

Solving Quadratic Equations - Lesson 1

Solving Quadratic Equations
(by Factorisation)

LI

- Solve quadratic equations by factorising.

SC

- Common factor, difference of two squares, quadratic trinomial.
- Solve simple (i.e. linear) equations.

Quadratic Expression :

$$ax^2 + bx + c$$

quadratic (x^2) term

x term

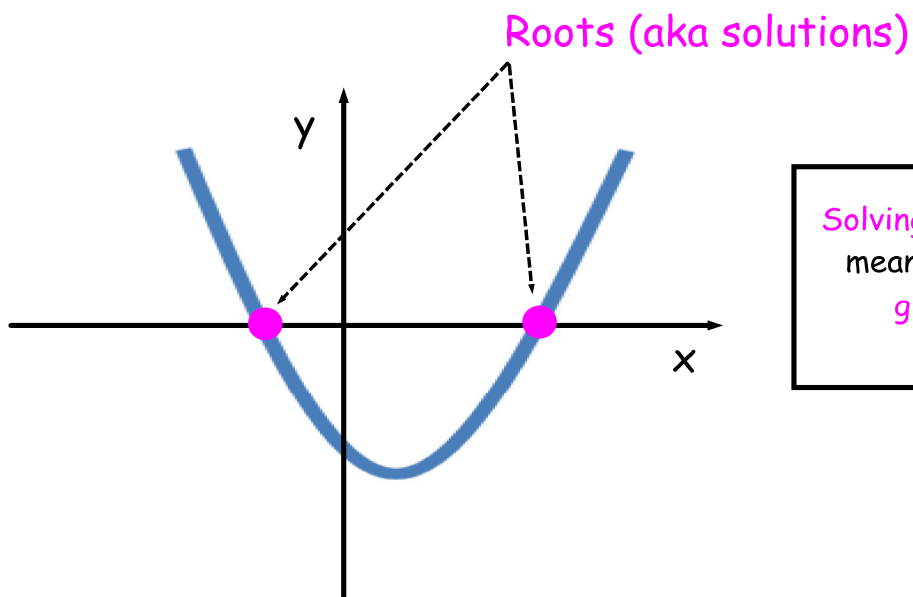
constant term

Quadratic Equation :

$$ax^2 + bx + c = 0$$

To **solve** a quadratic equation means
to find out which **x-values**
fit the equation

Graphical Interpretation



Solving a quadratic equation
means finding where the
graph crosses the
x-axis

If the product of 2 real numbers equals 0, then at least one of them must be 0

Solving Quadratics by Common Factor

Example 1

Solve $x(x - 3) = 0$ for x .

$$\begin{array}{c}
 x(x - 3) = 0 \\
 \swarrow \quad \searrow \\
 x = 0, \quad x - 3 = 0 \\
 \boxed{x = 0, \quad x = 3}
 \end{array}$$

Example 2

Find the roots of $x^2 + 4x = 0$.

$$\begin{array}{c}
 x^2 + 4x = 0 \\
 x(x + 4) = 0 \\
 \swarrow \quad \searrow \\
 x = 0, \quad x + 4 = 0 \\
 \boxed{x = 0, \quad x = -4}
 \end{array}$$

Example 3

Solve algebraically $14p - 6p^2 = 0$.


$$\begin{array}{c}
 14p - 6p^2 = 0 \\
 2p(7 - 3p) = 0 \\
 \swarrow \quad \searrow \\
 2p = 0, \quad 7 - 3p = 0 \\
 p = 0, \quad 3p = 7 \\
 \boxed{p = 0, \quad p = 7/3}
 \end{array}$$

Solving Quadratics by Difference of 2 SquaresExample 4

Obtain the roots of $4x^2 - 9 = 0$.

$$4x^2 - 9 = 0$$

$$(2x - 3)(2x + 3) = 0$$


$$2x - 3 = 0, \quad 2x + 3 = 0$$

$$2x = 3, \quad 2x = -3$$


$$x = 3/2, \quad x = -3/2$$

Example 5

Solve algebraically $121 - 9y^2 = 0$.

$$121 - 9y^2 = 0$$

$$(11 - 3y)(11 + 3y) = 0$$


$$11 - 3y = 0, \quad 11 + 3y = 0$$

$$3y = 11, \quad 3y = -11$$

$$y = 11/3, \quad y = -11/3$$

Solving Quadratics by Factorising TrinomialsExample 6

Solve $x^2 + 5x - 24 = 0$.

$$x^2 + 5x - 24 = 0$$

$$(x - 3)(x + 8) = 0$$

$$\begin{array}{ccc} & \swarrow & \searrow \\ x - 3 = 0, & & x + 8 = 0 \end{array}$$

$$x = 3, x = -8$$

Example 7

Find the roots of $2x^2 + 3x - 5 = 0$.

$$2x^2 + 3x - 5 = 0$$

$$(2x + 5)(x - 1) = 0$$

$$\begin{array}{ccc} & \swarrow & \searrow \\ 2x + 5 = 0, & & x - 1 = 0 \end{array}$$

$$x = -5/2, x = 1$$

Questions

1 Solve the following equations algebraically.

a $(x - 4)(x - 2) = 0$ **b** $3x(x + 4) = 0$ **c** $(2x - 3)(x + 2) = 0$

d $(2x + 3)(2x + 3) = 0$ **e** $x(2x + 7) = 0$ **f** $(2x - 1)(x + 7) = 0$

g $4x(5 - 3x) = 0$ **h** $(x + 1)(3x - 2) = 0$ **i** $12x(4 + 3x) = 0$

2 Find the roots of the following equations.

a $4x^2 - x = 0$ **b** $6x^2 + 9x = 0$ **c** $15x - 25x^2 = 0$ **d** $4x^2 - 10x = 0$

e $5x^2 - 5x = 0$ **f** $16x - 4x^2 = 0$ **g** $11x + x^2 = 0$ **h** $4x - 6x^2 = 0$

3 Solve the following quadratic equations algebraically.

a $4x^2 - 9 = 0$ **b** $25p^2 - 16 = 0$ **c** $4 - m^2 = 0$ **d** $x^2 - 81 = 0$

e $x^2 - 49 = 0$ **f** $9x^2 - 100 = 0$ **g** $121 - 81q^2 = 0$ **h** $64 - 4t^2 = 0$

4 Find the solutions to the following equations.

a $x^2 + 8x + 15 = 0$ **b** $t^2 - 4t + 3 = 0$ **c** $x^2 - 3x - 10 = 0$

d $x^2 - 5x + 6 = 0$ **e** $x^2 - 8x - 20 = 0$ **f** $z^2 + 14z + 45 = 0$

g $y^2 + 4y - 12 = 0$ **h** $w^2 + w - 6 = 0$ **i** $r^2 + 5r - 14 = 0$

5 Solve the following equations algebraically.

a $2r^2 + 3r + 1 = 0$ **b** $-t^2 + 7t - 12 = 0$ **c** $3s^2 - 4s - 4 = 0$

d $-2p^2 - 7p - 3 = 0$ **e** $3w^2 + 5w - 12 = 0$ **f** $-6x^2 + 31x - 5 = 0$

g $-12x^2 + 24x - 12 = 0$ **h** $2m^2 + 7m - 15 = 0$ **i** $5p^2 + 13p - 18 = 0$

6 Find the roots of the following quadratic equations.

a $p^2 + 4p = 0$ **b** $x^2 + 14x + 49 = 0$ **c** $2x^2 - 3x - 5 = 0$

d $36 - p^2 = 0$ **e** $18m + 12m^2 = 0$ **f** $-5x^2 - 38x - 21 = 0$

g $8x^2 - 50 = 0$ **h** $-6x^2 + 22x + 40 = 0$ **i** $98 - 32m^2 = 0$

j $6a^2 - 33a + 15 = 0$ **k** $12x^2 - 75 = 0$ **l** $5x^2 + 35x + 60 = 0$

Answers

1

a $x = 4$ or $x = 2$

b $x = 0$ or $x = -4$

c $x = \frac{3}{2}$ or $x = -2$

d $x = -\frac{3}{2}$

e $x = 0$ or $x = -\frac{7}{2}$

f $x = \frac{1}{2}$ or $x = -7$

g $x = 0$ or $x = \frac{5}{3}$

h $x = -1$ or $x = \frac{2}{3}$

i $x = 0$ or $x = -\frac{4}{3}$

2

a $x(4x - 1) = 0$
 $x = 0$ or $x = \frac{1}{4}$

b $3x(2x + 3) = 0$
 $x = 0$ or $x = -\frac{3}{2}$

c $5x(3 - 5x) = 0$
 $x = 0$ or $x = \frac{3}{5}$

d $2x(2x - 5) = 0$
 $x = 0$ or $x = \frac{5}{2}$

e $5x(x - 1) = 0$
 $x = 0$ or $x = 1$

f $4x(4 - x) = 0$
 $x = 0$ or $x = 4$

g $x(11 + x) = 0$
 $x = 0$ or $x = -11$

h $2x(2 - 3x) = 0$
 $x = 0$ or $x = \frac{2}{3}$

3

a $(2x + 3)(2x - 3) = 0$
 $x = -\frac{3}{2}$ or $x = \frac{3}{2}$

b $(5p - 4)(5p + 4) = 0$
 $p = \frac{4}{5}$ or $p = -\frac{4}{5}$

c $(2 - m)(2 + m) = 0$
 $m = 2$ or $m = -2$

d $(x - 9)(x + 9) = 0$
 $x = 9$ or $x = -9$

e $(x - 7)(x + 7) = 0$
 $x = 7$ or $x = -7$

f $(3x - 10)(3x + 10) = 0$
 $x = \frac{10}{3}$ or $x = -\frac{10}{3}$

g $(11 - 9q)(11 + 9q) = 0$
 $q = \frac{11}{9}$ or $q = -\frac{11}{9}$

h $(8 - 2t)(8 + 2t) = 0$
 $t = 4$ or $t = -4$

Answers

<p>4 a $(x + 5)(x + 3) = 0$ $x = -5$ or $x = -3$</p> <p>b $(t - 3)(t - 1) = 0$ $t = 3$ or $t = 1$</p> <p>c $(x - 5)(x + 2) = 0$ $x = 5$ or $x = -2$</p> <p>d $(x - 3)(x - 2) = 0$ $x = 3$ or $x = 2$</p> <p>e $(x - 10)(x + 2) = 0$ $x = 10$ or $x = -2$</p> <p>f $(z + 9)(z + 5) = 0$ $z = -9$ or $z = -5$</p> <p>g $(y + 6)(y - 2) = 0$ $y = -6$ or $y = 2$</p> <p>h $(w + 3)(w - 2) = 0$ $w = -3$ or $w = 2$</p> <p>i $(r + 7)(r - 2) = 0$ $r = -7$ or $r = 2$</p>	<p>5 a $(2r + 1)(r + 1) = 0$ $r = -\frac{1}{2}$ or $r = -1$</p> <p>b $-(t - 3)(t - 4) = 0$ $t = 3$ or $t = 4$</p> <p>c $(3s + 2)(s - 2) = 0$ $s = -\frac{2}{3}$ or $s = 2$</p> <p>d $-(p + 3)(2p + 1) = 0$ $p = -3$ or $p = -\frac{1}{2}$</p> <p>e $(3w - 4)(w + 3) = 0$ $w = \frac{4}{3}$ or $w = -3$</p> <p>f $-(x - 5)(6x - 1) = 0$ $x = 5$ or $x = \frac{1}{6}$</p> <p>g $-(12x^2 - 24x - 12) = 0$ $-(x - 1)(x - 1) = 0$ $x = 1$</p> <p>h $(2m - 3)(m + 5) = 0$ $m = \frac{3}{2}$ or $m = -5$</p> <p>i $(p - 1)(5p + 18) = 0$ $p = 1$ or $p = -\frac{18}{5}$</p>	<p>6 a $p(p + 4) = 0$ $p = 0$ or $p = -4$</p> <p>b $(x + 7)(x + 7) = 0$ $x = -7$</p> <p>c $(x + 1)(2x - 5) = 0$ $x = -1$ or $x = \frac{5}{2}$</p> <p>d $(6 + p)(6 - p) = 0$ $p = -6$ or $p = 6$</p> <p>e $6m(2m + 3) = 0$ $m = 0$ or $m = -\frac{3}{2}$</p> <p>f $-(x + 7)(5x + 3) = 0$ $x = -7$ or $x = -\frac{3}{5}$</p> <p>g $2(2x - 5)(2x + 5) = 0$ $x = \frac{5}{2}$ or $x = -\frac{5}{2}$</p> <p>h $-2(x - 5)(3x + 4) = 0$ $x = 5$ or $x = -\frac{4}{3}$</p> <p>i $-2(4m - 7)(4m + 7) = 0$ $m = \frac{7}{4}$ or $m = -\frac{7}{4}$</p> <p>j $3(a - 5)(2a - 1)$ $a = 5$ or $a = \frac{1}{2}$</p> <p>k $3(2x - 5)(2x + 5)$ $x = \frac{5}{2}$ or $x = -\frac{5}{2}$</p> <p>l $5(x + 3)(x + 4)$ $x = -3$ or $x = -4$</p>
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