Solving Quadratic Equations - Lesson 5

Solving Quadratic Equations in Context

LI

- Solve quadratic equations in real-life situations.

SC

- Make a quadratic equation from a contextual situation.
- Solve quadratics.

Jan 10-12:11

Quadratic Expression :

$$ax^2 + bx + c$$

quadratic (x^2) term

x term

constant term

Quadratic Equation :

$$ax^2 + bx + c = 0$$

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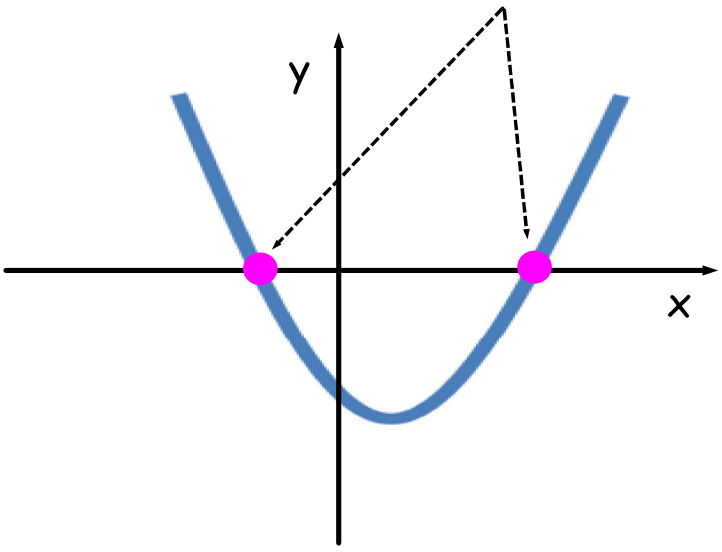
To solve a quadratic equation means to find out which x-values fit the equation

Graphical Interpretation

Roots (aka solutions)



Solving a quadratic equation means finding where the graph crosses the x-axis

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Example 1

The number of diagonals, d, in an n - sided polygon is given by,

$$d = \frac{n(n-3)}{2}$$

If a polygon has 35 diagonals, how many sides does it have ?

$$d = 35$$

$$d = \frac{n(n-3)}{2}$$

$$\frac{n(n-3)}{2} = 35$$

$$n(n-3) = 70$$

$$n^2 - 3n = 70$$

$$n^2 - 3n - 70 = 0$$

$$(n-10)(n+7) = 0$$

$$n = 10, \quad n = -7$$

As $n > 0$,

$$n = 10 \text{ sides}$$

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Example 2

A rectangle has sides of length $(x + 3)$ cm and $(2x - 1)$ cm.

If the area of the rectangle is $72\ cm^2$, find x and the dimensions of the rectangle.

$$A = L\,B$$

$$72 = (x + 3)(2x - 1)$$

$$(x + 3)(2x - 1) = 72$$

$$2x^2 + 6x - x - 3 = 72$$

$$2x^2 + 5x - 75 = 0$$

$$(2x + 15)(x - 5) = 0$$

$$x = -15/2, \quad x = 5$$

As $x > 0$,

$$x = 5 \text{ cm}$$

$$x + 3 = 5 + 3 = 8 \text{ cm}$$

$$2x - 1 = 2(5) - 1 = 9 \text{ cm}$$

Dimensions : 8 cm by 9 cm

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Questions

1 The total number of games, n, in a competition where each team plays every other team twice is given by:

$$n = t^2 - t$$

where t is the number of teams entered.

If the total number of games played in the competition was 110, how many teams were entered?

2 The distance, x metres, travelled by a model train is given by the formula:

$$x = t^2 - 4t + 1$$

where t is the time in seconds.

How many seconds does it take for the train to travel 22 metres?

3 The number of handshakes, h, in a room of n people, if each pair shakes hands once, is given by the formula:

$$h = \tfrac{1}{2}n(n-1), n > 1$$

If there are 36 handshakes, how many people are in the room?

4 John is 6 years older than Jim. The product of their ages is 135.

Let John's age = x years.

a Find an expression for Jim's age in terms of x.

b Find their respective ages algebraically.

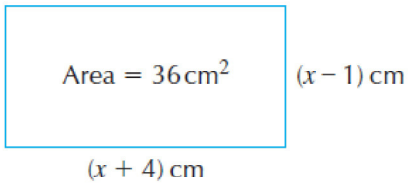
5 Sarah is 4 years younger than Dave. The product of their ages is 320.

Let Sarah's age = x years.

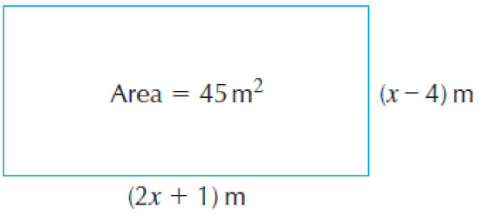
Find their respective ages algebraically.

6 The dimensions and areas of the following rectangles are given. In each case find x.

a

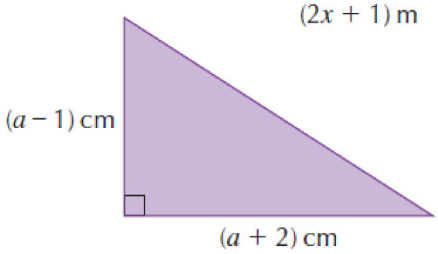


b



7 The area of this triangle is 14 cm².

Find the value of a.



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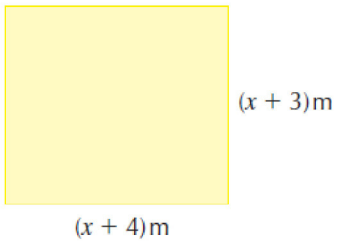
8 The areas of the following rectangles are equal.



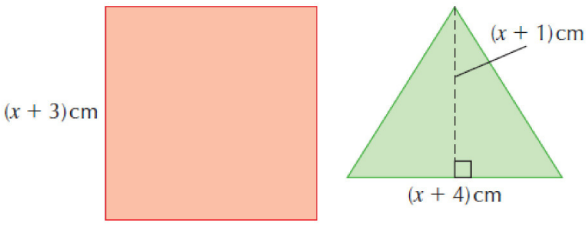
a Find the value of x.

b Calculate the area of one of the rectangles.

9 The area of the square is 37 cm² bigger than the area of the triangle.

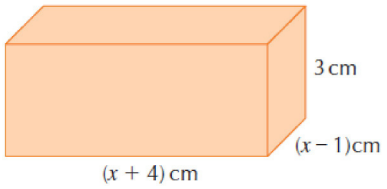
a Find the value of x.

b Find the area of each shape.



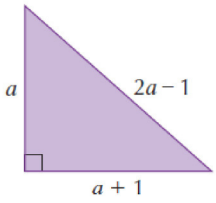
10 The volume of the following cuboid is 42 cm^3.

Find the value of x.

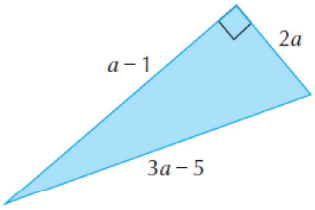


11 By using Pythagoras' theorem, find the value of a in each of the following right-angled triangles.

a

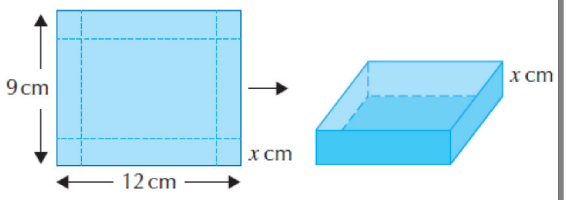


b



12 A rectangle measuring 12 cm × 9 cm has four squares, each of length x cm, cut from its corners. The sides are then folded up to create a box of height x cm.

a The length l of the base of the box, in cm, is given by $l = 12 - 2x$. Find a similar expression for the breadth, b.



The area of the base of the box is 54 cm^2.

b Show that $2x^2 - 21x + 27 = 0$.

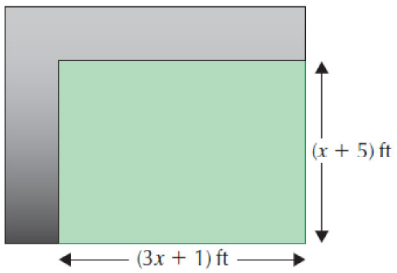
c Calculate the volume of the box.

13 A plan of Dave's garden is shown. The path is 2 ft wide.

a If the area of the lawn is twice the area of the path, show that $x^2 - 9 = 0$.

b Find the area of the lawn.

c Dave wants to re-pave the path at a cost of £3.50 per square foot. How much does the path cost?



14 A school currently has a rectangular playing field measuring 150 metres by 100 metres. Owing to an extension to the main building, plans have been made to reduce both the length and the breadth by x metres.

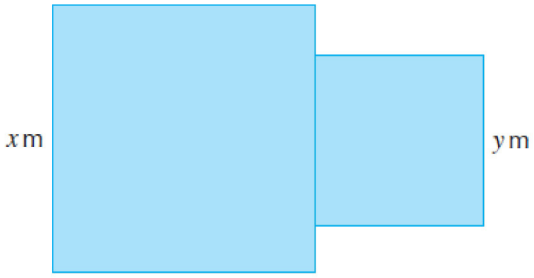
The new playing field is 75% of the area of the original.

a Show that $x^2 - 250x + 3750 = 0$.

b Find the dimensions of the new playing field, correct to the nearest metre.

15 A pond is made from two squares with lengths x metres and y metres respectively.

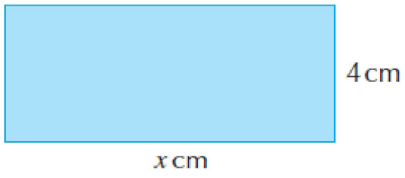
It has an area of $52\,m^2$ and a perimeter of 32 m.

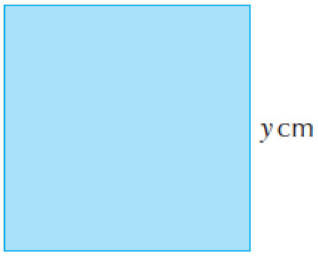


a Form two equations using the above information.

b Hence, find the values of x and y.

16 The perimeter of the following rectangle is equal to the perimeter of the square.





a Show that $x = 2y - 4$.

b Write an expression for the area of the rectangle in terms of y.

c The area of the square is $9\,cm^2$ bigger than the area of the rectangle.

Show that $y^2 - 8y + 7 = 0$.

d Find the dimensions of the square and the rectangle.

Answers

1 $110 = t^2 - t$

$(t - 11)(t + 10) = 0$

Since t is > 0, $t = 11$ teams

2 $22 = t^2 - 4t + 1$

$(t - 7)(t + 3) = 0$

since $t > 0$, $t = 7$ seconds

3 $36 = \frac{1}{2}n(n - 1)$

$(n - 9)(n + 8) = 0$

since $n > 1$, $n = 9$ people

4 **a** $x = y + 6$

$y = x - 6$, where y is Jim's age

b $(x - 6)x = 135$

$(x - 15)(x + 9) = 0$

since $x > 0$, $x = 15$ years and $y = 9$

5 Sarah's age $= x = 16$ years and so Dave's age is 20 years

6 **a** $x = 5$

b $x = 7$

7 $a = 5$

8 **a** $x = 5$

b area $= 72\ \text{m}^2$

9 **a** $x = 5$

b area of square $= 64\ \text{cm}^2$ and area of triangle $= 27\ \text{cm}^2$

10 $x = 3$

11 **a** $a = 3$

b $a = 6$

12 a $b = 9 - 2x$

b $(12 - 2x)(9 - 2x) = 54$

$4x^2 - 42x + 54 = 0$

$2x^2 - 21x + 27 = 0$

$x = 9$ or $x = \frac{3}{2}$ $x < 9$ so $x = \frac{3}{2}$

c $V = (12 - 2x)(9 - 2x)x$

$= 81\,\text{cm}^3$

13 a $(3x + 1)(x + 5) = 2\,[2(x + 5 + 2) + 2(3x + 1)]$

$3x^2 + 16x + 5 = 2(8x + 16)$

$3x^2 + 16x + 5 = 16x + 32$

$3x^2 - 27 = 0$

$x^2 - 9 = 0$

b $x^2 - 9 = 0$

so $x = 3$

area of lawn $= (3x + 1)(x + 5)$

$= 10 \times 8 = 80\,\text{ft}^2$

c area of path $= 40\,\text{ft}^2$

$40 \times £3.50 = £140$

14 a $A_1 = 150 \times 100$

$A_2 = (150 - x)(100 - x)$

$A_2 = 0.75 \times A_1$

$0.75 \times 150 \times 100 = (150 - x)(100 - x)$

$11250 = 15000 - 250x + x^2$

$x^2 - 250x + 3750 = 0$

b $x = 16.028\text{m}$

New dimensions are

$(150 - 16) \times (100 - 16)$

$= 134\text{m} \times 84\text{m}$

15 a $2x + y = 16$ and $x^2 + y^2 = 52$

b $x = 6\text{m}$, $y = 4\text{m}$

16 a $2x + 8 = 4y$

$x = 2y - 4$

b area of rectangle $= 4x = 4(2y - 4)$

$= 8y - 16$

c $y^2 = 4(2y - 4) + 9$

$y^2 - 8y + 7 = 0$

d $(y - 7)(y - 1) = 0$

$y = 1$ or $y = 7$

use $y = 7$ because $y = 1$ gives negative value for x

$y = 7$ gives $x = 10$

rectangle $= 10\,\text{cm} \times 4\,\text{cm}$

square $= 7\,\text{cm} \times 7\,\text{cm}$

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