Lourdes Secondary School

S1/2 Numeracy Methodology Booklet 2

Contents

How to use this booklet

The purpose of this document is to help you support your child's development in Numeracy and Maths.

There are worked examples with pictorial representation for each topic. If you would like further examples, explanations and the opportunity to try some yourself, there are videos available by scanning the given QR codes.

You will need the use of a QR Scanner to view the videos. This can be downloaded for free from the App Store. If you do not have access to a QR scanner a list of the YouTube video links can be found at the end of this document.

Curriculum for Excellence Levels

The table below is a guide to the Curriculum for Excellence Level at which a pupil should expect to see the topics covered within this booklet in their Primary or Mathematics class. However, please be aware that pupils may experience numeracy topics across the curriculum at different times and not always in the depth covered herein.

(Details of a Curriculum for Excellence can be found at https://education.gov.scot/)

Topic	Early	First	Second	Third	Fourth
Place Value	✓	✓	✓	✓	✓
Addition	✓	✓	✓	✓	✓
Subtraction	✓	✓	✓	✓	✓
Multiplication from 1 to 10		✓	✓	✓	✓
Multiplication by a multiple of 10		✓	✓	✓	✓
Long multiplication			✓	✓	✓
Division		✓	✓	✓	✓
Integers			✓	✓	✓
Order of operations			✓	✓	✓
Fractions	✓	✓	✓	✓	✓
Equivalent fractions		✓	✓	✓	✓
Fractions of a quantity		✓	✓	✓	✓
Mixed numbers and improper *fractions				✓	✓
Adding and subtracting fractions				✓	✓
Multiplying fractions					✓
Dividing fractions					✓
Decimals		✓	✓	✓	✓
Percentages			✓	✓	✓
Rounding			✓	✓	✓
Significant figures				✓	✓
Scientific notation				✓	✓
Ratio				✓	✓
Proportion				✓	✓
Time	✓	✓	✓	✓	✓
Angles			✓	✓	✓
Coordinates			✓	✓	✓
Scale and grid references			✓	✓	✓
Averages					✓
Graph Work		✓	✓	✓	✓
Equations					✓
Formulae					✓
Measurement	✓	✓	✓	✓	✓

Terminology and Methodology

We avoid the use of the word 'sum' to mean a maths question

Addition (+)

- sum of
- more than
- add
- total
- and
- plus
- increase
- altogether

Subtraction (-)

- less than
- take away
- minus
- subtract
- difference between
- reduce
- decrease

Equals (=)

- is equal to
- same as
- makes
- will be

Multiplication (×)

- multiply
- times
- product
- of

Division (÷)

- divide
- share equally
- split equally
- groups of
- per

The connection between fractions, percentages and decimals

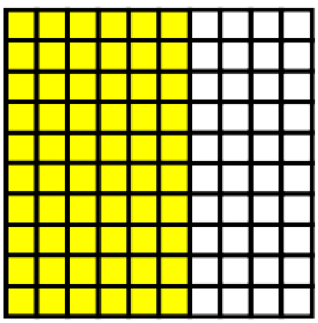
A quick reminder from our first Numeracy Booklet (Page 30 onwards) on percentages. As stated before, pupils are expected to know these commonly used percentages as fractions and decimals.

Percentage	Fraction	Decimal
1%	$\frac{1}{100}$	$0 \cdot 01$
10%	$\frac{1}{10}$	$0 \cdot 1$
20%	$\frac{1}{5}$	$0 \cdot 2$
$33\frac{1}{3}\%$	$\frac{1}{3}$	$0 \cdot 333... = 0 \cdot \dot{3}$
50%	$\frac{1}{2}$	$0 \cdot 5$
$66\frac{2}{3}\%$	$\frac{2}{3}$	$0 \cdot 666... = 0 \cdot \dot{6}$
75%	$\frac{3}{4}$	$0 \cdot 75$

Now we are going to look at converting between percentages, fractions and decimals which are not found in the above table.

Converting Percentages to Fractions and Decimals

Since a percentage is always out of one hundred, this can be written as a fraction with the denominator being 100.



60% means 60 out of 100

$$60\% = \frac{60}{100}$$

$$= \frac{3}{5}$$

simplify by dividing numerator and denominator by 20

Written Methodology

Example 1   As a fraction   45% = 45 out of 100

$$= \frac{45}{100}$$

$$= \frac{9}{20}$$

simplify by dividing numerator and denominator by 5

As a decimal

$$45\% = \frac{45}{100}$$

$$= 45 \div 100$$

When dividing by 100 we move all numbers 2 places to the right.

$$= 0{\cdot}45$$

Example 2   As a fraction   7% = 7 out of 100

$$= \frac{7}{100}$$

This fraction is already in its simplest form.

As a decimal

$$7\% = \frac{7}{100}$$

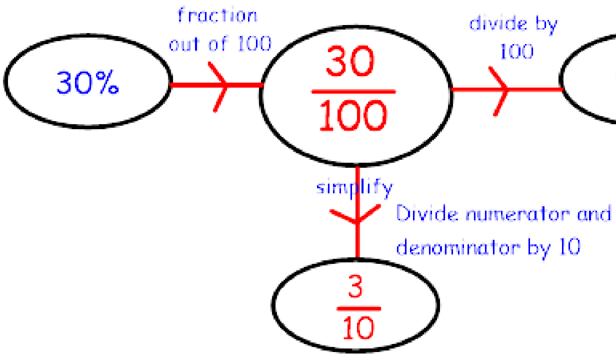
$$= 7 \div 100$$

$$= 0{\cdot}07$$

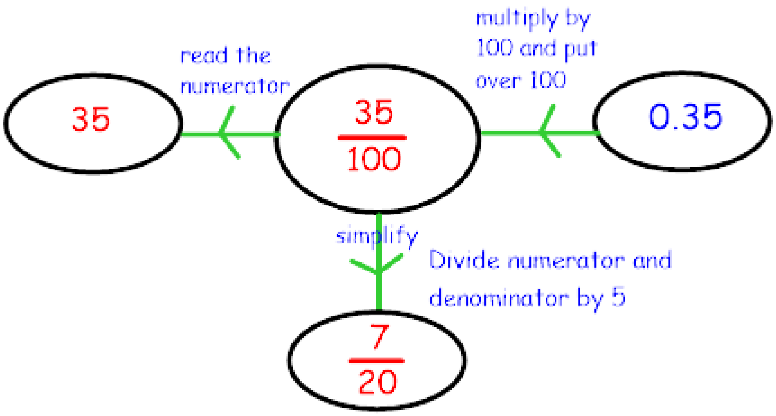
When dividing by 100 we move all numbers 2 places to the right.

Other Pictorial Method

<u>Example 1</u> Convert 30% into a fraction and decimal



<u>Example 2</u> Convert 0.35 into a fraction and percentage



Decimals

Multiplying Decimals

Can you:

(a) Multiply whole numbers?
(b) Divide by 10, 100, 1000 etc?

If you answered yes to both of these questions then you will be able to multiply decimals just as easily.

Written Methodology

1. **Question: $12 \times 0{\cdot}7$**

We can calculate 12×7

$12 \times 7 = 84$

$12 \times 0{\cdot}7 = 8{\cdot}4$ — We must divide our answer by 10

We have divided 7 by 10

2. **Question: $34 \times 0{\cdot}05$**

We can calculate 34×5

$34 \times 5 = 170$

$34 \times 0{\cdot}5 = 17{\cdot}0$ — We must divide our answer by 10

We have divided 5 by 10

$34 \times 0{\cdot}05 = 1{\cdot}70$ — We must divide our answer by 10 again

We have divided by 10 again

3.

<u>Option 1</u>

Question: $0{\cdot}52 \times 0{\cdot}067$

We can calculate 52×67

$52 \times 67 = 3\,484$

We have divided 52 by 10 $5{\cdot}2 \times 67 = 348{\cdot}4$ We must divide our answer by 10

We have divided 5•2 by 10 $0{\cdot}52 \times 67 = 34{\cdot}84$ We must divide our answer by 10

Since we have reached the first decimal we are looking for in our calculation, we can now deal with our second decimal in the given calculation.

We have divided 67 by 10 $0{\cdot}52 \times 6{\cdot}7 = 3{\cdot}484$ We must divide our answer by 10

We have divided 6•7 by 10 $0{\cdot}52 \times 0{\cdot}67 = 0{\cdot}3484$ We must divide our answer by 10

We have divided 0•67 by 10 $0{\cdot}52 \times 0{\cdot}067 = 0{\cdot}03484$ We must divide our answer by 10

<u>Option 2</u>

Question: $0{\cdot}52 \times 0{\cdot}067$

We can calculate 52×67

$52 \times 67 = 3\,484$

We have divided 52 by 100 $0{\cdot}52 \times 67 = 34{\cdot}84$ We must divide our answer by 100

Since we have reached the first decimal we are looking for in our calculation, we can now deal with our second decimal in the calculation.

We have divided 67 by 1000 $0{\cdot}52 \times 0{\cdot}067 = 0{\cdot}03484$ We must divide our answer by 1000

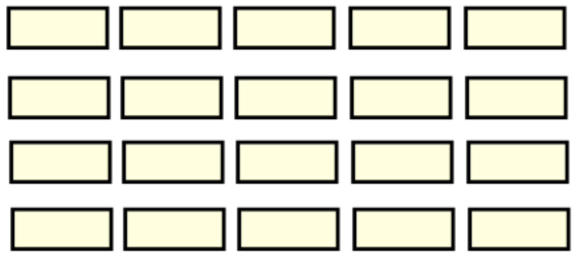
Dividing Decimals

Similar to multiplying decimals, if you can multiply by 10, 100, 1000 etc. and are able to divide whole numbers then you will easily be able to divide using decimals.

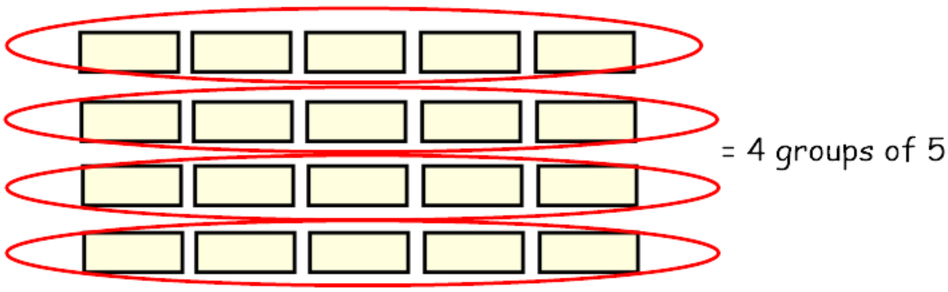
Let us first of all have a look at what it means to divide by a whole number.
If I asked you to show me what does:

$$20 \div 5$$ look like?

Here we have 20 wholes:



To get the answer to 20 ÷ 5 we need to split 20 into groups of 5 as shown below:

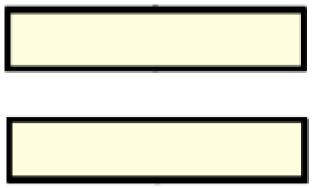


Therefore, 20 ÷ 5 = 4 since we can see there are **4** groups of 5.

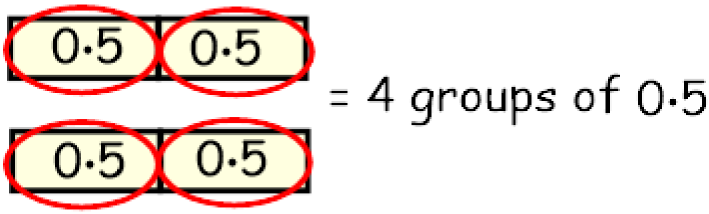
We can now apply this same method when dividing by decimals.

What does: $2 \div 0{\cdot}5$ look like?

Here we have 2 wholes:



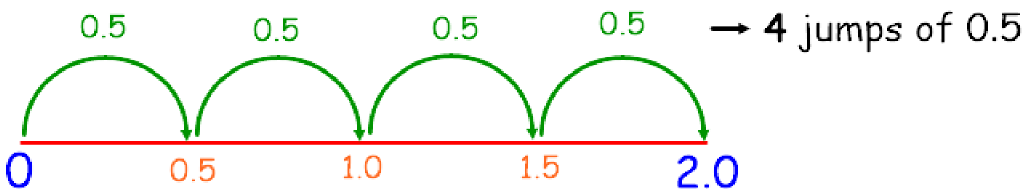
To get the answer to 2 ÷ 0.5 we need to split 2 into groups of 0.5 as shown below:



Therefore, 2 ÷ 0.5 = 4 since we can see there are 4 groups of 0.5.

Other Pictorial Method

We can also think about 2 ÷ 0.5 as how many jumps of 0.5 are there in 2?



Written Methodology

To divide by decimals we can also use equivalent fractions to help us.

Example 1 $2 \div 0{\cdot}5$

convert into a fraction

$$2 \div 0{\cdot}5$$

$$= \frac{2}{0{\cdot}5}$$

Create an equivalent fraction so that both the numerator and denominator are whole numbers

→multiply the numerator and denominator by 10

$$= \frac{20}{5}$$

$$= 4$$

Therefore, $2 \div 0{\cdot}5 = 4$

Example 2 $21 \div 0{\cdot}07$

convert into a fraction

$$21 \div 0{\cdot}07$$

$$= \frac{21}{0{\cdot}07}$$

Create an equivalent fraction so that both the numerator and denominator are whole numbers

→multiply the numerator and denominator by 100

$$= \frac{2100}{7}$$

$$= 300$$

Therefore, $21 \div 0{\cdot}07 = 300$

Example 3 $3{\cdot}6 \div 0{\cdot}09$

convert into a fraction

$$3{\cdot}6 \div 0{\cdot}09$$

$$= \frac{3{\cdot}6}{0{\cdot}09}$$

Create an equivalent fraction so that both the numerator and denominator are whole numbers

$\rightarrow$ multiply the numerator and denominator by 100

$$= \frac{360}{9}$$

$$= 40$$

Therefore, $3{\cdot}6 \div 0{\cdot}09 = 40$

Length

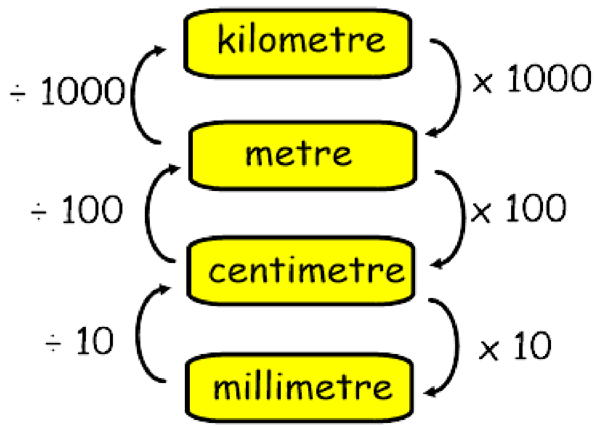
It is important that pupils are familiar with the different units used when measuring lengths, weights or volumes.

Conversions of length

Units that pupils will use in Maths to measure length are millimetres (mm), centimetres (cm), metres (m) and kilometres (km). The prefixes ‘milli’, ‘centi’ and ‘kilo’ refer to one thousandth, one hundredth and one thousand respectively and can be applied to many situations both within Maths and Numeracy as well as across other curricular areas.

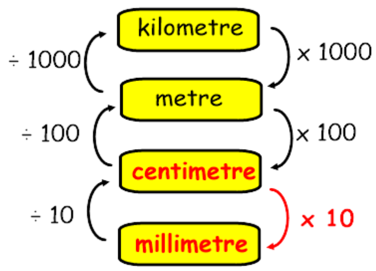
10 mm = 1 cm
100 cm = 1 m
1 000 m = 1 km

We can use this information to help us convert between the given units.



Example 1

Change 3.6 centimetres into millimetres

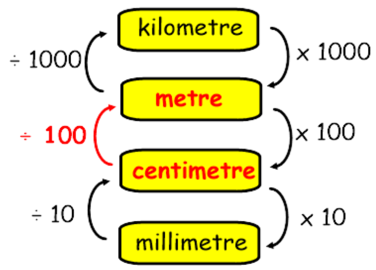


$$3 \cdot 6 \times 10 = 36$$

$$3 \cdot 6 \text{ cm} = 36 \text{ mm}$$

Example 2

Change 237 centimetres into metres

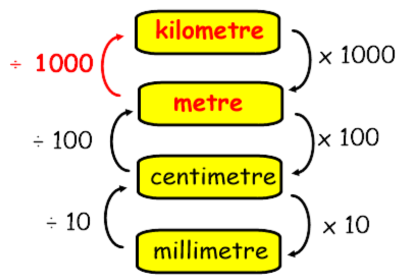


$$237 \div 100 = 2 \cdot 37$$

$$237 \text{ cm} = 2 \cdot 37 \text{ m}$$

Example 3

Change 2345 metres into kilometres

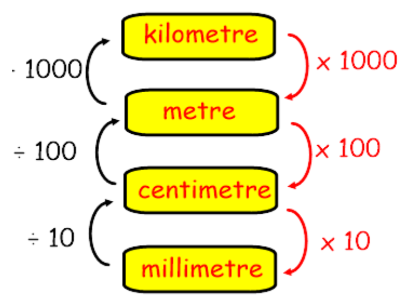


$$2345 \div 1000 = 2 \cdot 345$$

$$2345 \text{ m} = 2 \cdot 345 \text{ km}$$

Example 4

Change 5.4 kilometres to millimetres



$$5 \cdot 4 \times 1000 = 5\ 400$$

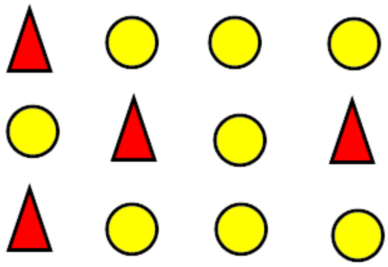
$$5\ 400 \times 100 = 540\ 000$$

$$540\ 000 \times 10 = 5\ 400\ 000$$

$$5 \cdot 4 \text{ km} = 5\ 400\ 000 \text{ mm}$$

Ratio

Ratios are used to show how things are shared. They show you how many of one thing there is compared to another.



This reads 4 to 8

The ratio of triangles to circles is 4 : 8

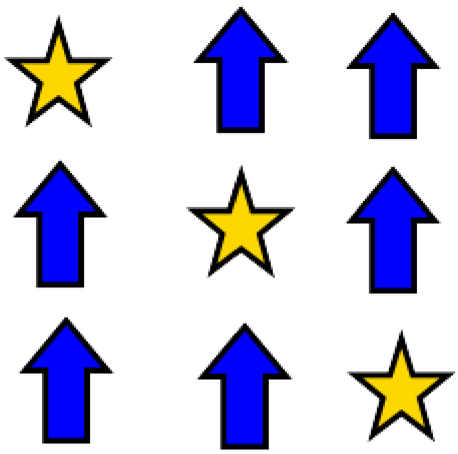
To separate quantities we use a colon

This reads 8 to 4

The ratio of circles to triangles is 8 : 4

As you can see, it matters which order we write our numbers.

Example 1



(1) What is the ratio of arrows to stars?

arrows : stars
6 : 3

(2) What is the ratio of stars to arrows?

stars : arrows
3 : 6

As you can see it can be useful to use a table to ensure the ratio is in the correct order.

Simplifying Ratios

We can simplify ratios in the same way that we simplify fractions by dividing all parts of the ratio by the highest common factor (HCF) of all of the terms. If no common factor exists then the ratio cannot be simplified.

<u>Example 1</u>

If we look back at our ratio of triangles to circles we got:

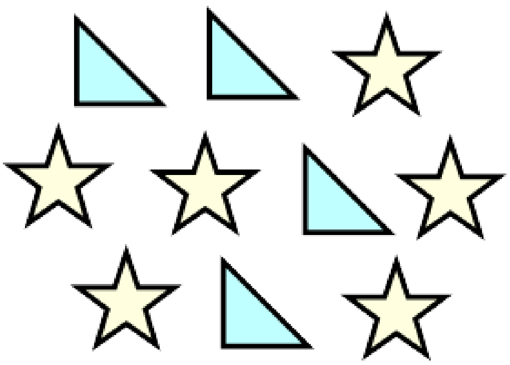
triangles : circles

$$\div 4 \quad 4 : 8 \quad \div 4$$
$$1 : 2$$

> The HCF of 4 and 8 is 4.
>
> This means that we divide both sides of the ratio by **4**.

<u>Example 2</u>

What is the ratio of stars to triangles in its simplest form?



stars : triangles

$$\div 2 \quad 6 : 4 \quad \div 2$$
$$3 : 2$$

> The HCF of 6 and 4 is 2.
>
> This means that we divide both sides of the ratio by **2**.

Example 3

In a class of 30 pupils, there are 18 boys and 12 girls.

What is the ratio of girls to boys, in its simplest form?

girls : boys

$$\div 6 \quad \begin{array}{c} 18 : 24 \\ \downarrow \\ 3 : 4 \end{array} \quad \div 6$$

The HCF of 18 and 24 is 6.

This means that we divide both sides of the ratio by **6**.

The ratio of girls to boys is 3:4

Example 4

In a jar of sweets there are 20 blue, 10 red and 15 yellow sweets.

What is the ratio of blue to red to yellow sweets, in its simplest form?

blue : red : yellow

$$\div 5 \quad \begin{array}{c} 20 : 10 : 15 \\ \downarrow \div 5 \\ 4 : 2 : 3 \end{array} \quad \div 5$$

The HCF of 20, 10 and 15 is 5.

This means that we divide all values in the ratio by **5**.

Ratio Calculations

Ratios can be used to calculate unknown quantities or to distribute an amount accordingly.

<u>Example 1</u>

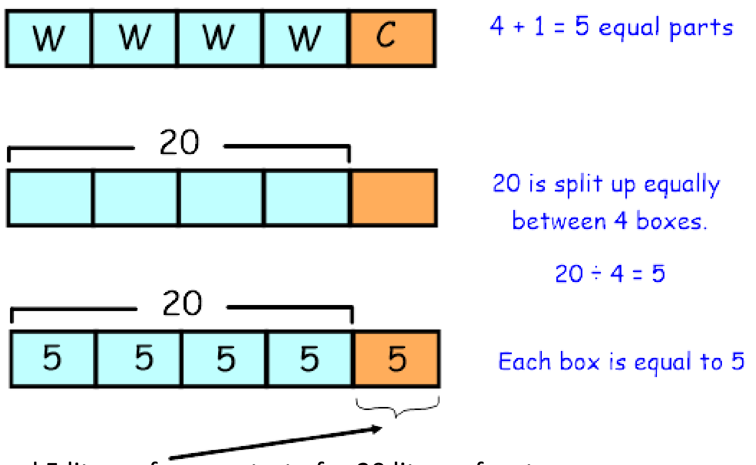
To make a diluting juice drink it is suggested the ratio of water to concentrate is 4:1.

How much concentrate is required for 20 litres of water?

Pictorial

water : concentrate

4 : 1



Answer: You need 5 litres of concentrate for 20 litres of water.

Written Methodology

water : concentrate

4 : 1

We know the new amount of water is 20, so we can write this directly under the water part of the ratio. → 20 :

Just like when dealing with equivalent fractions, you must do the same to both sides to keep the ratio equivalent.

20 divided by 4 is 5.

This tells us that we have multiplied 4 by 5 to get 20 therefore, we must also multiply the 1 on the right hand side by 5 to get our answer.

water : concentrate

$$\times 5 \left(\begin{array}{c} 4 : 1 \\ 20 : \mathbf{5} \end{array} \right) \times 5$$

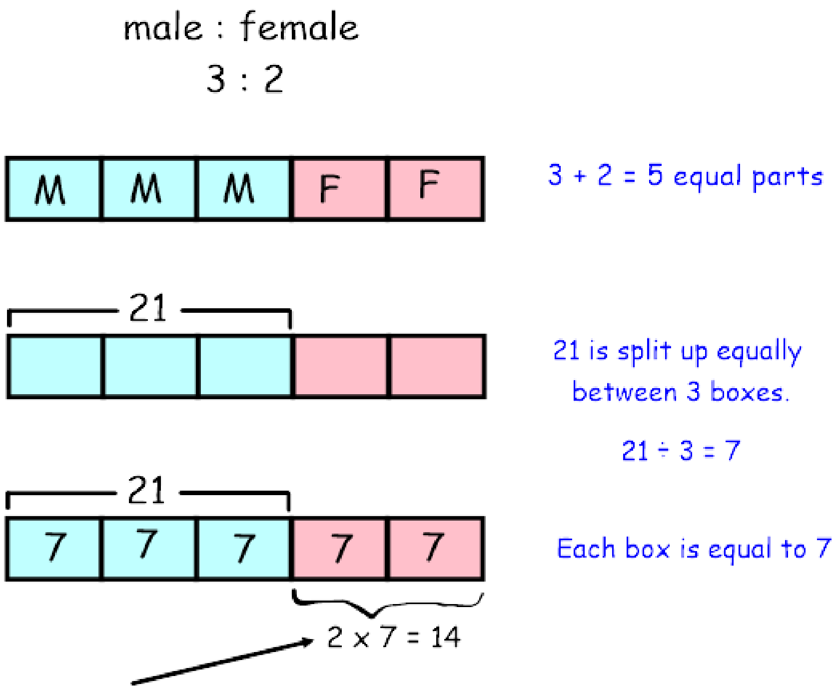
Answer: You need 5 litres of concentrate for 20 litres of water.

Example 2

The ratio of males to female teachers in a school is 3 : 2.

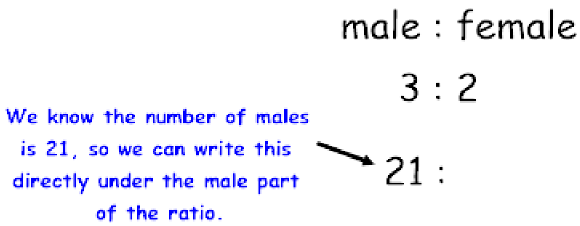
If there are 21 male teachers, how many teachers are female?

Pictorial



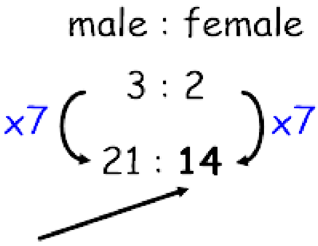
Answer: There are 14 female teachers.

Written Methodology



21 divided by 3 is 7.

This tells us that we have multiplied 3 by 7 to get 21 therefore, we must also multiply the 2 on the right hand side by 7 to get our answer.



Answer: There are 14 female teachers.

Sharing Ratio

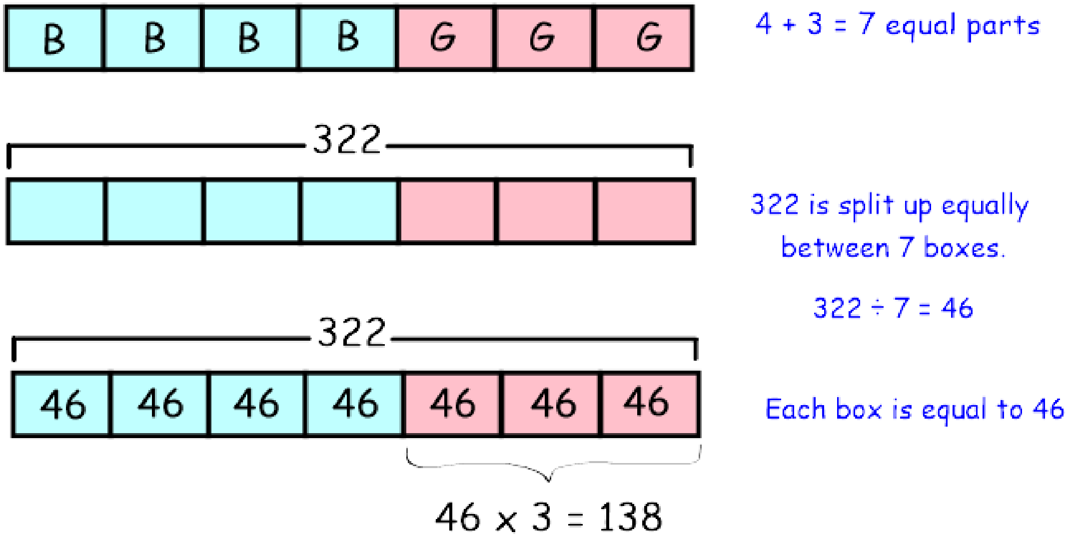
Example 1

The ratio of boys to girls in second year is 4 : 3. If there are 322 pupils, how many girls are there?

Pictorial

boys : girls

4 : 3



Answer: There are 138 girls.

Written Methodology

boys : girls

4 : 3

4 + 3 = 7 equal parts

Calculate the value of 1 part = 322 ÷ 7 = 46

boys : girls

$$\times 46 \left(\begin{array}{c} 4 : 3 \\ 184 : 138 \end{array} \right) \times 46$$

Answer: There are 138 girls.

Proportion

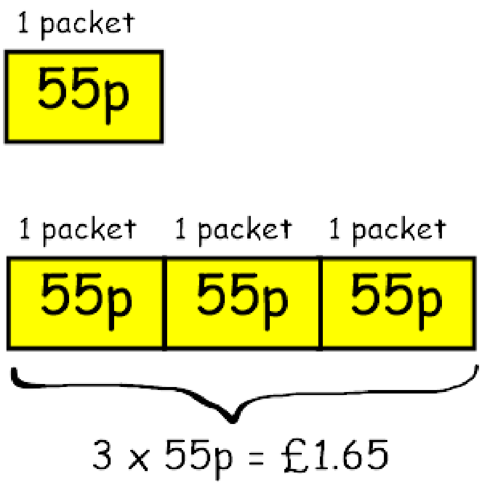
Direct Proportion

Two quantities are said to be in direct proportion if they increase or decrease in the same ratio.

Example 1

If one packet of crisps costs 55p, how much will three packets cost?

Pictorial



Answer: Three packets will cost £1.65 (165p).

Written Methodology

1 packet : 3 packets

$$\times 55 \quad \begin{matrix} 1 : 3 \\ 55 : \end{matrix}$$

We have multiplied 1 by 55 therefore,we must also multiply the 3 on the right hand side by 55.

1 packet : 3 packets

$$\times 55 \quad \begin{matrix} 1 : 3 \\ 55 : 165 \end{matrix} \quad \times 55$$

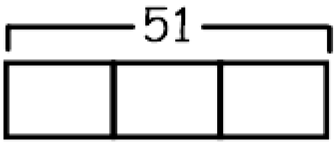
Answer: Three packets will cost £1.65 (165p).

Sometimes it is necessary to work out the value of a single item before working out multiple items.

<u>Example 2</u>

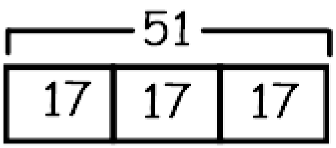
The weight of 3 rubbers is 51 grams. What is the weight of 8 rubbers?

Pictorial

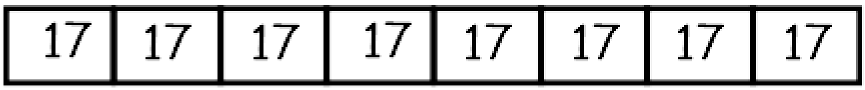


51 is split up equally between 3 boxes.

$51 \div 3 = 17$



Each box is equal to 17



8 boxes = $8 \times 17 = 136$

Answer: 8 rubbers weigh 136 grams.

Written Method

no of rubbers : weight (g)

$\div 3$ 3 : 51
1 :

To get 1 rubber we have divided 3 by 3 therefore, we must also divide the 51 on the right hand side by 3.

no of rubbers : weight (g)

$\div 3$ 3 : 51 $\div 3$
1 : 17
$\times 8$ 8 : 136 $\times 8$

To get 8 rubbers we have multiplied 1 by 8 therefore, we must also multiply the 17 on the right hand side by 17.

Answer: 8 rubbers weigh 136 grams.

Example 3

A 75 millilitre (ml) bottle of an isotonic drink contains 21 calories.

How many calories would there be in 500ml of the isotonic drink?

For this particular example you would not want to draw out 75 boxes and then 500 boxes. The easiest way would be to use the written methodology as shown in the previous example.

Written Methodology

volume (ml) : calories (kcal)

$$\begin{array}{rcl} & 75 : 21 & \\ \div 75 & & \div 75 \\ & 1 : 0.28 & \\ \times 500 & 500 : 140 & \times 500 \end{array}$$

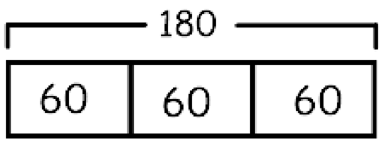
Answer: 500ml of isotonic drink contains 140 calories.

Indirect Proportion

If two quantities are inversely proportional to each other then as the value of one increases the value of the other decreases.

<u>Example 1</u>
It takes three men 1 hour to paint a garden fence.
Working at the same speed, how long would it take 4 men to paint the same fence?

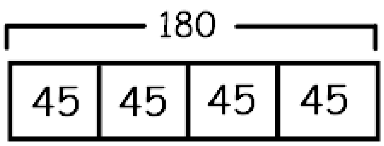
Pictorial



3 boxes each representing 60 minutes

3 x 60 = 180

1 man would take 180 minutes altogether



180 is split up equally between 4 boxes.

180 ÷ 4 = 45

4 men would take 45 minutes each

Written Methodology

3 men = 60 minutes

1 man = 60 x 3 (it would take him 3 times as long on his own)

= 180 minutes

4 men = 180 ÷ 4

= 45 minutes

Example 2

Four friends went to a restaurant for dinner and they agreed to pay £32.70 each for the bill.

Since it was one of their birthdays, they decided to split the bill between only three of them.

How much did they end up paying each?

Pictorial

$$\overbrace{\boxed{32.70}\boxed{32.70}\boxed{32.70}\boxed{32.70}}^{£130.80}$$

4 boxes each representing £32.70

$4 \times 32.70 = £130.80$

1 person would pay £130.80 altogether

$$\overbrace{\boxed{43.60}\boxed{43.60}\boxed{43.60}}^{£130.80}$$

£130.80 is split up equally between 3 boxes.

$130.80 \div 3 = 43.60$

3 people would pay £43.60 each

Written Methodology

Total bill for 4 people $= 4 \times 32.70$

$= £130.80$

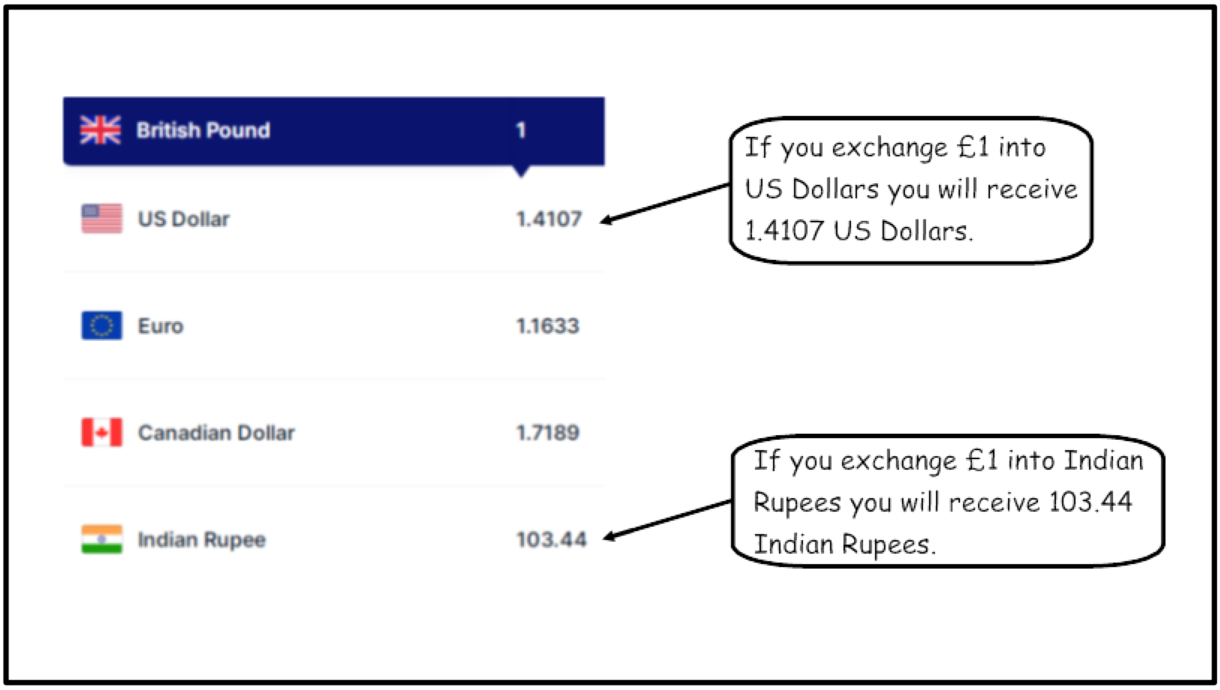
Split between 3 people $= 130.80 \div 3$

$= £43.60$

Foreign Exchange

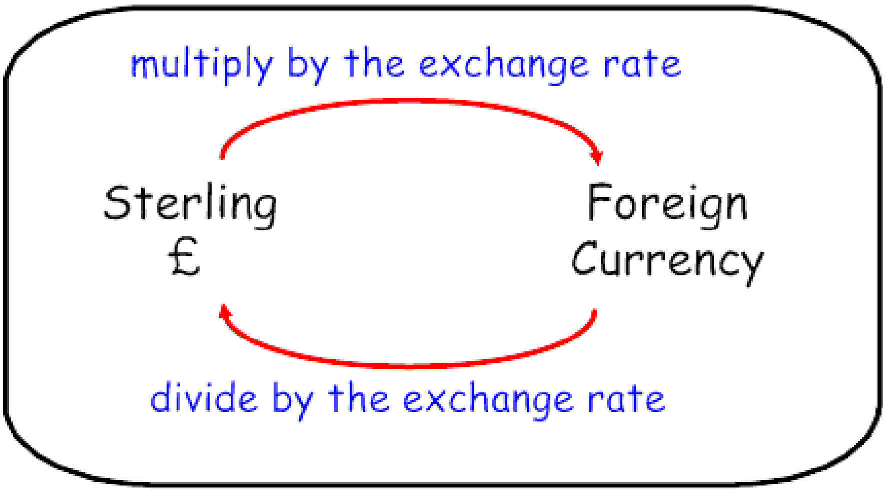
Foreign Exchange is the conversion of one currency into a different currency.

The exchange rate will tell you the amount of foreign currency you will receive for every one pound (£ sterling). The exchange rate changes *constantly* throughout the day.



To change £ Sterling into another foreign currency we ***multiply*** the required amount by that country's exchange rate.

To change back into £ Sterling from another foreign currency we ***divide*** the required amount by that country's exchange rate.



Example 1

Using these exchange rates, convert £100 into Euros

British Pound	1
US Dollar	1.4107
Euro	1.1633
Canadian Dollar	1.7189
Indian Rupee	103.44

	Pounds (£)	:	Exchange (Euros)	
× 100	1	:	1.1633	× 100
	100	:	€116.33	

Example 2

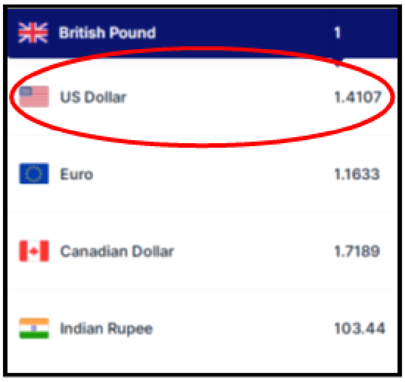
Using these exchange rates, convert £300 into Canadian Dollars

British Pound	1
US Dollar	1.4107
Euro	1.1633
Canadian Dollar	1.7189
Indian Rupee	103.44

	Pounds (£)	:	Exchange (Can. Dollars)	
× 300	1	:	1.7189	× 300
	300	:	$515.67	

Example 3

Using these exchange rates, convert \$340 (US Dollars) into pounds.



British Pound	1
US Dollar	1.4107
Euro	1.1633
Canadian Dollar	1.7189
Indian Rupee	103.44

Exchange (US \$)	:	Pounds (£)
340	:	£241.02

÷ 1.4107

Example 4

Which one is the best deal for the fitness tracker?

Scotland	France	USA
£100	114 Euros	\$162

British Pound	1
US Dollar	1.4107
Euro	1.1633
Canadian Dollar	1.7189
Indian Rupee	103.44

We need to convert each price into pounds to allow us to compare the price in each of the countries.

Exchange (US \$)	:	Pounds (£)
162	:	£114.84

÷ 1.4107

Exchange (Euros)	:	Pounds (£)
114	:	£98

÷ 1.1633

The best deal is from France for 114 Euros which converts into £98 which is less than £100 (Scotland) and £114.84 (USA).

Time

Pupils will need to recall basic facts about time:

1 year	= 365 days (366 in a leap year)
	= 52 weeks
	= 12 months

1 day	= 24 hours
1 hour	= 60 minutes
1 minute	= 60 seconds

Time is measured using either the 12 or 24 hour clock.

<u>12 hour clock</u>

a.m ⟶ midnight to noon

p.m ⟶ noon to midnight

<u>24 hour clock</u>

Always contains **four** digits

00:00 ⟶ 23:59

Midnight ⟶ 00:00

12 noon ⟶ 12:00

The hours after 12 noon are:

1pm ⟶ 13:00

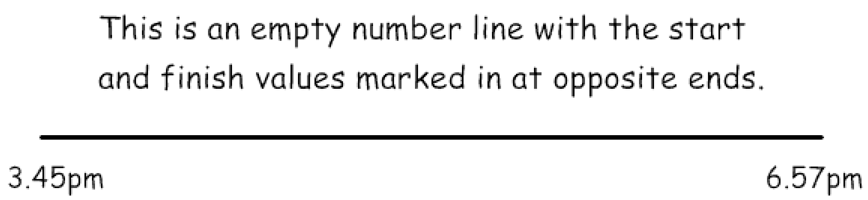
2pm ⟶ 14:00 etc

Time Intervals

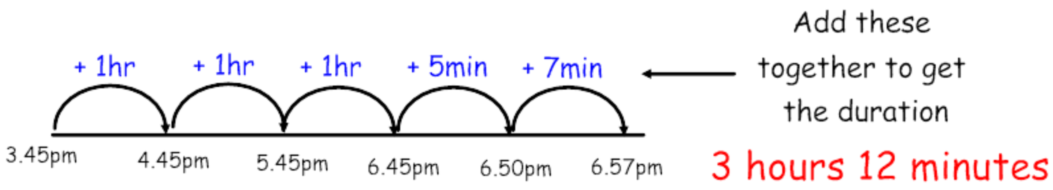
We use an empty number line and addition to teach time.

Example 1

How long is it from 3.45pm to 6:57pm?

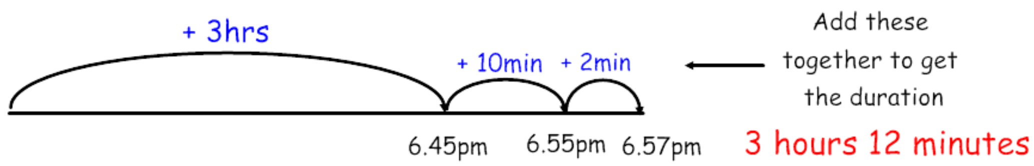


We then add on hours/minutes and move along the line until we reach the end time.



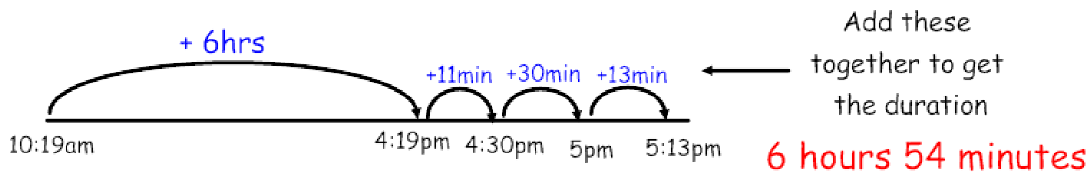
Answer: 3 hours and 12 minutes

You can decide to move along the empty number line in bigger increments. It is a personal choice on how you move from your starting time to the finishing time.



Example 2

How long is it from 10:19pm to 5:13am?



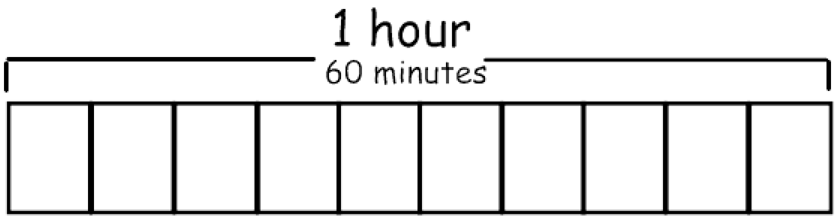
Answer: 6 hours and 54 minutes.

Converting minutes into hours

Converting minutes to hours can be shown pictorially using a bar model as shown below.

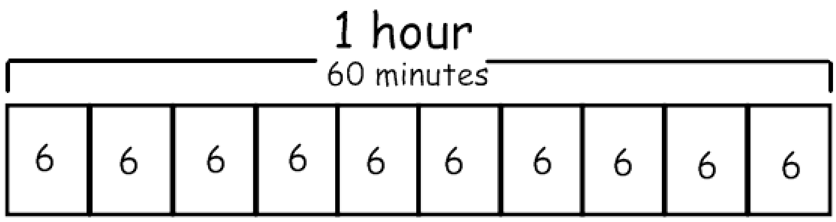
Pictorial

We know that one hour is equal to sixty minutes



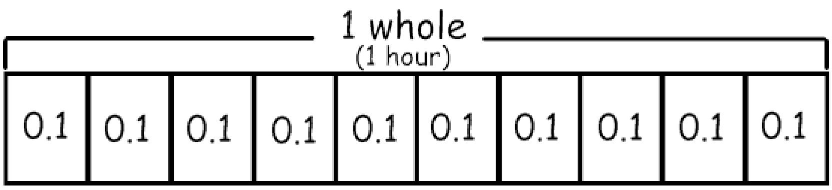
The bar model above has been split up into 10 equal parts.

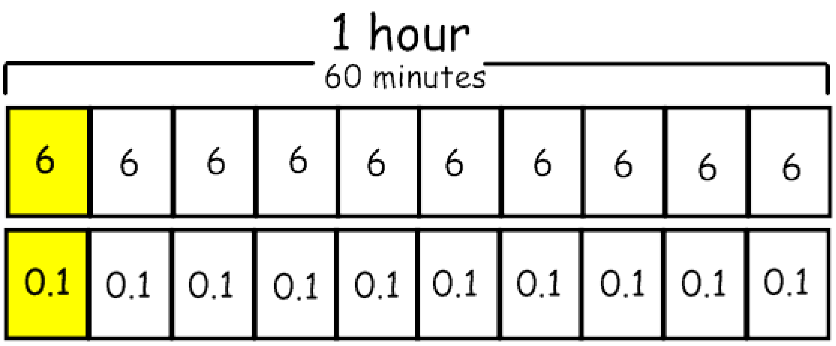
Each part represents 6 minutes since 60 ÷ 10 = 6.



We can also show this as 1 hour (1 whole) being split up into 10 equal parts.

Each part represents 0.1 of an hour since 1 ÷ 10 = 0.1.

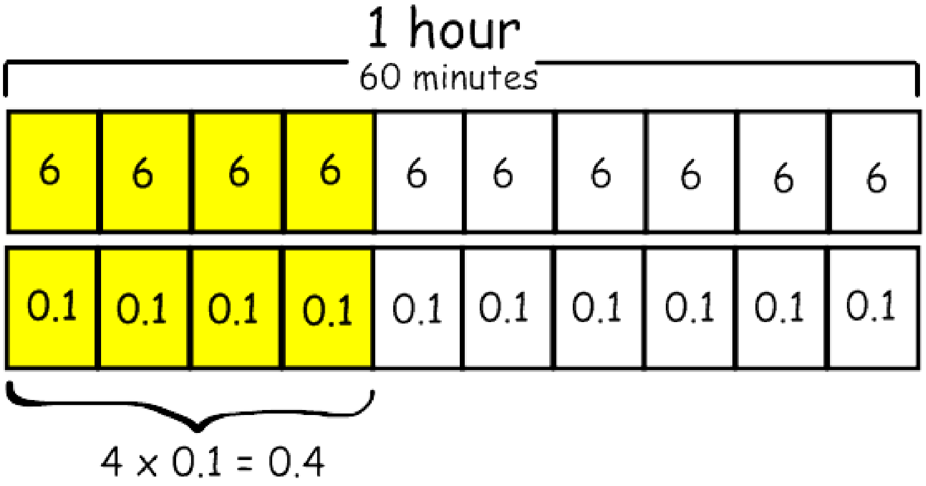




This proves that 6 minutes is equal to 0.1 hours.

Example 1

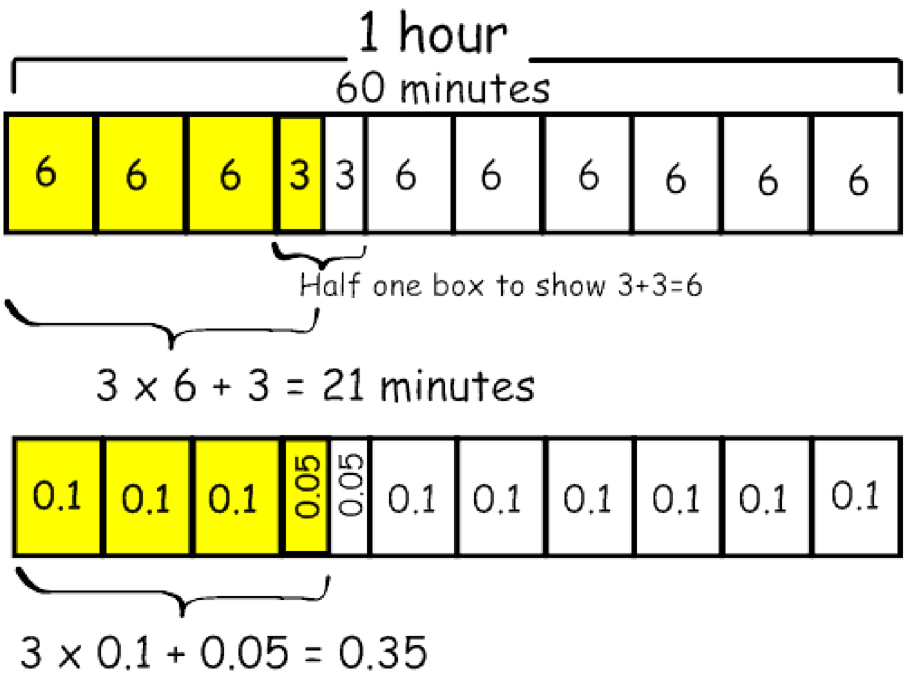
Convert 24 minutes into hours.



Answer: 24 minutes = 0.4 hours

Example 2

Convert 21 minutes into hours.



Answer: 21 minutes = 0.35 hours

Written Methodology

Converting minutes to hours can also be completed by writing the number of minutes as a fraction of an hour.

<u>Example 1</u>

Convert 24 minutes into hours.

24 minutes can be written as a fraction of 1 hour.

24 out of 60 minutes.

$$\frac{24}{60} = 24 \div 60$$

2 steps
* divide by 10 then
* divide by 6

step 1 $24 \div 10 = 2.4$

step 2 $2.4 \div 6 = 0.4$

<u>Answer:</u> 24 minutes = 0.4 hours

<u>Example 2</u>

Convert 21 minutes into hours.

21 minutes can be written as a fraction of 1 hour.

21 out of 60 minutes.

$$\frac{21}{60} = 21 \div 60$$

2 steps
* divide by 10 then
* divide by 6

step 1 $21 \div 10 = 2.1$

step 2 $2.1 \div 6 = 0.35$

<u>Answer:</u> 21 minutes = 0.35 hours

Example 3

Convert 3 hours and 39 minutes into hours.

39 minutes can be written as a fraction of 1 hour.

39 out of 60 minutes.

$\frac{39}{60} = 39 \div 60$ ← 2 steps
* divide by 10 then
* divide by 6

step 1 $39 \div 10 = 3.9$

step 2 $3.9 \div 6 = 0.65$ ← Then add this to 3 whole hours

Answer: 3 hours and 39 minutes = 3.65 hours

Converting hours into minutes

When you convert minutes to hours you divide by sixty.

To convert in the opposite direction, hours to minutes we do the opposite operation, multiply by sixty.

Written Methodology

<u>Example 1</u>

Convert 0.3 hours into hours and minutes.

0.3 hours

0 hours　　　0.3×60

When multiplying by 60 we
can do this in two steps:

$60 = 10 \times 6$

$0.3 \times 10 = 3$　　and　　$3 \times 6 = 18$

Multiply by 10 and then multiply by 6

<u>Answer:</u>　　**0.3 hours =** 0 hours and 18 minutes

Example 2

Convert 0.65 hours into minutes

0.65 hours

0 hours   0.65 x 60

When multiplying by 60 we
can do this in two steps:

60 = 10 x 6

0.65 x 10 = 6.5   and   6.5 x 6 = 39

Multiply by 10 and then multiply by 6

Answer: **0.65 hours =** 0 hours and 39 minutes

Example 3

Convert 4.45 hours into hours and minutes.

4.45 hours

4 hours   0.45 x 60

When multiplying by 60 we
can do this in two steps:

60 = 10 x 6

0.45 x 10 = 4.5   and   4.5 x 6 = 27

Multiply by 10 and then multiply by 6

Answer: **4.45 hours =** 4 hours and 27 minutes

Calculating Distance, Speed and Time.

When calculating speed, distance and time we use the strategies we have already looked at within this booklet on ratio. We tend to stay clear of using the DST triangle that some of you may be familiar with.

Calculating Distance

What does 40 mph actually mean?

This means that something travels 40 miles every hour.

Knowing this can then help us to calculate the distance travelled for a given amount of time.

<u>Example 1</u>

A car travels for 2 hours at a speed of 30mph.

How far does it travel?

	time	:	distance
	1hr	:	30 miles
x2			
	2hrs	:	

We have multiplied 1 by 2 to get 2, we must also multiply the 30 on the right hand side by 2.

	time	:	distance	
	1hr	:	30 miles	
x2				x2
	2hrs	:	60 miles	

<u>Answer:</u> The car travels 60 miles.

Example 2

A car travels for 3 and a half hours at a speed of 40km/h.

How far does it travel?

time	:	distance
1hr	:	40 miles
3hrs	:	

×3

We have multiplied 1 by 3 to get 3, we must also multiply the 40 on the right hand side by 3.

time	:	distance
1hr	:	40 miles
3hrs	:	120 miles

×3 ×3

time	:	distance
1hr	:	40 miles
0.5hrs	:	

÷2

We have divided 1 by 2 to get 0.5, we must also divide the 40 on the right hand side by 2.

time	:	distance
1hr	:	40 miles
0.5hrs	:	20 miles

÷2 ÷2

To get the final answer we add together the distance travelled for 3 hours and 0.5 hours.

Answer: The car travels 140 miles. (120 + 20)

Calculating Speed

We know that 50mph means that something travels fifty miles every one hour.

If we use ratio to show how far something travels in one hour then this is the equivalent to calculating the average speed.

<u>Example 1</u>

A car travels 180 miles in 3 hours.

What is the average speed of the car?

$$\begin{array}{cccc} & \text{time} & : & \text{distance} \\ \div 3 & 3 & : & 180 \text{ miles} \\ & 1 & : & \end{array}$$

We have divided 3 by 3 to get 1, we must also divide the 180 on the right hand side by 3.

$$\begin{array}{ccccc} & \text{time} & : & \text{distance} & \\ \div 3 & 3 & : & 180 \text{ miles} & \\ & 1 & : & 60 \text{ miles} & \end{array}$$

We have shown that the car travels 60 miles in one hour.

Therefore, the average speed is 60 mph.

<u>Answer:</u> The average speed of the car is 60 mph.

Example 2

A cyclist travels 14 miles in 20 minutes.

What is the average speed of the cyclist?

	time	:	distance
×3	20 minutes	:	14 miles
	1hour	:	

We have multiplied 20 minutes by 3 to get 1 hour (60 minutes).
We must also multiply the 14 on the right hand side by 3.

	time	:	distance	
×3	3	:	14 miles	×3
	1	:	42 miles	

We have shown that the cyclist travels 42 miles in one hour.

Therefore, the average speed is 42 mph.

Answer: The average speed of the cyclist is 42 mph.

Example 3

A car travels 150 miles in 2 and a half hours.

What is the average speed of the car?

	time	:	distance
÷5	2.5 hours	:	150 miles
	0.5 hours	:	

There are five 30 minutes in 2.5 hours so we have divided 2.5 hours by 5 to get 0.5 hours (30 minutes).

We must also divide the 150 on the right hand side by 5.

	time	:	distance	
÷5	2.5 hours	:	150 miles	÷5
	0.5 hours	:	30 miles	

Now that we have the distance covered in 30 minutes we can multiply both sides by 2 to get the distance covered in 1 hour.

	time	:	distance	
x2	0.5 hours	:	30 miles	x2
	1 hour	:	60 miles	

We have shown that the car travels 60 miles in one hour.

Therefore, the average speed is 60 mph.

Answer: The average speed of the car is 60 mph.

Calculating Time

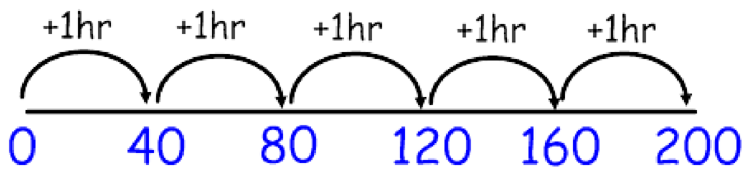
We will continue to use ratio to help us calculate the time that something takes to travel at a given speed and distance.

<u>Example 1</u>

A car travels 200 miles at an average speed of 40mph. How long does the journey take?

Pictorial Method

We know that 40mph means that the car will travel forty miles every one hour. We can then show this pictorially by counting how many jumps of 40 it would take to get from 0 miles to 200 miles.



There are 5 jumps of 40 to get from 0 miles to 200 miles.

<u>Answer:</u> The journey will take 5 hours

Written Methodology

40 mph → 40 miles every one hour

Time	:	Distance
1hr	:	40
	:	200

$$40 \xrightarrow{\times 5} 200$$

Is 200 a multiple of 40?

200 ÷ 40 = 5

Yes

We have multiplied 40 by 5 to get 200. We must also multiply the 1 on the left hand side by 5.

	Time	:	Distance	
× 5	1 hr	:	40	× 5
	5 hrs	:	200	

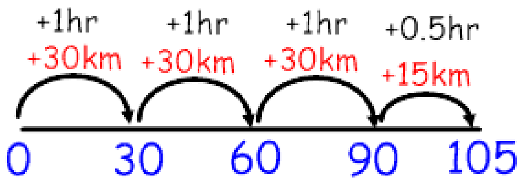
Answer: The journey takes 5 hours.

Example 2

A cyclist travels 105 kilometres at an average speed of 30km/hr.

How long are they cycling for?

Pictorial Method



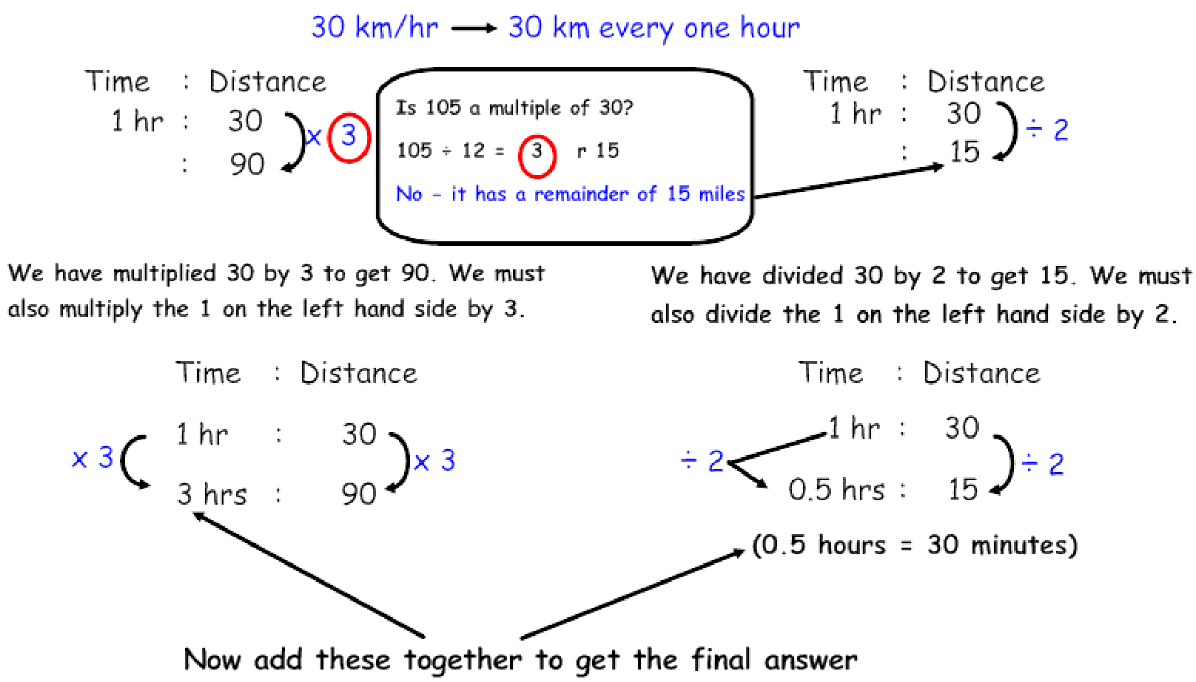
There are 3 full jumps of 30 to get from 0 kilometres to 90 kilometres.

If we were to add on another 30 it would take us over 105. Therefore, we have added on 15 (half of 30).

There are 3 and a half jumps of 30 to get from 0 km to 105 km.

Answer: The journey takes 3 hours and 30 minutes.

Written Methodology



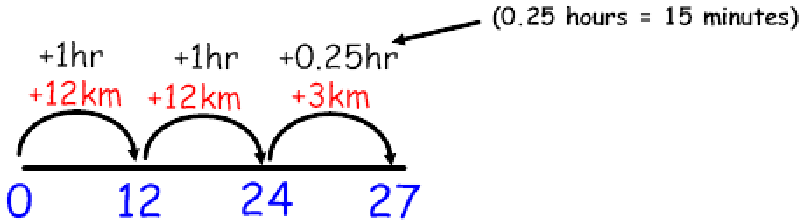
Answer: The journey takes 3 hours and 30 minutes.

Example 3

Sean runs 27 kilometres at an average speed of 12 km/hr.

How long was he running for?

Pictorial Method



There are 2 full jumps of 12 to get from 0 km to 24 km.

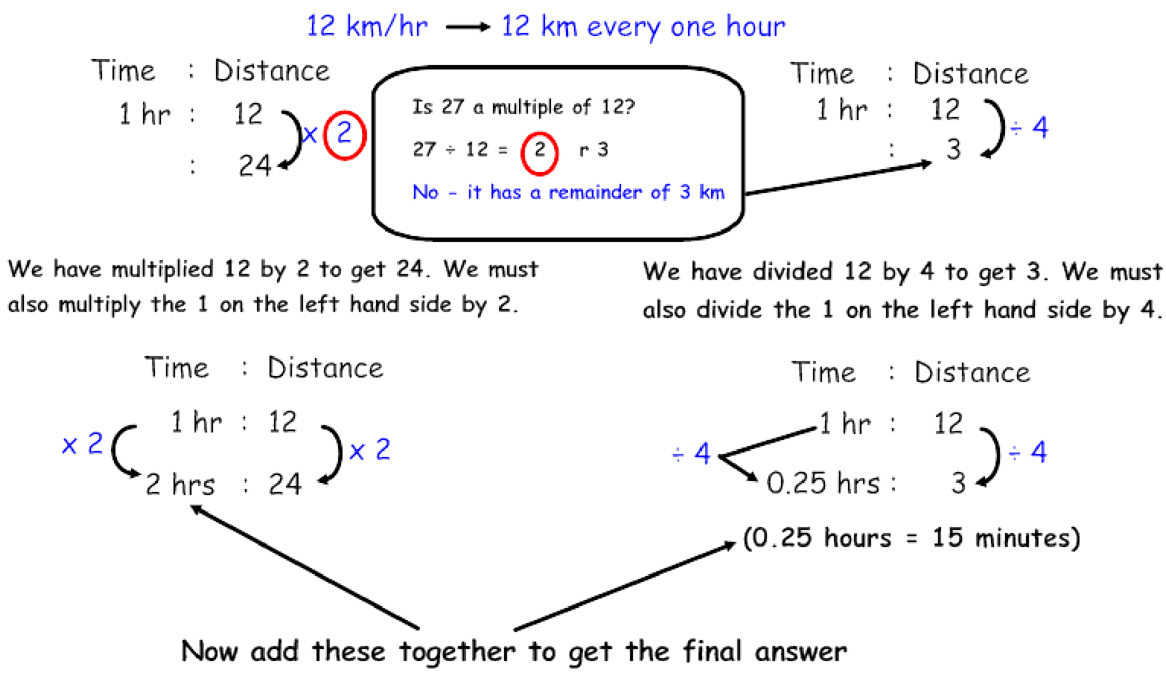
If we were to add on another 12 it would take us over 27.

Therefore, we have added on 3 (one quarter of 12).

There are 2 and one quarter jumps of 12 to get from 0 km to 27 km.

Answer: The journey takes 2 hours and 15 minutes.

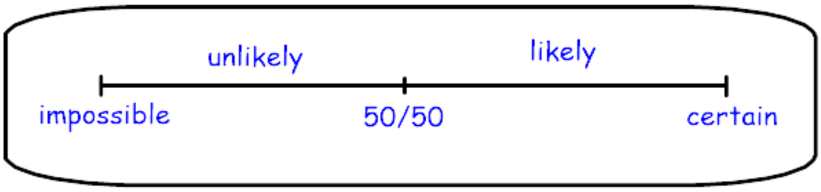
Written Methodology



Answer: The journey takes 2 hours and 15 minutes.

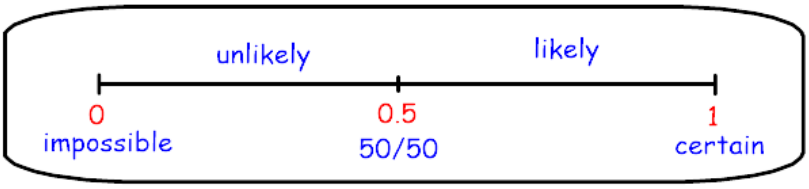
Probability

Probability is a measure of the likelihood of an event happening.



- We measure probability on a scale of zero to one.
- Probability of 0 means that the event is impossible.
- Probability of 1 means that the event is certain.

The greater the probability, the more likely an event will occur.



To calculate a value for the probability we use the following equation:

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{All possible outcomes}}$$

Example 1

There is a bag of 13 marbles.

There are 3 blue marbles, 8 red marbles and 2 green marbles.

What is the probability that you choose a red marble?

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{All possible outcomes}}$$

$$= \frac{8}{13}$$

8 — No. of red marbles

13 — No. of marbles altogether

The probability of choosing a red marble is $\frac{8}{13}$.

Example 2

What is the probability of rolling a die and landing on an even number?

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{All possible outcomes}}$$

$$= \frac{3}{6}$$

3 — No. of even numbers (2,4,6)

6 — No. of numbers on the die

$$= \frac{1}{2} \quad \text{(simplest form)}$$

We can also convert our answer for probability from a fraction into a decimal. We have previously learned how to do this in the '***Decimals section'*** at the beginning of this booklet.

This is very useful when we want to compare probabilities against each other.

<u>Example 3</u>

There are two raffles being held in school.
The S1 raffle has 100 tickets available, while the S2 raffle has 80 tickets available.

Mary bought 23 tickets for the S1 raffle and Scotty bought 18 tickets for S2 raffle.

Who has a better chance of winning?

S1 Raffle (Mary) = $\frac{23}{100}$

No of tickets Mary bought

Total no. of tickets available

$= 23 \div 100$

$= 0.23$

S2 Raffle (Scotty) = $\frac{18}{80}$

No of tickets Scotty bought

Total no. of tickets available

$= 18 \div 80$

$= 0.225$

Answer: Mary has a better chance of winning since 0.23 is greater than 0.225.

<u>YouTube Video Links</u>

Topic	YouTube Link
Converting between fractions, decimals and percentages	
Multiplying decimals	
Dividing Decimals	
Conversion of lengths	
Simplifying ratio	
Ratio calculations	
Sharing ratio	
Proportion	
Time intervals	
Converting hours to minutes	
Converting minutes to hours	
Calculating Distance	
Calculating Speed	
Calculating Time	