E. Trigonometric identities

 $\sin x$ Remember: $\sin^2 x + \cos^2 x = 1$; and $\tan x =$ $\cos x$

Exercise 5

1. Simplify the following using the above 2 identities:

(a)
$$2\sin^2 x^\circ + 2\cos^2 x^\circ$$

(b)
$$5\cos^2 x^{\circ} + 5\sin^2 x^{\circ}$$

(c)
$$\frac{3\sin x^{\circ}}{\cos x^{\circ}}$$

(d)
$$\frac{5\sin x^{\circ}}{2\cos x^{\circ}}$$

2. Write down a simple expression, identical to:

(a)
$$1 - \sin^2 x^{\circ}$$

(b)
$$1 - \cos^2 x^{-1}$$

(a)
$$1 - \sin^2 x^{\circ}$$
 (b) $1 - \cos^2 x^{\circ}$ (c) $\tan x^{\circ} \cos x^{\circ}$

(d)
$$\frac{\sin x^{\circ}}{\cos x^{\circ}}$$

3. Simplify:

(a)
$$\frac{1-\cos^2 x^{\circ}}{\sin^2 x^{\circ}}$$

$$(b) \quad \frac{1-\sin^2 x^{\circ}}{2\cos^2 x^{\circ}}$$

(c)
$$\frac{\sin^2 x^{\circ}}{\cos^2 x^{\circ}}$$

(d)
$$\frac{1-\sin^2 x^{\circ}}{\cos x^{\circ}}$$

(e)
$$\frac{1-\cos^2 x^{\circ}}{5\sin x^{\circ}}$$

(f)
$$\tan^2 x^{\circ} (1 - \sin^2 x^{\circ})$$

4. Prove the following trigonometric identities:

(a)
$$3 - 3\sin^2 x^\circ = 3\cos^2 x^\circ$$

(b)
$$5 - 5\cos^2 x^{\circ} = 5\sin^2 x^{\circ}$$

(c)
$$\sqrt{1-\cos^2 x^\circ} = \sin x^\circ$$

(d)
$$\tan x^{\circ} \sqrt{1 - \sin^2 x^{\circ}} = \sin x^{\circ}$$

(e)
$$\frac{1-\cos^2 x^{\circ}}{1-\sin^2 x^{\circ}} = \tan^2 x^{\circ}$$

(f)
$$\frac{1-\sin^2 x^{\circ}}{1-\cos^2 x^{\circ}} = \frac{1}{\tan^2 x^{\circ}}$$

Answers

Exercise 5

- 1. (a) 2

- (b) 5 (c) $3\tan x$ (d) $\frac{5}{2}\tan x$
- 2. (a) $\cos^2 x^{\circ}$ (b) $\sin^2 x^{\circ}$ (c) $\sin x^{\circ}$ (d) $\tan x^{\circ}$

- 3. (a) 1

- (b) $\frac{1}{2}$ (c) $\tan^2 x^{\circ}$ (d) $\cos x^{\circ}$ (e) $\frac{1}{5}\sin x^{\circ}$ (f) $\sin^2 x^{\circ}$
- 4. All proofs.