Trig Equations

3. Rearranging

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T o

Solve trig equations which involve rearranging

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F or

Rearrange equation

Use calculator to find one solution

Use symmetry of trig graphs to find other solutions

OR

Use CAST to find other solutions

Example 1

Solve $2\sin x^\circ - 1 = 0, \quad 0 \le x \le 360$

$$2\sin x^\circ = 1$$

$$\sin x^\circ = 0.5$$

$$x = \sin^{-1}(0.5)$$

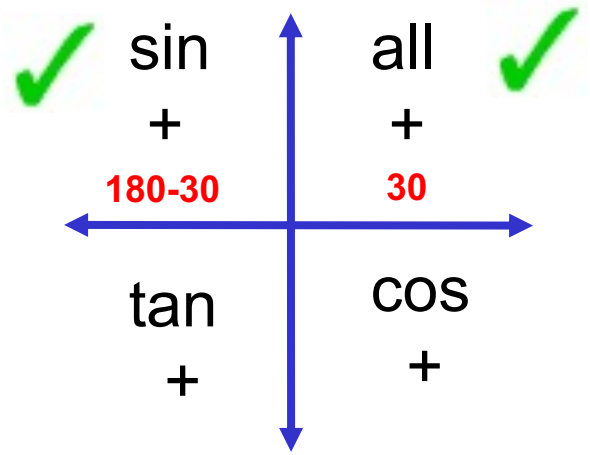
$$x = 30^\circ \text{ (calculator)}$$

or

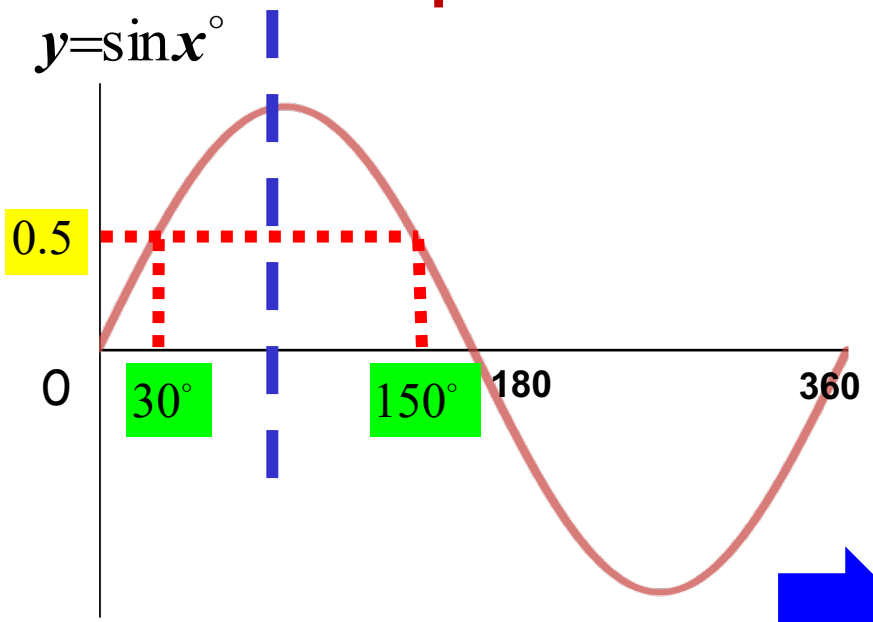
$$x = 180^\circ - 30^\circ = 150^\circ$$

$$x = 30^\circ \text{ or } 150^\circ$$

CAST



Graph



Example 2

Solve $5\cos x^\circ + 2 = 0, \quad 0 \le x \le 360$

$$5\cos x^\circ = -2$$

$$\cos x^\circ = -0.4$$

$$\cos x^\circ = 0.4$$

$$x = \cos^{-1}(0.4)$$

$$x = 66.4^\circ \text{ (calculator)}$$

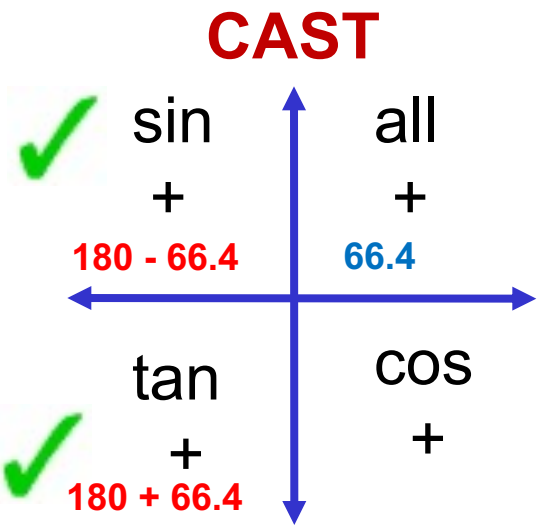
$$x = \cos^{-1}(-0.4)$$

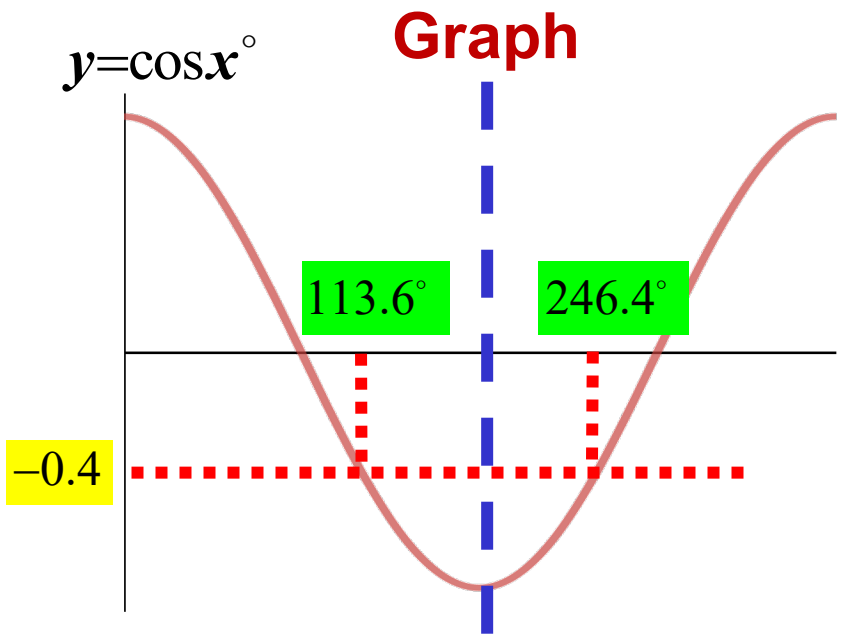
$$x = 180^\circ - 66.4^\circ = 113.6^\circ$$

or

$$x = 180^\circ + 66.4^\circ = 246.4^\circ$$

$$x = 113.6^\circ \text{ or } 246.4^\circ$$





Example 3

Solve $8\tan x^\circ + 3 = 0, \quad 0 \le x \le 360$

$$8\tan x^\circ = -3$$

$$\tan x^\circ = -0.375$$

$$\tan x^\circ = 0.375$$

$$x = \tan^{-1}(0.375)$$

$$x = 20.6^\circ \text{ (calculator)}$$

$$x = \tan^{-1}(-0.375)$$

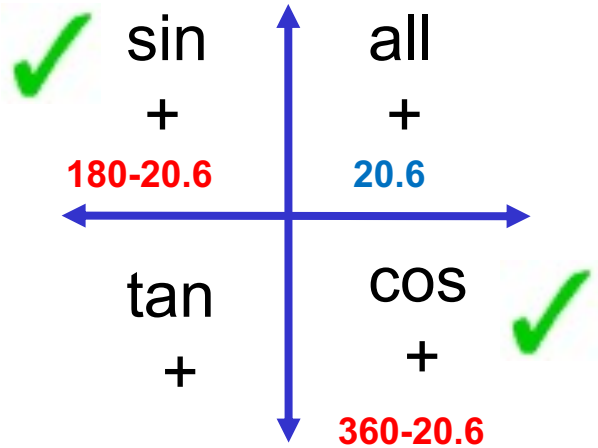
$$x = 180^\circ - 20.6^\circ = 159.4^\circ$$

or

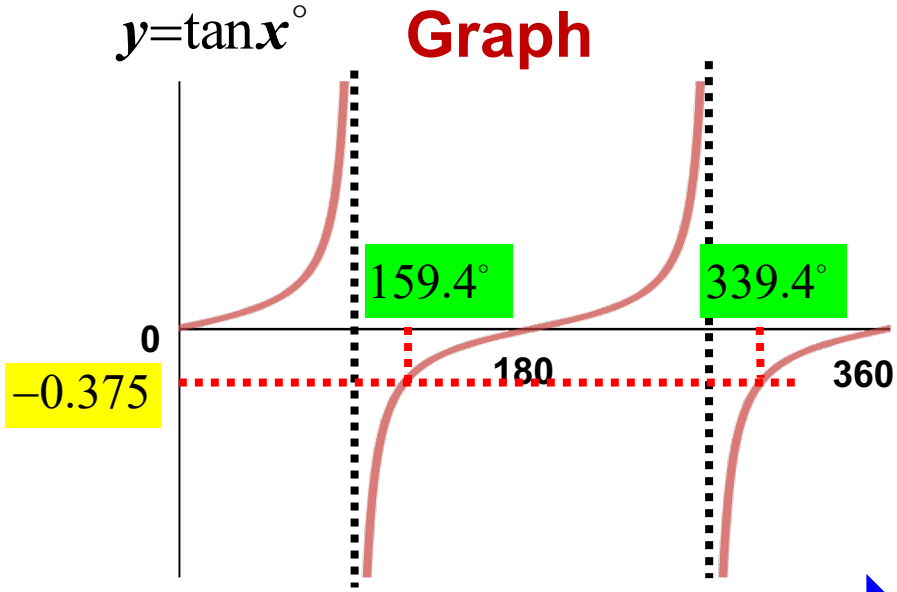
$$x = 360^\circ - 20.6^\circ = 339.4^\circ$$

$$x = 159.4^\circ \text{ or } 339.4^\circ$$

CAST



Graph



Extra Examples

Solve

$3\sin x^\circ - 1 = 0, \quad 0 \le x \le 360$	$x = 19.5$ *or* 160.5
$5\cos x^\circ - 3 = 0, \quad 0 \le x \le 360$	$x = 53.1$ *or* 306.9
$3\tan x^\circ + 2 = 0, \quad 0 \le x \le 360$	$x = 146.3$ *or* 326.3
$7\sin x^\circ + 3 = 0, \quad 0 \le x \le 360$	$x = 205.4$ *or* 334.6
$4\cos x^\circ + 1 = 0, \quad 0 \le x \le 360$	$x = 104.5$ *or* 255.5
$3\tan x^\circ - 8 = 0, \quad 0 \le x \le 360$	$x = 69.4$ *or* 249.4
$10\sin x^\circ = 3, \quad 0 \le x \le 360$	$x = 17.5$ *or* 162.5
$4\cos x^\circ + 9 = 6, \quad 0 \le x \le 360$	$x = 138.6$ *or* 221.4
$7\tan x^\circ + 8 = 5, \quad 0 \le x \le 360$	$x = 156.8$ *or* 336.8

Example 1

Harder Examples

Solve

$$7\sin x^\circ - 1 = 2\sin x^\circ + 2, \quad 0 \le x \le 360$$

+1 +1

$$7\sin x^\circ = 2\sin x^\circ + 3$$

$-2\sin x^0$ $-2\sin x^0$

$$5\sin x^\circ = 3$$

÷ 5 ÷ 5

$$\sin x^\circ = 0.6$$

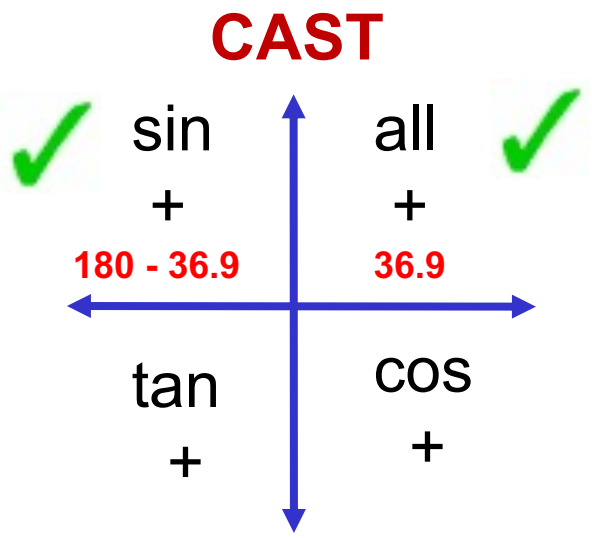
$$x = \sin^{-1}(0.6)$$

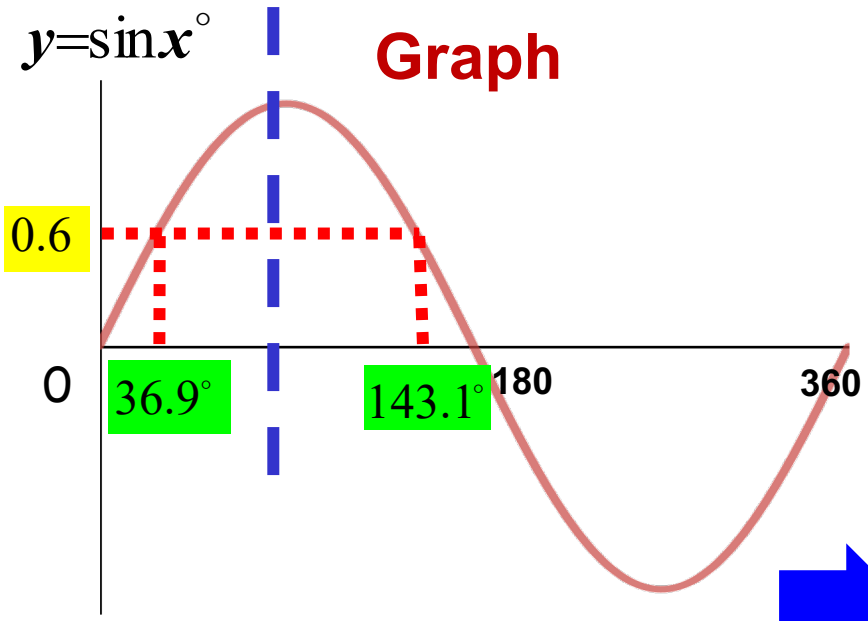
$$x = 36.9^\circ \text{ (calculator)}$$

or

$$x = 180^\circ - 36.9^\circ = 143.1^\circ$$

$x = 36.9^\circ$ or 143.1°





Example 2

Solve

$$8\cos x^\circ - 5 = 3\cos x^\circ - 2, \quad 0 \le x \le 360$$

$$8\cos x^\circ = 3\cos x^\circ + 3$$

$$5\cos x^\circ = 3$$

$$\cos x^\circ = 0.6$$

$$x = \cos^{-1}(0.6)$$

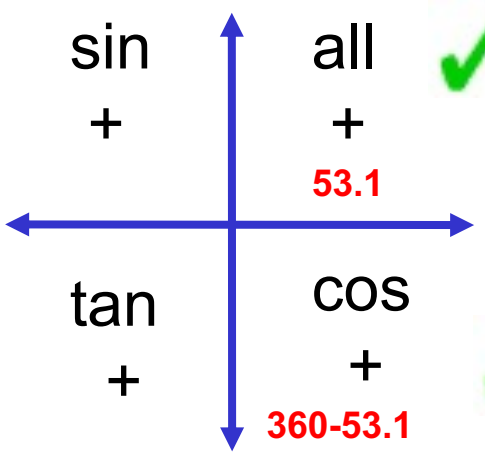
$$x = 53.1^\circ \text{ (calculator)}$$

or

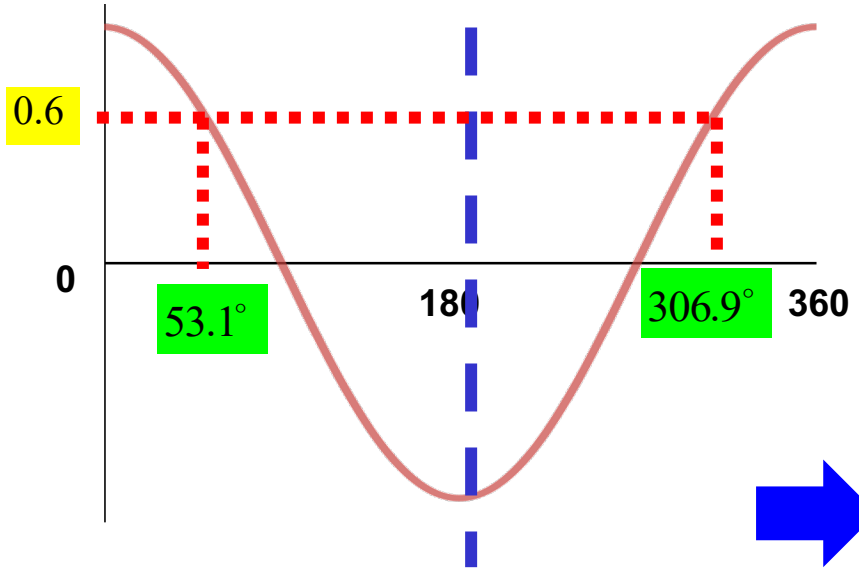
$$x = 360^\circ - 53.1^\circ = 306.9^\circ$$

$$x = 53.1^\circ \textit{ or } 306.9^\circ$$

CAST



Graph



Example 3

Solve

$$7\tan x^\circ + 3 = 4\tan x^\circ - 2, \quad 0 \le x \le 360$$

$$7\tan x^\circ = 4\tan x^\circ - 5$$

$$3\tan x^\circ = -5$$

$$\tan x^\circ = -1.667$$

$$x = \tan^{-1}(1.667)$$

$x = 59.0$ (calculator)

$$x = \tan^{-1}(-1.667)$$

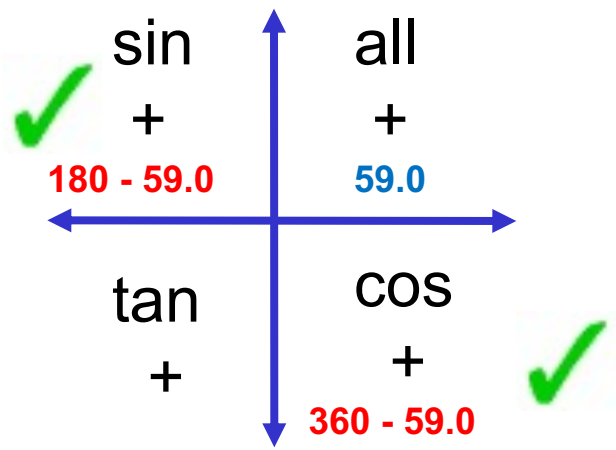
$$x = 180^\circ - 59.0^\circ = 121.0^\circ$$

or

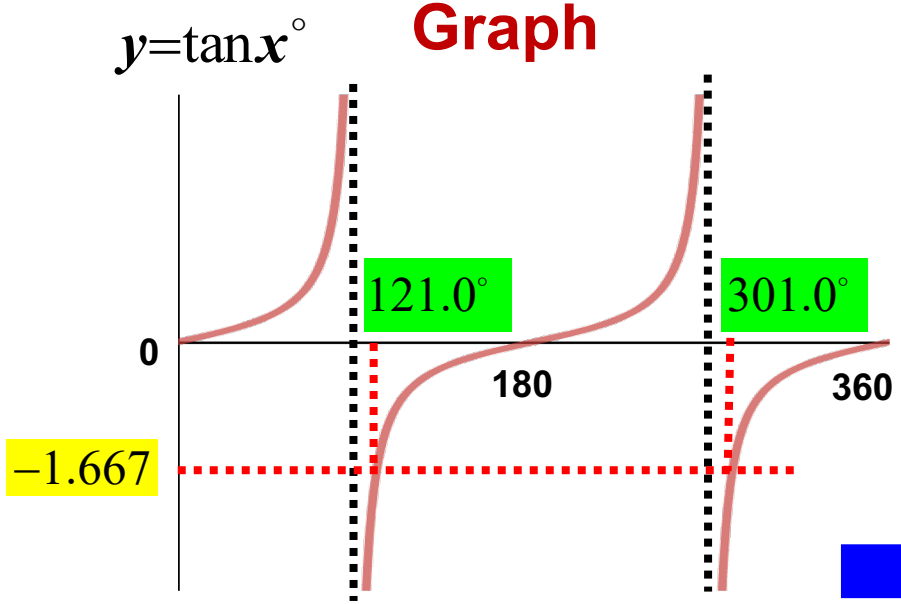
$$x = 360^\circ - 59.0^\circ = 301.0^\circ$$

$$x = 121.0^\circ \text{ or } 301.0^\circ$$

CAST



Graph



Example 4

Solve $4\cos^2 x^\circ = 1,\ 0 \le x \le 360$

$$\cos^2 x^\circ = 0.25$$

$$\cos x^\circ = \sqrt{0.25}$$

$$\cos x^\circ = 0.5 \ \textit{or}\ -0.5$$

$$x = \cos^{-1}(0.5)$$

$$x = 60^\circ \ (\text{calculator})$$

or

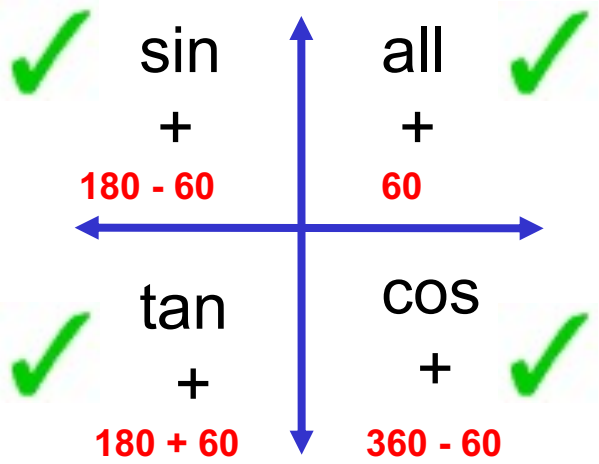
$$x = 360^\circ - 60^\circ = 300^\circ$$

$$x = \cos^{-1}(-0.5)$$

$$x = 180^\circ - 60^\circ = 120^\circ$$

or

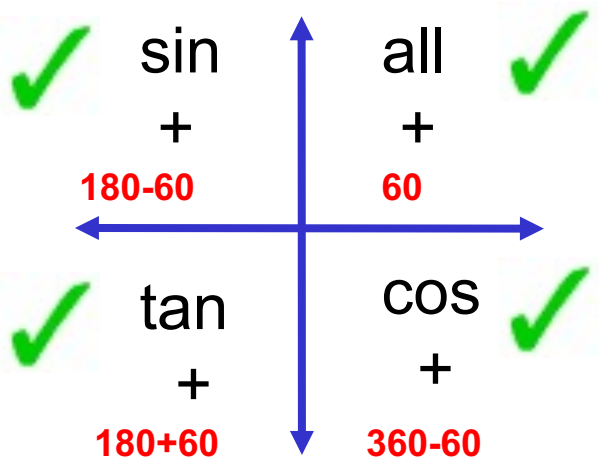
$$x = 180^\circ + 60^\circ = 240^\circ$$



$$x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$

Example 5

Solve $4\sin^2 x^\circ - 3 = 0, \quad 0 \le x \le 360$



$$4\sin^2 x^\circ = 3$$

$$\sin^2 x^\circ = \frac{3}{4}$$

$$\sin x^\circ = \sqrt{\frac{3}{4}}$$

$$\sin x^\circ = 0.866 \text{ or } -0.866$$

$$x = \sin^{-1}(0.866)$$

$$x = 60^\circ \text{ (calculator)}$$

or

$$x = 180^\circ - 60^\circ = 120^\circ$$

$$x = \sin^{-1}(-0.866)$$

$$x = 180^\circ + 60^\circ = 240^\circ$$

or

$$x = 360^\circ - 60^\circ = 300^\circ$$

$$x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$