of p for which the equation $px^2 - 2x + 1 = 0$ has no real roots.

8 Find the values of k for which $kx^2 - 10x + k = 0$ has equal roots.

9 Show that the graphs $y = x^2 - 3x + 2$ and $y = 2x - 9$ do not intersect.

10 For what values of k do the following equations have equal roots?

a $x^2 + (k - 5)x + 4k = 0$ **b** $x^2 = k(2x - 5)$

Answers

1 **a** $a = 1,\ b = 6,\ c = 9$
$b^2 - 4ac = 0$: equal roots

b $a = 3,\ b = -4,\ c = 2$
$b^2 - 4ac = -8$: no real roots

c $a = 1,\ b = 5,\ c = -1$
$b^2 - 4ac = 29$: two real roots

d $a = 5,\ b = 4,\ c = 9$
$b^2 - 4ac = -164$: no real roots

e $a = -1,\ b = -2,\ c = 4$
$b^2 - 4ac = 20$: two real roots

f $a = 9,\ b = -6,\ c = 1$
$b^2 - 4ac = 0$: equal roots

2 **a** $a = 3,\ b = 4,\ c = 0$
$b^2 - 4ac = 16$: two points of contact

b $a = 1,\ b = \frac{1}{2},\ c = \frac{1}{4}$
$b^2 - 4ac = -\frac{3}{4}$: no points of contact

c $a = 2,\ b = 0,\ c = -9$
$b^2 - 4ac = 72$: two points of contact

d $a = 1,\ b = 4,\ c = 6$
$b^2 - 4ac = -8$: no points of contact

e $a = -1,\ b = 3,\ c = 0$
$b^2 - 4ac = 9$: two points of contact

f $a = 5,\ b = -4,\ c = -3$
$b^2 - 4ac = 76$: two points of contact

3 **a** $a = 1,\ b = 1,\ c = 3$
$b^2 - 4ac = -11$: no real roots

b $a = 1,\ b = -4,\ c = 1$
$b^2 - 4ac = 12$: two real roots

c $a = 1,\ b = -8,\ c = 16$
$b^2 - 4ac = 0$: two equal roots

d $a = 4,\ b = -7,\ c = -2$
$b^2 - 4ac = 81$: two real roots

e $a = 4,\ b = 4,\ c = 1$
$b^2 - 4ac = 0$: two equal roots

f $a = 1,\ b = 2,\ c = 1$
$b^2 - 4ac = 0$: two equal roots

4 $a = 1,\ b = 3,\ c = 5$
$b^2 - 4ac = -11$: no real roots

5 $a = 1,\ b = -4,\ c = -7$
$b^2 - 4ac = 44$: two real roots

6 $k \leq 2$

7 $p > 1$

8 $k = \pm 5$