Lourdes Secondary School

LOURDES

S1

Numeracy Methodology Booklet 1

Contents

How to use this booklet

The purpose of this document is to help you support your child's development in Numeracy and Maths.

There are worked examples with pictorial representation for each topic. If you would like further examples, explanations and the opportunity to try some yourself, there are videos available by scanning the given QR codes.

You will need the use of a QR Scanner to view the videos. This can be downloaded for free from the App Store.

Curriculum for Excellence Levels

The table below is a guide to the Curriculum for Excellence Level at which a pupil should expect to see the topics covered within this booklet in their Primary or Mathematics class. However, please be aware that pupils may experience numeracy topics across the curriculum at different times and not always in the depth covered herein.

(Details of a Curriculum for Excellence can be found at https://education.gov.scot/)

Topic	Early	First	Second	Third	Fourth
Place Value	✓	✓	✓	✓	✓
Addition	✓	✓	✓	✓	✓
Subtraction	✓	✓	✓	✓	✓
Multiplication from 1 to 10		✓	✓	✓	✓
Multiplication by a multiple of 10		✓	✓	✓	✓
Long multiplication			✓	✓	✓
Division		✓	✓	✓	✓
Integers			✓	✓	✓
Order of operations			✓	✓	✓
Fractions	✓	✓	✓	✓	✓
Equivalent fractions		✓	✓	✓	✓
Fractions of a quantity		✓	✓	✓	✓
Mixed numbers and improper *fractions				✓	✓
Adding and subtracting fractions				✓	✓
Multiplying fractions					✓
Dividing fractions					✓
Decimals		✓	✓	✓	✓
Percentages			✓	✓	✓
Rounding			✓	✓	✓
Significant figures				✓	✓
Scientific notation				✓	✓
Ratio				✓	✓
Proportion				✓	✓
Time	✓	✓	✓	✓	✓
Angles			✓	✓	✓
Coordinates			✓	✓	✓
Scale and grid references			✓	✓	✓
Averages					✓
Graph Work		✓	✓	✓	✓
Equations					✓
Formulae					✓
Measurement	✓	✓	✓	✓	✓

Terminology and Methodology

We avoid the use of the word ‘sum’ to mean a maths question

Addition (+)

- sum of
- more than
- add
- total
- and
- plus
- increase
- altogether

Subtraction (-)

- less than
- take away
- minus
- subtract
- difference between
- reduce
- decrease

Equals (=)

- is equal to
- same as
- makes
- will be

Multiplication (×)

- multiply
- times
- product
- of

Division (÷)

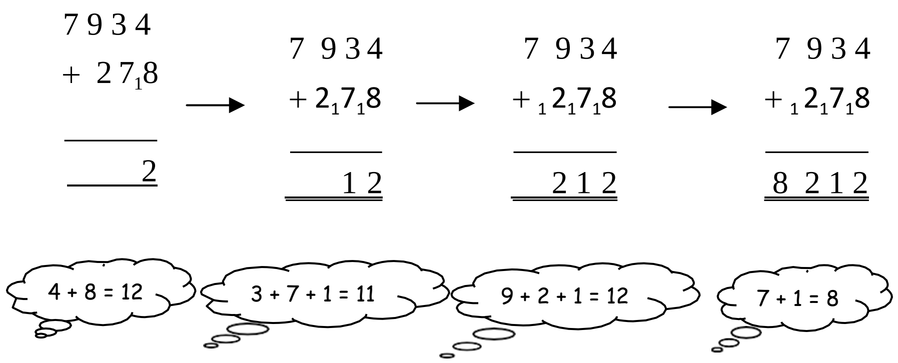
- divide
- share equally
- split equally
- groups of
- per

Addition

Written Methodology

When adding numbers, ensure that they are lined up correctly according to their place value. Start at the right hand side, write down the units and then carry tens to the next column on the left.

Example Add 7934 and 278



Mental Strategies

This is not an exhaustive list of mental strategies for addition and which strategy you choose may vary depending on the question.

Example Find 38 + 57

Method 1 Add tens, add units and add together.

30 + 50 = 80 8 + 7 = 15 80 + 15 = 95.

Method 2 Split one number into units and tens and then add in two steps.

38 + 50 = 88 88 + 7 = 95.

Method 3 Round up one number to the next ten and then subtract.

38 + 60 = 98 (60 is 3 too many so now subtract 3)

98 – 3 = 95

Subtraction

Written Methodology

After aligning digits by their place value we use decomposition for subtraction; we do not use “borrow and pay back”.

Example Subtract 425 from 9143

$$\begin{array}{r} {}^{8}\cancel{9}\;{}^{1}1\;{}^{3}\cancel{4}\;{}^{1}3 \\ -\quad 4\;\;2\;\;5 \\ \hline 8\;\;7\;\;1\;\;8 \\ \hline \end{array}$$

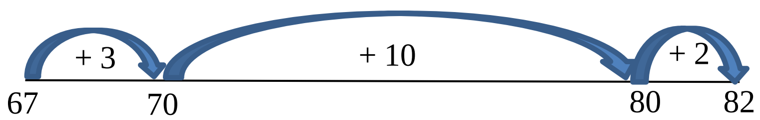
This subtraction was completed using the following steps:

1) In the units column try 3 subtract 5 – (using this method the top number must be larger than the bottom number)
2) From the tens column we have exchanged 1 ten for 10 units (i.e. 40 becomes 30 + 10 units)
3) The 10 units are added on to the units column to make 13 – 5 = 8
4) The subtraction in the tens column is now 3 – 2 = 1.
5) We repeat this process in the hundreds column to complete the subtraction.

Other Methods

Example **Calculate** 82 - 67

Method 1 “Count On”



3 + 10 + 2 = 15.

Method 2 Decompose the number to be subtracted

67 = 60 + 7 so subtract 60 then subtract 7

82 – 60 = 22 22 – 7 = 15.

Multiplication from 1 to 10

It is essential that every pupil can recall basic multiplication tables from 1 to 10 readily.

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

$8 \times 7 = 56$

Methods

Example Calculate 27×8

Method 1

$$\begin{array}{r} 2\ 7 \\ \times\ {}_{5}8 \\ \hline 2\ 1\ 6 \end{array}$$

Method 2 Decompose any number larger than one digit.

$27 = 20 + 7$ so multiply 8 by 20 then 8 by 7 and add

$20 \times 8 = 160$ $7 \times 8 = 56$

Therefore $27 \times 8 = 160 + 56$
$= 216$

	20	7
8	160	56

160 + 56 = 216

Method 3 Round to the nearest 10.

$30 \times 8 = 240$ but 30 is three lots of 8 too many so subtract $3 \times 8 = 24$
$240 - 24 = 216$.

Multiplication by 10,100 and 1000

For multiplication by 10, 100 and 1000 the digits move to the <u>left</u> by 1, 2 and 3 places, respectively. If there is no number under a column then ***a zero is added to keep the place value.***

Tens of thousands	Thousands	Hundreds	Tens	Units	
			5	3	
		5	3	0	(×10)
	5	3	0	0	(×100)
5	3	0	0	0	(×1000)

The pattern continues in this way for powers of 10 larger than 1000.

Please note, we do not say "add a zero" for multiplication.

Zeros are needed to fill the empty spaces in the hundreds, tens and units columns, otherwise when the number is written without the column headings it will appear as a different number

Multiplication by a multiple of 10

It is relatively easy to multiply by 10, 100, 1000 etc (see page 6). Therefore, given a calculation which involves multiplication by a multiple of 10 (40, 600, 3000 etc) we can quickly complete our calculation in two steps by deconstructing the number. This is often referred to as the "two step method".

Method 1

1. Calculate 14×50

   $50 = 5 \times 10$ so

   $14 \times 5 = 70$ and $70 \times 10 = 700$

   Multiply by 5 Multiply by 10

2. Calculate 300×6000

   $6000 = 6 \times 1000$ so

   $300 \times 6 = 1\ 800$ and $1\ 800 \times 1000 = 1\ 800\ 000$

   Multiply by 6 Multiply by 1000

Method 2

$$
\begin{array}{r}
14 \\
\times\ \ 50 \\
\hline
{}_{2}00 \\
+\ 700 \\
\hline
700
\end{array}
$$

Long Multiplication

For long multiplication we multiply by the units and then by the tens before adding the resulting answers. (For numbers larger than two digits the process continues in the same way.)

Method 1

1. Calculate 65×27

$$\begin{array}{rl} 6\,5 & \\ \underline{\times\,2\,7} & \\ 4\,5\,5 & \longrightarrow 65 \times 7 \\ +\,\underline{1\,3\,0\,0} & \longrightarrow 65 \times 20 \\ \underline{\underline{1\,7\,5\,5}} & \end{array}$$

2. Calculate 413×59

$$\begin{array}{rl} 4\,1\,3 & \\ \underline{\times\ \ 5\,9} & \\ 3\,7\,1\,7 & \longrightarrow 413 \times 9 \\ +\,\underline{2\,0\,6\,5\,0} & \longrightarrow 413 \times 50 \\ 2\,4\,3\,6\,7 & \end{array}$$

Method 2

1. Calculate 65×27

Split the numbers up into tens and units:

65 = 60 + 5

27 = 20 + 7

×	60	5
20	1200	100
7	420	35

20 × 5 = 100

20 × 60 = 1200

7 × 60 = 420

7 × 5 = 35

Then finally add your answers together to get your final answer.

$65 \times 27 = 1755$

2. Calculate 413 × 59

Split the numbers up into hundreds, tens and units:

413 = 400 + 10 + 3

59 = 50 + 9

50 × 10 = 500

50 × 3 = 150

50 × 400 = 20 000

×	400	10	3
50	20 000	500	150
9	3 600	90	27

9 × 400 = 3 600

9 × 3 = 27

9 × 10 = 90

Then finally add your answers together to get your final answer.

413 × 59 = 24 367

Division by a single digit

Division can be considered as the reverse process of multiplication. For example,

$$2 \times 3 = 6 \qquad \text{so} \qquad 6 \div 2 = 3 \qquad \text{and} \qquad 6 \div 3 = 2\,.$$

Essentially division tells us how many times one number (the divisor) goes into another (the dividend). From above, 2 goes into 6 three times and 3 goes into 6 two times. The answer from a division is called the quotient. For more difficult divisions we use the division sign:

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\,\text{dividend}} \end{array}$$

We start from the left of the dividend and calculate how many times the divisor divides it. This number is written above the division sign. If the divisor does not divide the digit exactly then the number left over is called the remainder. If there is a remainder then it is carried over to the next column and the process is repeated. If there is no remainder then we move directly on to the next column.

At the end of the dividend if there is still a remainder then in the early stages pupils are expected to write the remainder after the quotient, however, as a pupil progresses we expect the dividend to be written as a decimal and the remainder to be carried into the next column. It is important that digits are lined up carefully.

Examples

1. 8 divided by 2 is 4

$$\begin{array}{r} 4\; \\ 2\overline{)\,8\;6} \end{array} \longrightarrow \begin{array}{r} 4\;3 \\ 2\overline{)\,8\;6} \end{array} \longleftarrow \text{6 divided by 2 is 3 with no remainder}$$

2. $$\begin{array}{r} 0\;3\;5 \\ 5\overline{)\,1\,{}^{1}7\,{}^{2}5} \end{array}$$

3. $$\begin{array}{r} 0\;7\;2\;3 \\ 7\overline{)\,5\,{}^{5}0\,{}^{1}6\,{}^{2}1} \end{array}$$

4. $$\begin{array}{r} 1\;4\;\text{r}\;3 \\ 4\overline{)\,5\,{}^{1}9} \end{array} \quad \text{(early stages)} \qquad\qquad \begin{array}{r} 1\;4\cdot7\;5 \\ 4\overline{)\,5\,{}^{1}9\cdot{}^{3}0\,{}^{2}0} \end{array} \quad \text{(later stages)}$$

Long Division

We can think of division as a multiplication.

15 ÷ 5   can also been seen as   5 times what gives you 15?

15 ÷ 5 = **3**   5 x **3** = 15

We apply this method when we are completing long division.

Examples

1. 1014 ÷ 39

39 x 'what' = 1014 → write out the answers to

x10 x 5 x1

39 x 10 = 390
39 x 5 = 195
39 x 1 = 39

$$\left.\begin{array}{l} 39\boxed{\times 10} = 390 \\ 39\boxed{\times 10} = 390 \end{array}\right\} 780$$

Can we add on another 390?   No

So let's add on 195

$39\boxed{\times 5} = 195 \}$ 780 + 195 = 975

Can we add on another 195?   No

So let's add on 39

$39\boxed{\times 1} = 39$ 975 + 39 = 1014

How many times have we multiplied 39?

Add together the values that we have multiplied by.
(the numbers in the boxes 10 + 10 + 5 + 1)

1014 ÷ 39 = 26

Division by 10, 100 and 1000

For division by 10, 100 and 1000 the digits move to the right by 1, 2 and 3 places, respectively.

$$\longrightarrow$$

Tens of thousands	Thousands	Hundreds	Tens	Units	
5	3	0	0	0	
	5	3	0	0	(÷10)
		5	3	0	(÷100)
			5	3	(÷1000)

The pattern continues in this way for powers of 10 larger than 1000. Please note, we do not say "take away a zero" or "move the decimal point" for division.

This method can then also be used for decimal numbers.

Division by a multiple of 10

Long division is no longer part of the Curriculum, however pupils are expected to divide by multiples of 10. The method is similar to that outlined in the previous page.

Examples

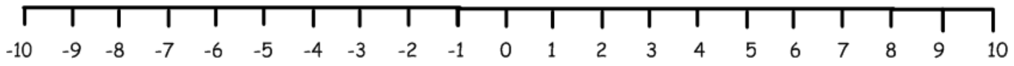
2. Calculate $1020 \div 60$

$60 = 6 \times 10$ so divide by 6 and divide by 10 (in any order).

$1020 \div 10 = 102$ and then $\begin{array}{r|l} & 0\ 1\ 7 \\ \hline 6 & 1\ {}^{1}0\ {}^{4}2 \end{array}$ therefore $1020 \div 60 = 17$

Integers

Despite common misconceptions in young children, the first number is not zero! The set of numbers known as integers comprises positive and negative whole numbers and the number zero. Negative numbers are below zero and are written with a negative sign, “ – ”. Integers can be represented on a number line.



Integers are used in a number of real life situations including temperature, profit and loss, height below sea level and golf scores.

Adding and Subtracting Integers

Consider 2 + 3. Using a number line this addition would be “start at 2 and move right 3 places”. Whereas 2 – 3 would be “start at 2 and move left 3 places”. Picturing a number line may help pupils extend their addition and subtraction to integers.

Examples

Addition

1. $3 + 5$ start at 3 and move
$= 8$ right 5 places.

2. $-4 + 7$ start at −4 and move
$= 3$ right 7 places.

Now consider 6 + (-8). Here we read “start at 6 and prepare to move right but then change direction to move left 8 places because of the (-8). Therefore, $6 + (-8) = -2$ and we could rewrite this calculation as

3. $6 + (-8)$
$= 6 - 8$
$= -2$

Subtraction

1. $6 - 2$ start at 6 and move
$= 4$ left 2 places.

2. $-7 - 4$ start at -7 and move
$= -11$ left 4 places.

Similarly, 3 – (-5) can be read as “start at 3 and prepare to move left but instead change direction because of the (-5) and move right 5 places”. So 3 – (-5) = 8 and we can write our calculation as

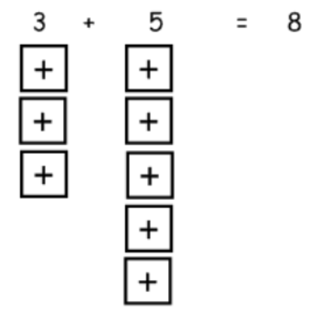
3. $3 - (-5)$
$= 3 + 5$
$= 8$

Method 2 (+/- tiles)

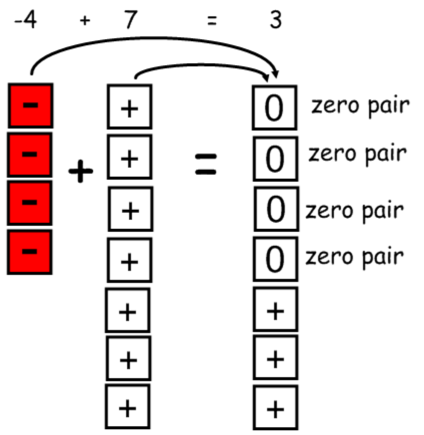
A positive tile is worth positive 1 and a negative tile is worth negative 1. Therefore, when you add one positive tile and one negative tile you get zero. We call this a ‘zero pair'.

Adding Integers

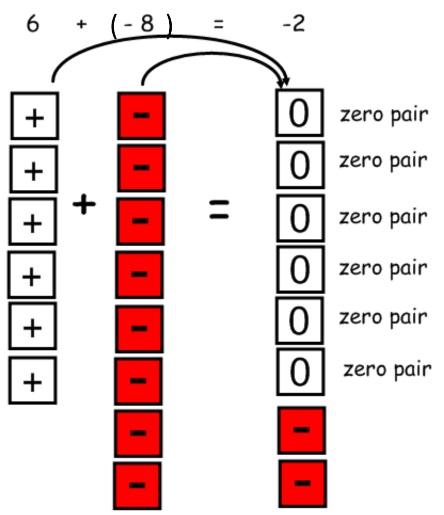
1.



2.

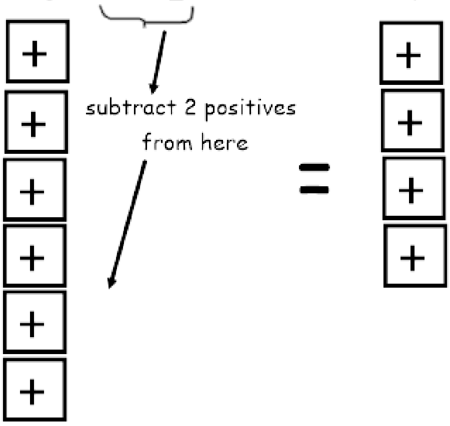


3.

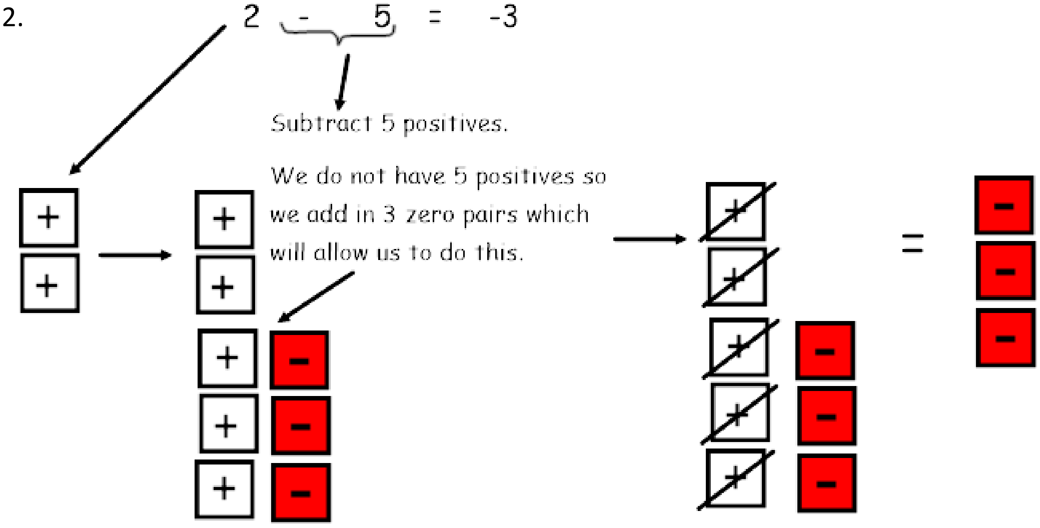


Subtracting Integers

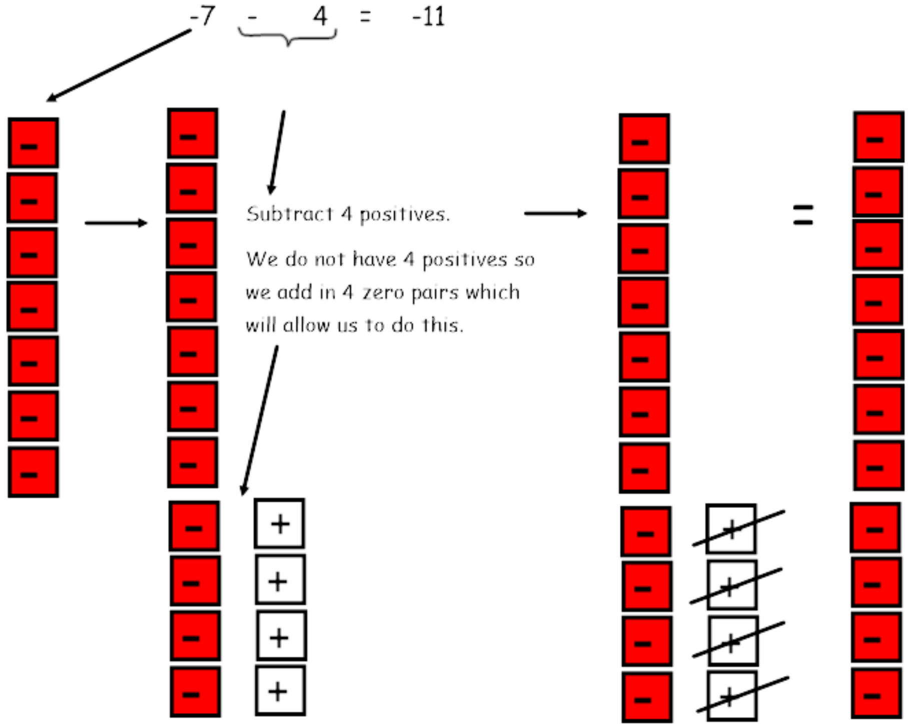
1. 6 - 2 = 4



2. 2 - 5 = -3



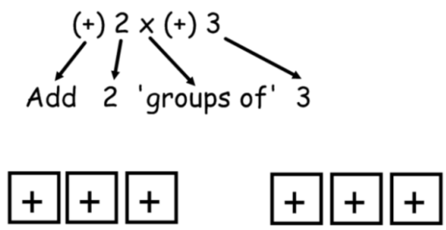
3. -7 - 4 = -11



Multiplying and Dividing Integers

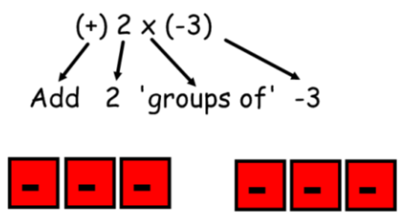
Multiplying Integers

1. 2×3



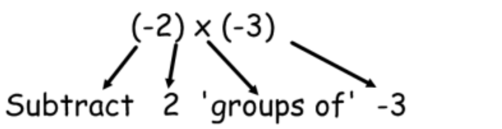
= 6

2. 2×-3



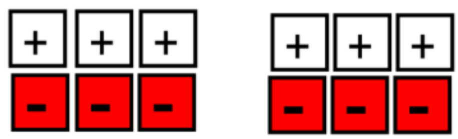
= -6

3. -2×-3

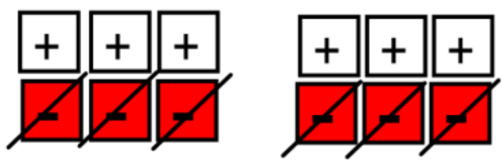


Since we cannot subtract from something that is not there,

we will show 2 groups of 3 **zero pairs**



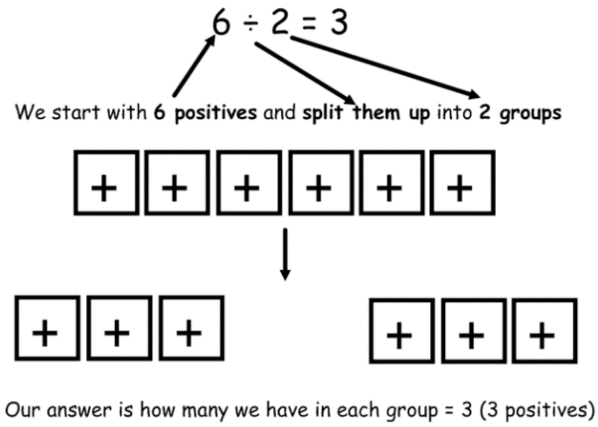
We can now subtract 2 groups of -3



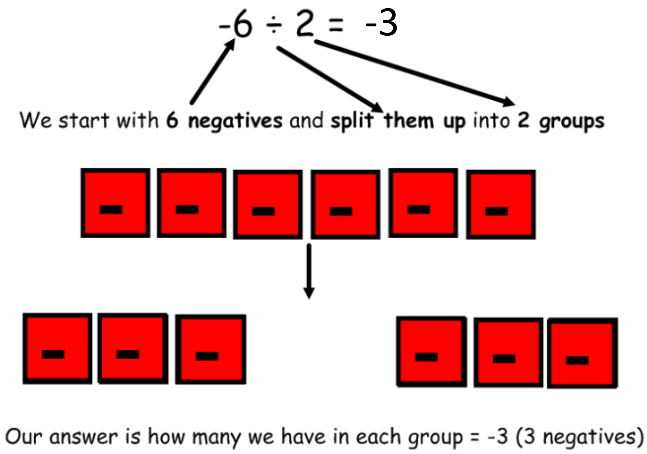
= 6

Dividing Integers

1. $6 \div 2 = 3$



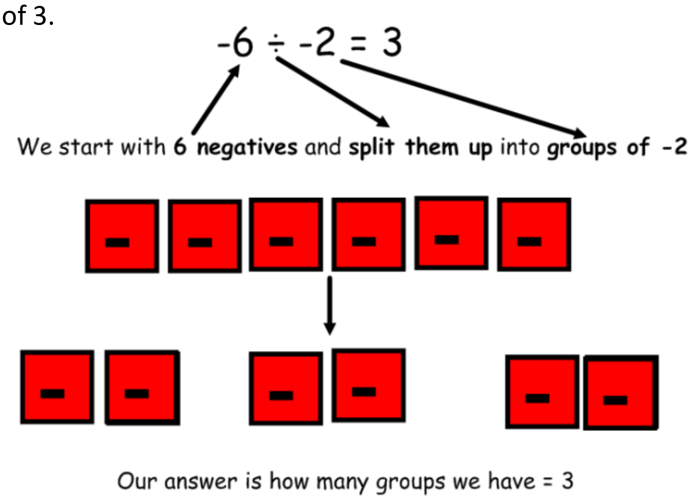
2. $-6 \div 2 = -3$



3. $-6 \div -2 = 3$

How many groups of the second number can be made from the first number?

This question is different since we cannot split something up into negative groups. Therefore we say to ourselves, how many groups of negative 2 can you make from negative 6? This will give you your answer of 3.



Once the pupils are comfortable with using the above method, they will soon realise for themselves that the following happens.

If the signs of the integers are the same, the product (the answer from multiplying) and quotient (the answer from dividing) is positive.

1. 4 x 5 = 20 2. -7 x (-3) = 21 3. 40 ÷ 5 = 8 4. -10 ÷ (-2) = 5

If the signs of the integers are different, the product (the answer from multiplying) and quotient (the answer from dividing) is negative.

1. 5 x (-5) = -25 2. -7 x 5 = -35 3. 40 ÷ (-10) = -4 4. -20 ÷ (4) = -5

Fractions

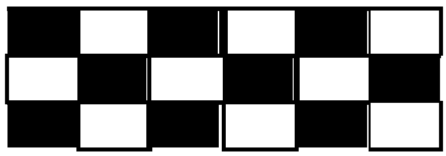
A fraction tells us how much of a quantity we have. Every fraction has two parts, a numerator (top) and a denominator (bottom).

Examples

1. numerator $\rightarrow \dfrac{2}{5} \leftarrow$ denominator

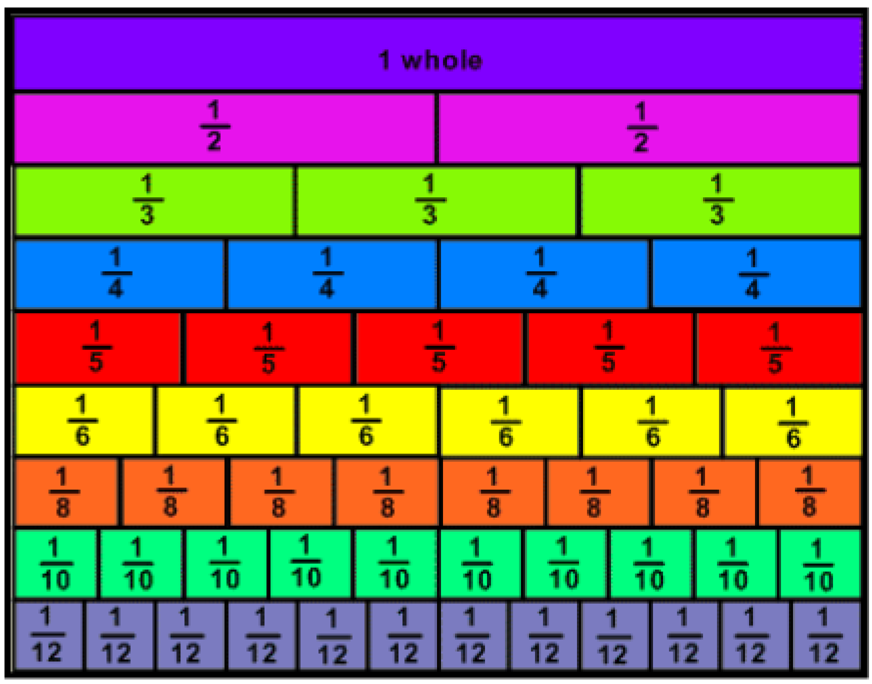
This is the fraction two fifths which represents two out of five.

2. What fraction of the rectangle is shaded?



There are 9 parts shaded out of 18 so the fraction is $\dfrac{9}{18}$

When creating and working with equivalent fractions, all pupils will be issued with a copy of the fraction wall shown below:



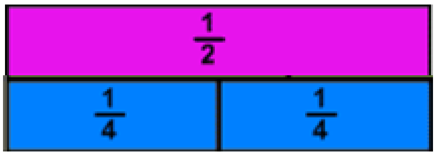
Equivalent Fractions

To find equivalent fractions we multiply or divide both the numerator and the denominator of the fraction by the same number.

If you look at the fraction wall you can easily see the equivalent fractions.

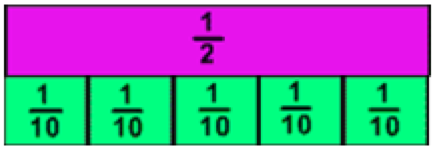
1.

$$\underset{\times 2}{\overset{\times 2}{\frac{1}{2} = \frac{2}{4}}}$$



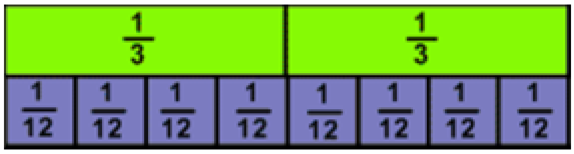
2.

$$\underset{\times 5}{\overset{\times 5}{\frac{1}{2} = \frac{5}{10}}}$$



3.

$$\underset{\times 4}{\overset{\times 4}{\frac{2}{3} = \frac{8}{12}}}$$



4.

$$\underset{\times 7}{\overset{\times 7}{\frac{5}{11} = \frac{35}{77}}}$$

Simplifying Fractions

To simplify fractions we divide both the numerator and the denominator of the fraction by the same number. (Ideally this is the highest common factor, HCF, of the numbers, i.e. the highest number which can divide each number with no remainder.) We always aim to write fractions in the simplest possible form.

1.

Simplify $\frac{6}{10}$

Write out all factors of 6 and 10

6	10
1 x 6	1 x 10
2 x 3	2 x 5
All factors→1, 2, 3, 6	All factors→1, 2, 5, 10

highest common factor (hcf) = 2

$$\frac{6}{10} \xrightarrow{\div 2} \frac{3}{5}$$

2.

Simplify $\frac{12}{28}$

Write out all factors of 12 and 28

12	28
1 x 12	1 x 28
2 x 6	2 x 14
3 x 4	4 x 7
All factors→1, 2, 3, 4, 6, 12	All factors→1, 2, 4, 7, 14, 28

highest common factor (hcf) = 4

$$\frac{12}{28} \xrightarrow{\div 4} \frac{3}{7}$$

More complicated examples are explained in the 'Simplifying Fractions' video, accessed by scanning here:

Fraction of a quantity

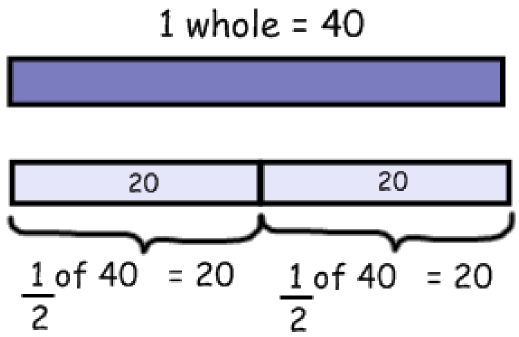
Throughout fraction and percentages work the word “of” represents a multiplication.

To find a fraction of a quantity you divide by the denominator and multiply by the numerator. If it is possible to simplify the fraction first by finding an equivalent fraction then this will make the calculation easier.

1. $\frac{1}{2}$ of 40

$= 40 \div 2$

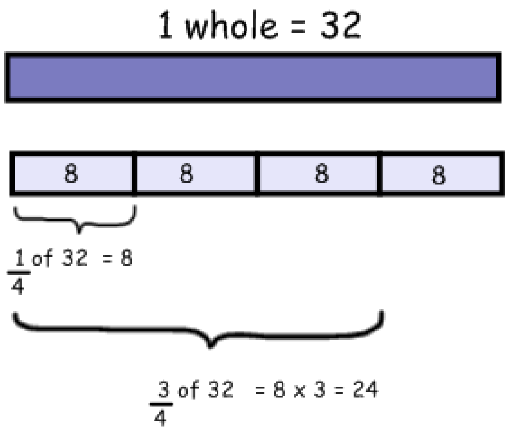
$= 20$



2. Calculate $\frac{3}{4}$ of 32

$\frac{1}{4}$ of 32 $= 32 \div 4 = 8$

$\frac{3}{4}$ of 32 $= 8 \times 3 = 24$



Mixed Numbers and Improper Fractions

If the numerator of a fraction is smaller than the denominator then the fraction is called a proper fraction.

If the numerator is larger than the denominator then the fraction is an improper fraction.

If a number comprises a whole number and a fraction then it is called a mixed number.

Examples

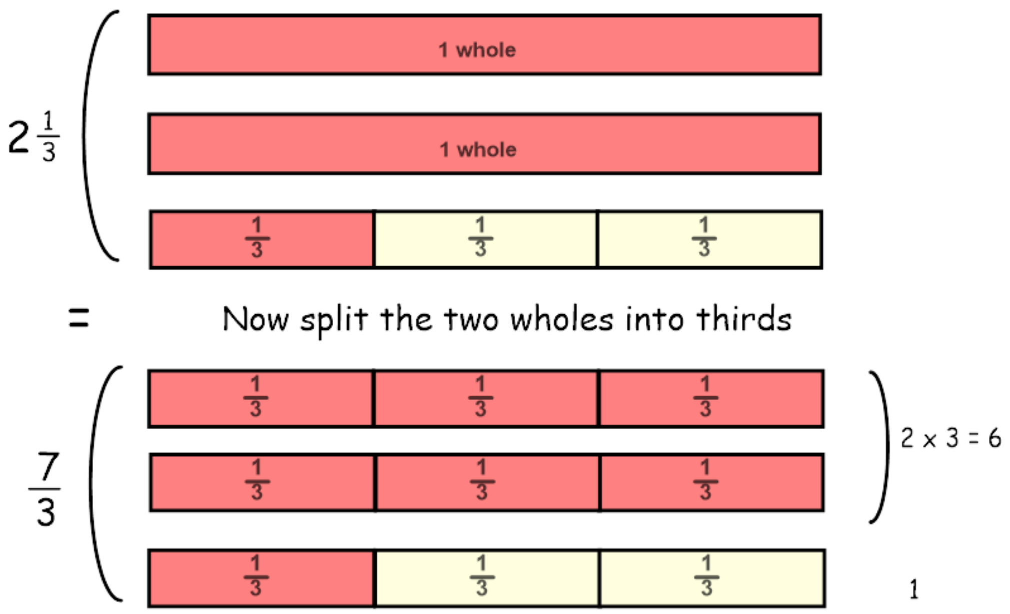
$\frac{1}{3}$ is a proper fraction $\frac{9}{4}$ is an improper fraction $3\frac{4}{5}$ is a mixed number

Conversion Between Mixed Numbers and Improper Fractions

To write a mixed number as an improper fraction we convert the whole number to a fraction and then add the existing fraction on.

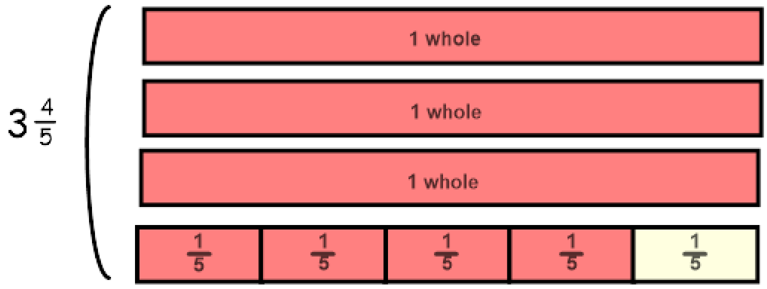
Mixed Numbers to Improper Fractions

1. Convert $2\frac{1}{3}$ into an improper fraction.



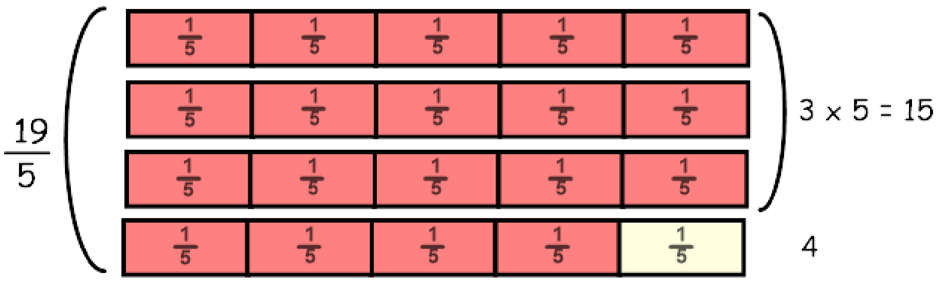
$$2\frac{1}{3} = \frac{7}{3}$$

2. Convert $3\frac{4}{5}$ into an improper fraction.



=

Now split the three wholes into fifths



$$3\frac{4}{5} = \frac{19}{5}$$

3. Convert $12\frac{5}{7}$ into an improper fraction.

Calculate how many sevenths are in 12.

$12 \times 7 = 84$ sevenths

Now add on the fraction.

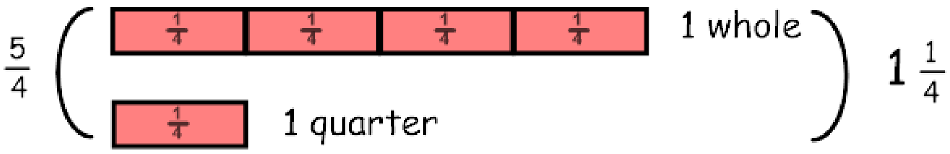
84 sevenths + 5 sevenths = 89 sevenths

$$12\frac{5}{7} = \frac{89}{7}$$

Improper Fraction to Mixed Number

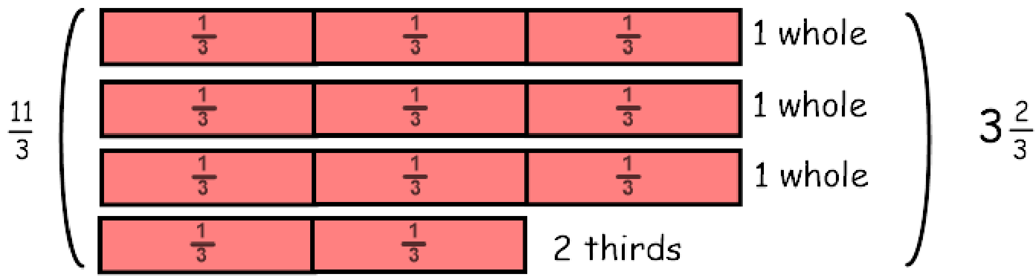
To write an improper fraction as a mixed number we calculate how many whole numbers there are (by considering multiples of the denominator) and then the remaining fraction part.

1. Convert $\frac{5}{4}$ into a mixed number.



$$\frac{5}{4} = 1\frac{1}{4}$$

2. Convert $\frac{11}{3}$ into a mixed number.



$$\frac{11}{3} = 3\frac{2}{3}$$

3. Convert $\frac{19}{4}$ into a mixed number

19 ÷ 4 = 4 remainder 3

The whole number in your answer

The number of quarters left over

$$\frac{19}{4} = 4\frac{3}{4}$$

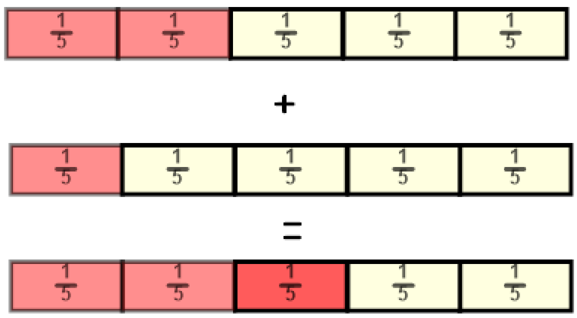
Adding and Subtracting Fractions

When adding and subtracting fractions, the denominators of the fractions **<u>must be the same</u>**. If the denominators are different then we find the lowest common multiple of the denominators (the lowest number in both times tables) and use equivalent fractions (see page 20).

1.

$$\frac{2}{5} + \frac{1}{5}$$

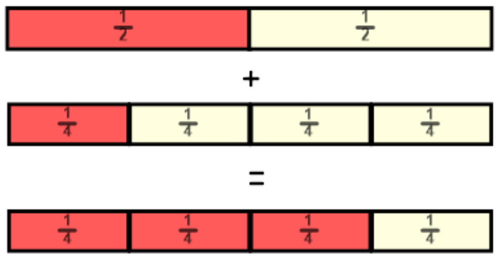
$$= \frac{3}{5}$$



2.

$$\frac{1}{2} + \frac{1}{4}$$

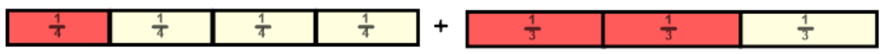
$$= \frac{3}{4}$$



(As you can see is equivalent to $\frac{2}{4}$)

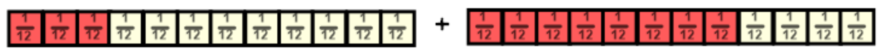
3.

$$\frac{1}{4} + \frac{2}{3}$$

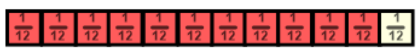


The lowest common multiple of 3 and 4 is 12.
Write both fractions as equivalent twelfths

$$= \frac{3}{12} + \frac{8}{12}$$



$$= \frac{11}{12}$$



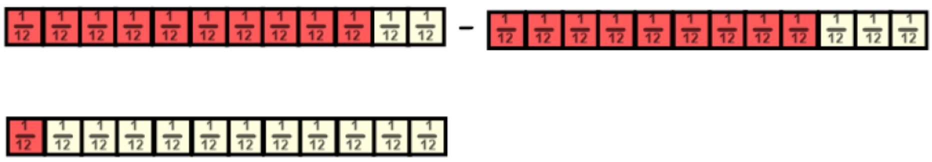
4.

$$\frac{5}{6} - \frac{3}{4} \quad (\times 2,\ \times 3)$$

$$= \frac{10}{12} - \frac{9}{12}$$

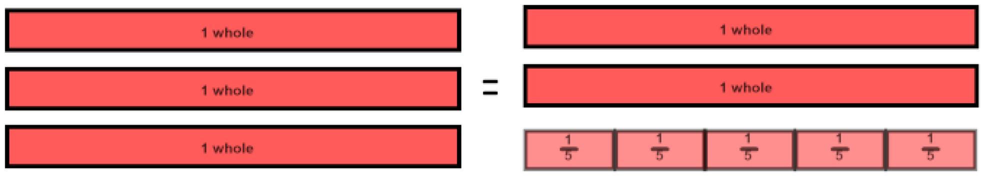
$$= \frac{1}{12}$$

The lowest common multiple of 6 and 4 is 12.
Write both fractions as equivalent twelfths.



5.

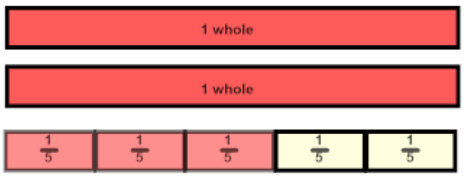
$$3 - \frac{2}{5}$$



Change 1 whole into fifths

Now we can easily subtract two fifths.

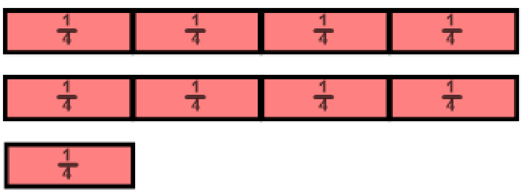
$$= 2\frac{3}{5}$$



6.

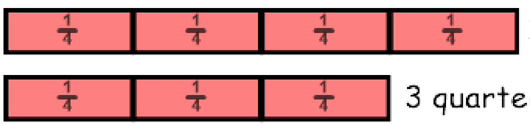
$$\frac{9}{4} - \frac{2}{4}$$

$\frac{9}{4}$



Subtract 2 quarters

$\frac{7}{4}$



1 whole

3 quarters

$1\frac{3}{4}$

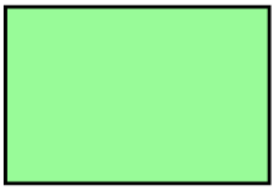
$$\frac{9}{4} - \frac{2}{4}$$

$$= \frac{7}{4}$$

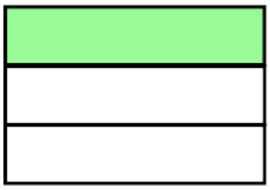
$$= 1\frac{3}{4}$$

Multiplying Fractions

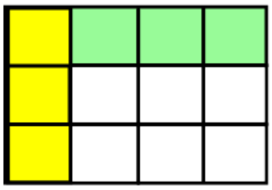
1. $\frac{1}{3} \times \frac{1}{4}$



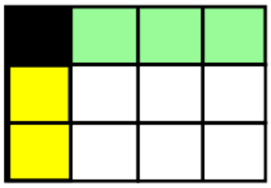
When multiplying a fraction by another fraction, start with one whole and then split it up to show the first fraction of the multiplication (thirds in this case, so divide by three).



Now you split the third into quarters (divide by four)

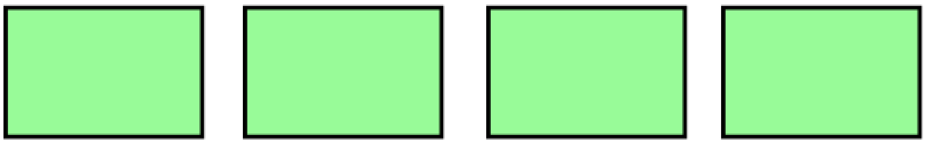


One quarter of one third is shown as the dark shaded section, one twelfth.



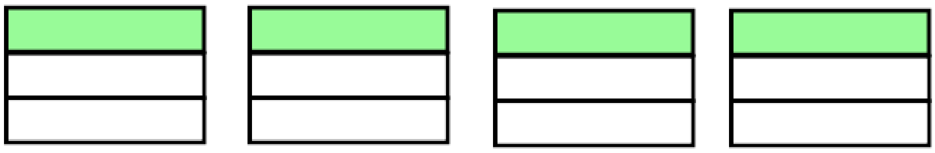
$$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

2. $4 \times \frac{1}{3}$



When multiplying a whole number by a fraction, start by showing the whole number.

Now split each whole into thirds (divide by three) to find one third of each whole.



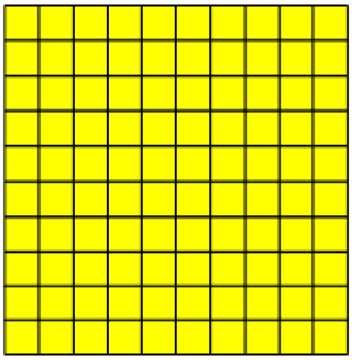
We have four lots of one third shaded.

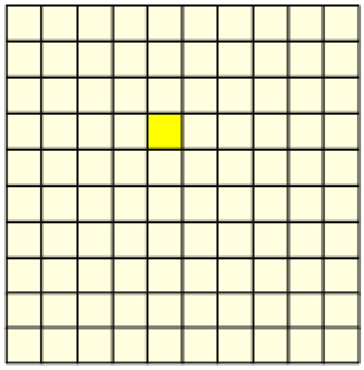
$$4 \times \frac{1}{3} = \frac{4}{3}$$

$$= 1\frac{1}{3}$$ (For converting between improper fractions and whole numbers see page 25)

Percentages

Percent refers to "out of 100".





One hundred hundredths = 1 whole = 100%

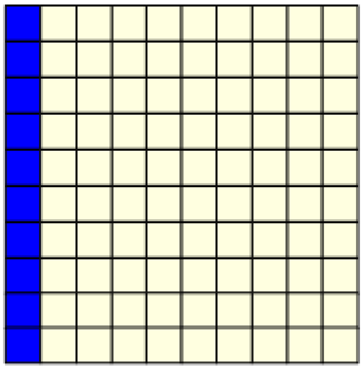
One hundredth = $\frac{1}{100}$ = 1%

Pupils are expected to know these commonly used percentages as fractions and decimals:

Percentage	Fraction	Decimal
1%	$\frac{1}{100}$	$0 \cdot 01$
10%	$\frac{1}{10}$	$0 \cdot 1$
20%	$\frac{1}{5}$	$0 \cdot 2$
$33\frac{1}{3}\%$	$\frac{1}{3}$	$0 \cdot 333\ldots = 0 \cdot \dot{3}$
50%	$\frac{1}{2}$	$0 \cdot 5$
$66\frac{2}{3}\%$	$\frac{2}{3}$	$0 \cdot 666\ldots = 0 \cdot \dot{6}$
75%	$\frac{3}{4}$	$0 \cdot 75$

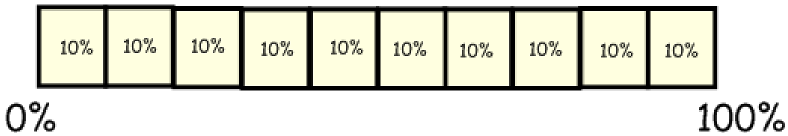
Finding a Percentage (non-calculator)

Finding 10% of a given value



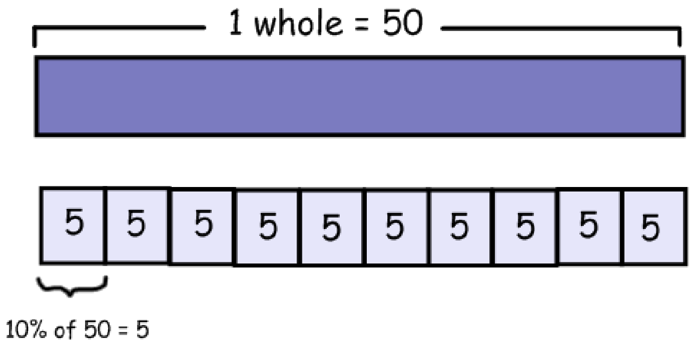
$$10\% = \frac{1}{10}$$

Using the same method from 'Fraction of a quantity' (Page 22) we know that when finding one tenth of a given value we divide by 10.



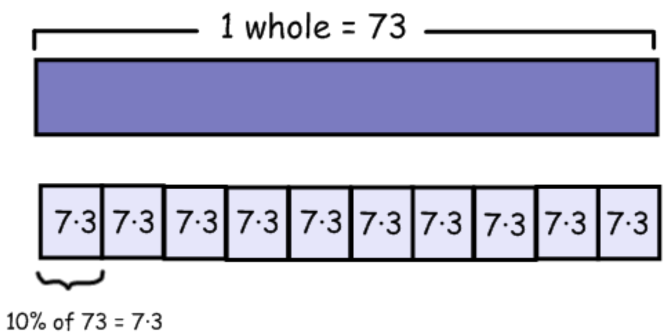
1. Find 10% of 50

$$10\% \text{ of } 50 = 50 \div 10$$
$$= 5$$



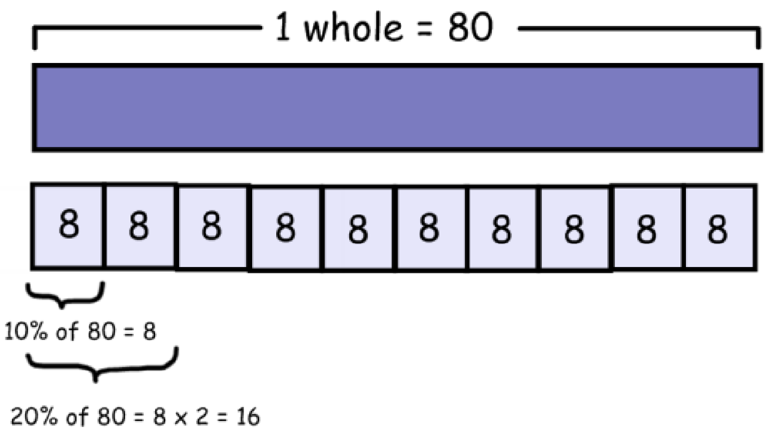
2. Find 10% of 73

$$10\% \text{ of } 73 = 73 \div 10$$
$$= 7{\cdot}3$$



Finding multiples of 10% of a given value

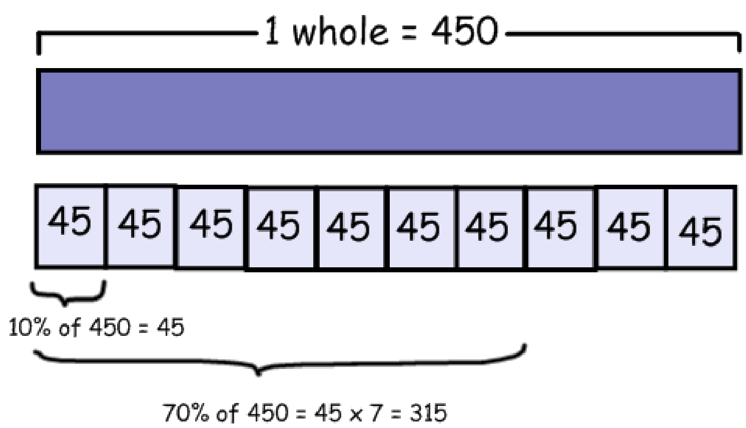
1. Find 20% of 80



10% of 80 = 80 ÷ 10
= 8

20% of 80 = 8 x 2
= 16

2. Find 70% of 450



10% of 450 = 450 ÷ 10
= 45

70% of 450 = 45 x 7
= 315

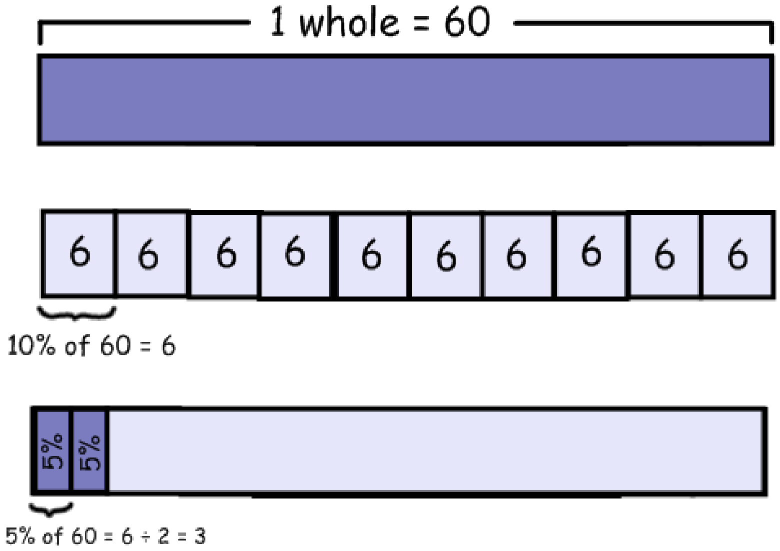
Finding 5% of a given value

To calculate 5% of a given quantity, find 10% and then divide your answer by 2
(10% ÷ 2 = 5%)

1. Find 5% of 60

   10% of 60 = 60 ÷ 10
   = 6

   5% of 60 = 6 ÷ 2
   = 3



Finding 1% of a given value (and multiples of 1%)

When we want to calculate 1% of a given quantity we divide by 100.

Using the same method from page 12; When dividing by 100 the digits move to the right by 2 places.

$$100\% \xrightarrow{\div 100} 1\%$$

1. Find 1% of 4000

1% of 4000 = 4000 ÷ 100
= 40

Thousands	Hundreds	Tens	Units
4	0	0	0
		4	0

2. Find 1% of 375

1% of 375 = 375 ÷ 100
= 3·75

Hundreds	Tens	Units	.	Tenths	Hundredths
3	7	5	·		
		3	·	7	5

3. Find 3% of 750

1% of 750 = 750 ÷ 100
= 7·5

Hundreds	Tens	Units	.	Tenths
7	5	0	·	
		7	·	5

3% of 750 = 7·5 x 3
= 22·5

4. Find 7% of 65

1% of 65 = 65 ÷ 100
= 0·65

Hundreds	Tens	.	Tenths	Hundredths
6	5	·		
	0	·	6	5

7% of 65 = 0·65 x 7
= 4·55

Finding any percentage of a given value

We can now use the above skills to calculate any percentage of a given quantity.

1. Find 13% of 420 Split 13% into 10% + 3%

10% of 420 = 420 ÷ 10
= 42

1% of 420 = 420 ÷ 100
= 4·2

x3

3% of 420 = 4·2 x 3
= 12·6

13% of 420 = 10% of 420 + 3% of 420
= 42 + 12·6
= 54·6

2. Find 37% of 710 Split 37% into 30% + 7%

10% of 710 = 710 ÷ 10
= 71

x3

30% of 710 = 71 x 3
= 213

1% of 710 = 710 ÷ 100
= 7·1

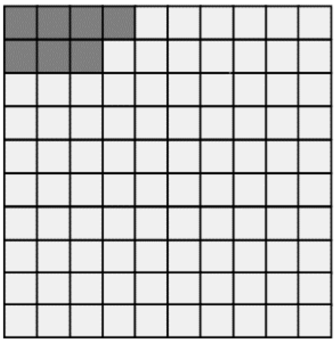
x7

7% of 710 = 7·1 x 7
= 49·7

37% of 710 = 30% of 710 + 7% of 710
= 213 + 49·7
= 262·7

Percentages on a Calculator

When calculating a percentage on a calculator we convert the percentage to a decimal by dividing it by 100.



7% of this grid is shaded

As a fraction this is $\frac{7}{100}$

$\frac{7}{100} = 7 \div 100 = 0{\cdot}07$

1. Find 7% of 150

7% as a fraction

$$\frac{7}{100} \times 150$$

Divide 7 by 100 to change into a decimal

$$= 0{\cdot}07 \times 150$$

$$= 10{\cdot}5$$

2. Find 81% of £35

81% as a fraction

$$\frac{81}{100} \times 35$$

Divide 81 by 100 to change into a decimal

$$= 0{\cdot}81 \times 35$$

$$= £28{\cdot}35$$

A as a percentage of B

To represent something as a percentage of something else we first of all write it as a fraction, then a decimal and then finally multiply by 100.

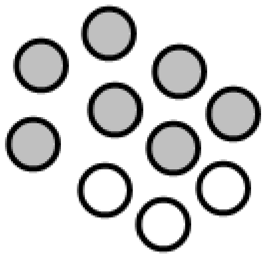
We can complete these calculations without a calculator if we can write an equivalent fraction (see page 20) with the new denominator being 100, as shown below.

1. What percentage of the marbles are grey?

Number of grey marbles

Number of marbles altogether

$$\frac{7}{10} \overset{\times 10}{=} \frac{70}{100} \quad (\times 10)$$



Therefore, 70% of the marbles are grey.

We will need to use a calculator when we cannot write an equivalent fraction.

2. John got 11 out of 15 for a recent Maths assessment.
   What was his score as a percentage?

write the score as a fraction — change to a decimal — change to a percentage

$$\frac{11}{15} = 11 \div 15 \times 100$$

$$= 73 \cdot 3\% \text{ to 1 decimal place}$$

3. What is £12 as a percentage of £48?

Write as a fraction — Change to a decimal — Change to a percentage

$$\frac{12}{48} = 12 \div 48 \times 100$$

$$= 25\%$$

Reverse the change

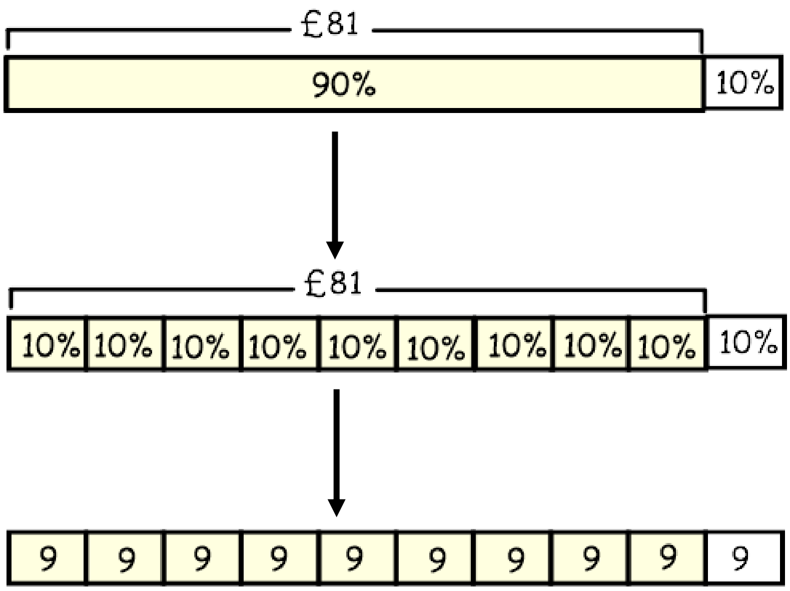
To help us find the original value of something after a percentage increase or decrease we use a bar model. The original value of something is always going to be 100%.

1. I bought a jacket for £81 in the sale. It had 10% off.

   How much did it originally cost?

   The original value has decreased, so we **subtract** this from 100%.

   What % did I get it for? 100% - 10% = 90%



81 ÷ 9 = 9

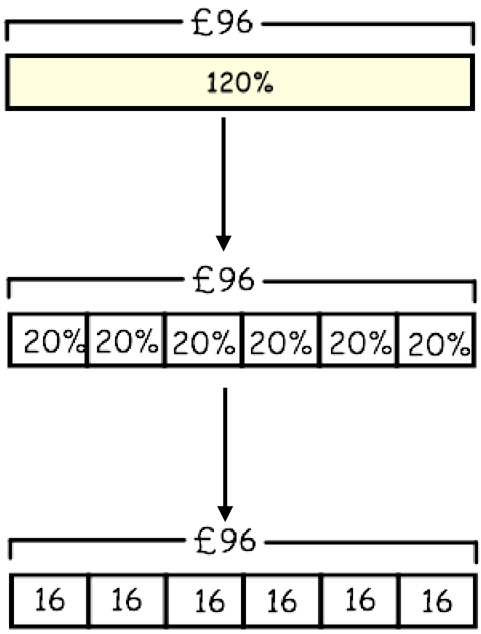
We have shown that each 10% is worth 9.

Therefore, original price is 10 x 9= £90

2. The gas company have increased their monthly charges by 20%. The average gas bill is now £96 per month. How much was the original average gas bill?

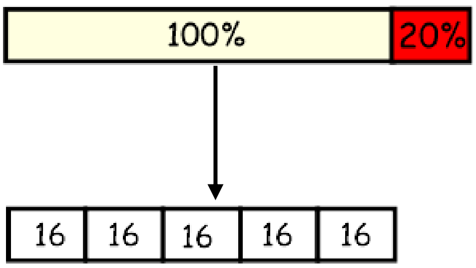
The original value has increased, so we **add** this to 100%.

What % are the monthly bills? 100% + 20% = 120%



96 ÷ 6 = 16

What do you have to do to get 100% → subtract 20%



We have shown that 20% is worth 16.

Therefore, original price is 5 x 16= £80.