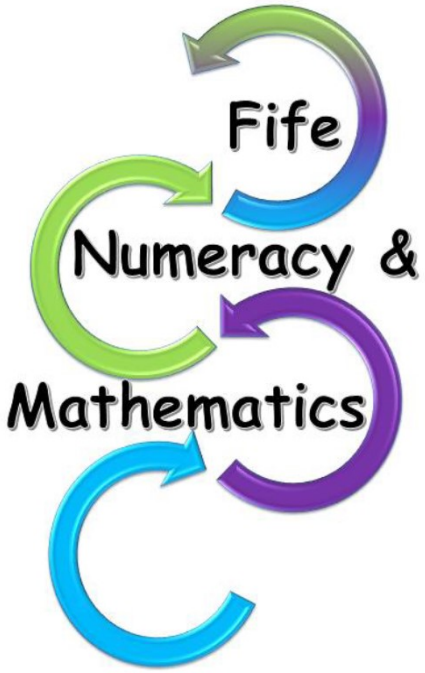
A Better Connected Directorate

Establishing the Approach to the Effective Teaching of Numeracy Across Children's Services



The Importance of Numeracy Skills

Being numerate helps us to function responsibly in everyday life and contribute effectively to society. It increases our opportunities within the world of work and establishes foundations which can be built upon through lifelong learning. Numeracy is not only a subset of mathematics; it is also a life skill which permeates and supports all areas of learning, allowing young people access to the wider curriculum.

The National Context: Policy and Practice

All schools, working with their partners, need to have strategies to ensure that all children and young people develop high levels of numeracy skills through their learning across the curriculum. These strategies will be built upon a shared understanding amongst staff of how children and young people progress in numeracy and of good learning and teaching in numeracy.

Numeracy Across Learning Principles and Practices

Education Scotland (2008)

Fife's Approach: Developing a Conceptual Understanding in Numeracy

Sitting within Fife's wider Teaching and Learning strategy, the mathematical pedagogical position of Fife, backed up by years of Worldwide research, is that conceptual understanding must come before procedural learning. Conceptual understanding is knowing more than isolated facts and methods. The successful learner understands mathematical ideas, and has the ability to transfer their knowledge into new situations while applying and adapting it to new contexts.

Conceptual Knowledge is:

"Learning mathematics with understanding, actively building new knowledge from experience and prior knowledge."

Principles and Standards for School Mathematics (2000)

"Ideas, relationships, connections, or having a *'sense'* of something."

Barr, Doyle et. Al. (2003)

"Learning that involves understanding and interpreting concepts and the relationships between concepts."

Arslan (2010)

"...learning in which students take the reins of their own learning and are able to apply their thinking to new contexts and situations."

Hattie (2017)

National and Fife Policy Documentation

Fife Numeracy & Mathematics Progression Documentation (2017)

First Steps Mathematics (2013), Department of Education Western Australia

Mathematics Recovery, Southern Cross University, New South Wales

Teaching Number Series (2011), Robert Wright

Australian Curriculum, Assessment and Reporting Authority

Education Endowment Foundation (2016)

Mathematics & Numeracy Benchmarks (2017), Education Scotland

CfE: Numeracy Across Learning: Principles and Practice (2008), Education Scotland

CfE: Numeracy & Mathematics: Experiences & Outcomes (2008), Education Scotland

What does the effective teaching of numeracy look like across Fife's learning communities?

"Good numeracy is the best protection against unemployment, low wages and poor health"

Andreas Schleicher, The Organisation for Economic Co-operation

Effective teaching of numeracy is inclusive with all learners' needs taken into account. The use of flexible/fluid groups, maths talk, enquiry, collaborative learning and problem solving are all encouraged. Learners' mind-sets and attitudes should be considered and nurtured to ensure positivity while building resilience. Research shows that, practitioners who employ effective questioning techniques which lead to responsive feedback being given to learners, increase attainment levels. Numeracy experiences should be planned with the needs of individuals and groups of children and young people at the centre.

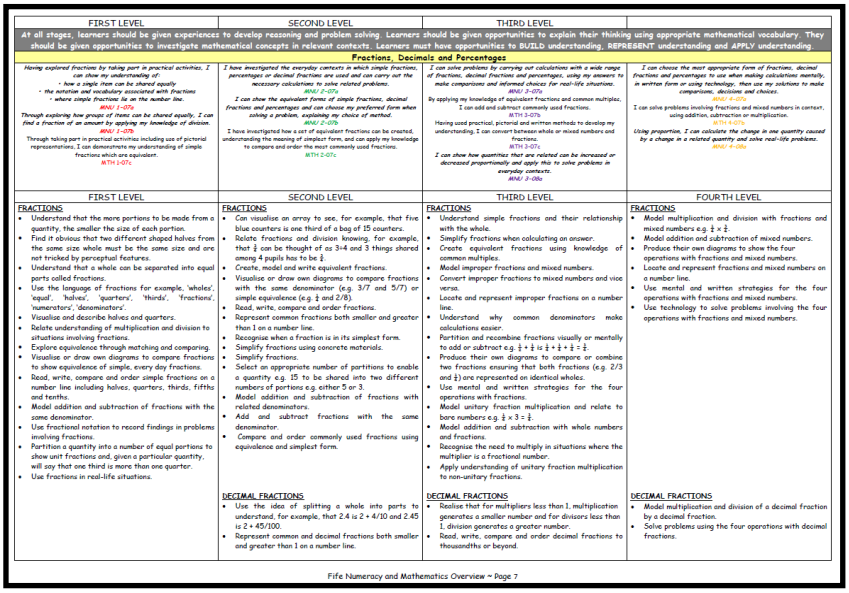
How Does the Fife Numeracy & Mathematics Documentation Support This Vision?

The Fife Numeracy and Mathematics Documentation has been developed to support practitioners in their effective planning and tracking of pupil progress in their mathematical learning journeys. They promote the mathematical pedagogical positon of Fife that conceptual understanding must come before procedural learning, while providing practitioners with the tools to enable them to effectively and comprehensively implement the approach in their everyday teaching and learning. The documentation covers the Broad General Education in its entirety from Early through to Fourth Level, including links to National 5, to support and promote breadth and depth of understanding along with appropriate pace and challenge to meet the needs of all learners. The support materials exist as three separate documents; Overviews, Progression Pathways and Records of Understanding.

Overviews

The Overviews provide a developmental progression, illustrating how learners build, represent and apply their understanding. They offer staff a clear outline of key aspects within and across both curriculum organisers and levels. By using the overviews, practitioners are able to easily identify where learners have been and where they are currently, while planning next steps in learning.

There are two Overview documents – Early to Third Level and First to Fourth Level.



FIRST LEVEL | SECOND LEVEL | THIRD LEVEL

At all stages, learners should be given experiences to develop reasoning and problem solving. Learners should be given opportunities to explain their thinking using appropriate mathematical vocabulary. They should be given opportunities to investigate mathematical concepts in relevant contexts. Learners must have opportunities to BUILD understanding, REPRESENT understanding and APPLY understanding.

Fractions, Decimals and Percentages

Having explored fractions by taking part in practical activities, I can show my understanding of:
- how a single item can be shared equally
- the notation and vocabulary associated with fractions
- where simple fractions lie on the number line.

MNU 1-07a

Through exploring how groups of items can be shared equally, I can find a fraction of an amount by applying my knowledge of division.

MNU 1-07b

Through taking part in practical activities including use of pictorial representations, I can demonstrate my understanding of simple fractions which are equivalent.

MTH 1-07c

I have investigated the everyday contexts in which simple fractions, percentages or decimal fractions are used and can carry out the necessary calculations to solve related problems.

MNU 2-07a

I can show the equivalent forms of simple fractions, decimal fractions and percentages and can choose my preferred form when solving a problem, explaining my choice of method.

MNU 2-07b

I have investigated how a set of equivalent fractions can be created, understanding the meaning of simplest form, and can apply my knowledge to compare and order the most commonly used fractions.

MTH 2-07c

I can solve problems by carrying out calculations with a wide range of fractions, decimal fractions and percentages, using my answers to make comparisons and informed choices for real-life situations.

MNU 3-07a

By applying my knowledge of equivalent fractions and common multiples, I can add and subtract commonly used fractions.

MTH 3-07b

Having used practical, pictorial and written methods to develop my understanding, I can convert between whole or mixed numbers and fractions.

MTH 3-07c

I can show how quantities that are related can be increased or decreased proportionally and apply this to solve problems in everyday contexts.

MNU 3-08a

FIRST LEVEL

FRACTIONS
- Understand that the more portions to be made from a quantity, the smaller the size of each portion.
- Find it obvious that two different shaped halves from the same size whole must be the same size and are not tricked by perceptual features.
- Understand that a whole can be separated into equal parts called fractions.
- Use the language of fractions for example, 'wholes', 'equal', 'halves', 'quarters', 'thirds', 'fractions', 'numerators', 'denominators'.
- Visualise and describe halves and quarters.
- Relate understanding of multiplication and division to situations involving fractions.
- Explore equivalence through matching and comparing.
- Visualise or draw own diagrams to compare fractions to show equivalence of simple, every day fractions.
- Read, write, compare and order simple fractions on a number line including halves, quarters, thirds, fifths and tenths.
- Model addition and subtraction of fractions with the same denominator.
- Use fractional notation to record findings in problems involving fractions.
- Partition a quantity into a number of equal portions to show unit fractions and, given a particular quantity, will say that one third is more than one quarter.
- Use fractions in real-life situations.

SECOND LEVEL

FRACTIONS
- Can visualise an array to see, for example, that five blue counters is one third of a bag of 15 counters.
- Relate fractions and division knowing, for example, that $\frac{1}{4}$ can be thought of as 3÷4 and 3 things shared among 4 pupils has to be $\frac{1}{4}$.
- Create, model and write equivalent fractions.
- Visualise or draw own diagrams to compare fractions with the same denominator (e.g. 3/7 and 5/7) or simple equivalence (e.g. $\frac{1}{2}$ and 2/8).
- Read, write, compare and order fractions.
- Represent common fractions both smaller and greater than 1 on a number line.
- Recognise when a fraction is in its simplest form.
- Simplify fractions using concrete materials.
- Simplify fractions.
- Select an appropriate number of partitions to enable a quantity e.g. 15 to be shared into two different numbers of portions e.g. either 5 or 3.
- Model addition and subtraction of fractions with related denominators.
- Add and subtract fractions with the same denominator.
- Compare and order commonly used fractions using equivalence and simplest form.

DECIMAL FRACTIONS
- Use the idea of splitting a whole into parts to understand, for example, that 2.4 is 2 + 4/10 and 2.45 is 2 + 45/100.
- Represent common and decimal fractions both smaller and greater than 1 on a number line.

THIRD LEVEL

FRACTIONS
- Understand simple fractions and their relationship with the whole.
- Simplify fractions when calculating an answer.
- Create equivalent fractions using knowledge of common multiples.
- Model improper fractions and mixed numbers.
- Convert improper fractions to mixed numbers and vice versa.
- Locate and represent improper fractions on a number line.
- Understand why common denominators make calculations easier.
- Partition and recombine fractions visually or mentally to add or subtract e.g. $\frac{1}{4} + \frac{1}{4}$ is $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$.
- Produce their own diagrams to compare or combine two fractions ensuring that both fractions (e.g. 2/3 and $\frac{1}{2}$) are represented on identical wholes.
- Use mental and written strategies for the four operations with fractions.
- Model unitary fraction multiplication and relate to bare numbers e.g. $\frac{1}{4} \times 3 = \frac{3}{4}$.
- Model addition and subtraction with whole numbers and fractions.
- Recognise the need to multiply in situations where the multiplier is a fractional number.
- Apply understanding of unitary fraction multiplication to non-unitary fractions.

DECIMAL FRACTIONS
- Realise that for multipliers less than 1, multiplication generates a smaller number and for divisors less than 1, division generates a greater number.
- Read, write, compare and order decimal fractions to thousandths or beyond.

FOURTH LEVEL

FRACTIONS
- Model multiplication and division with fractions and mixed numbers e.g. $\frac{1}{2} \times \frac{1}{2}$.
- Model addition and subtraction of mixed numbers.
- Produce their own diagrams to show the four operations with fractions and mixed numbers.
- Locate and represent fractions and mixed numbers on a number line.
- Use mental and written strategies for the four operations with fractions and mixed numbers.
- Use technology to solve problems involving the four operations with fractions and mixed numbers.

DECIMAL FRACTIONS
- Model multiplication and division of a decimal fraction by a decimal fraction.
- Solve problems using the four operations with decimal fractions.

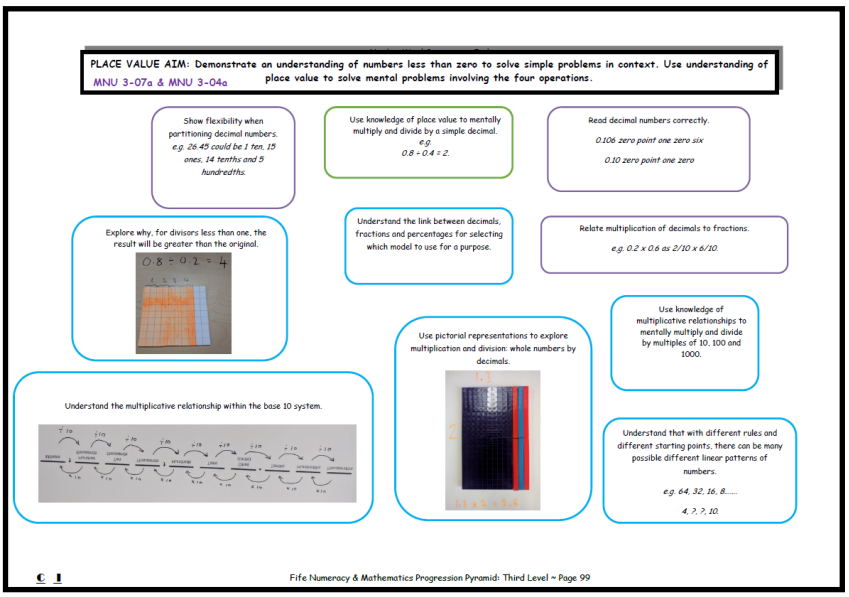
Fife Numeracy and Mathematics Overview ~ Page 7

Progression Pathways

The Fife Numeracy and Mathematics Progression Pathways determine a clear set of learning experiences and outcomes from each of the three curriculum organisers within Numeracy and Mathematics, namely:

- Number, Money and Measure
- Shape, Position and Movement
- Information Handling

The Pathways assist practitioners in their planning of effective pedagogy in conceptual understanding of mathematics to meet individual learners' needs.



PLACE VALUE AIM: Demonstrate an understanding of numbers less than zero to solve simple problems in context. Use understanding of place value to solve mental problems involving the four operations.

MNU 3-07a & MNU 3-04a

Show flexibility when partitioning decimal numbers.
e.g. 26.45 could be 1 ten, 15 ones, 14 tenths and 5 hundredths.

Use knowledge of place value to mentally multiply and divide by a simple decimal.
e.g.
0.8 ÷ 0.4 = 2.

Read decimal numbers correctly.
0.106 zero point one zero six
0.10 zero point one zero

Explore why, for divisors less than one, the result will be greater than the original.

Understand the link between decimals, fractions and percentages for selecting which model to use for a purpose.

Relate multiplication of decimals to fractions.
e.g. 0.2 x 0.6 as 2/10 x 6/10.

Use knowledge of multiplicative relationships to mentally multiply and divide by multiples of 10, 100 and 1000.

Use pictorial representations to explore multiplication and division: whole numbers by decimals.

Understand the multiplicative relationship within the base 10 system.

Understand that with different rules and different starting points, there can be many possible different linear patterns of numbers.
e.g. 64, 32, 16, 8.....
4, ?, ?, 10.

Fife Numeracy & Mathematics Progression Pyramid: Third Level ~ Page 99

Progressions Pathways (cont.)

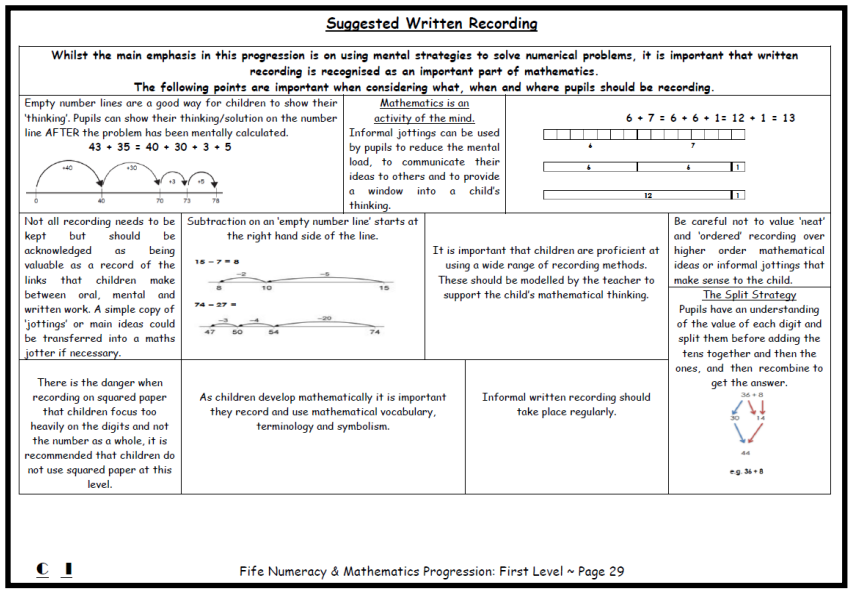
The purpose of the document is to provide a continuum of learning within a level and progression through levels. The developmental stages of learning in numeracy are clearly documented and this will support teachers when identifying starting points for learners. The National Benchmarks are embedded within the document.

Each strand is shown as a pathway, demonstrating how learning and teaching progress within this. The skills at the base of the pathway are required to be understood for further learning to be built upon. Learners will progress through the pathways as and when they are ready and able to do so, following a path which best meets their learning needs.

The 'Suggested Written Recording' section explores ways in which learners might record and represent their mathematical thinking. (They highlight the importance of allowing learners to build links between their mental and written methods, allowing for greater opportunities for them to use and apply their strategies. Once conceptual understanding is embedded this leads to more formal methods of recording.)

The 'Points to Consider' provide detailed descriptions of the key ideas or concepts and provide clear and concise guidance for teachers. Common learner misconceptions are also highlighted within this section.

The Progression Pathways, as a document in its entirety, focus on developing increasingly sophisticated and refined conceptual understanding, fluency, logical reasoning, analytical thought and problem solving skills



Suggested Written Recording

Whilst the main emphasis in this progression is on using mental strategies to solve numerical problems, it is important that written recording is recognised as an important part of mathematics.
The following points are important when considering what, when and where pupils should be recording.

Empty number lines are a good way for children to show their 'thinking'. Pupils can show their thinking/solution on the number line AFTER the problem has been mentally calculated.
43 + 35 = 40 + 30 + 3 + 5

Mathematics is an activity of the mind. Informal jottings can be used by pupils to reduce the mental load, to communicate their ideas to others and to provide a window into a child's thinking.

6 + 7 = 6 + 6 + 1= 12 + 1 = 13

Not all recording needs to be kept but should be acknowledged as being valuable as a record of the links that children make between oral, mental and written work. A simple copy of 'jottings' or main ideas could be transferred into a maths jotter if necessary.

Subtraction on an 'empty number line' starts at the right hand side of the line.

It is important that children are proficient at using a wide range of recording methods. These should be modelled by the teacher to support the child's mathematical thinking.

Be careful not to value 'neat' and 'ordered' recording over higher order mathematical ideas or informal jottings that make sense to the child.

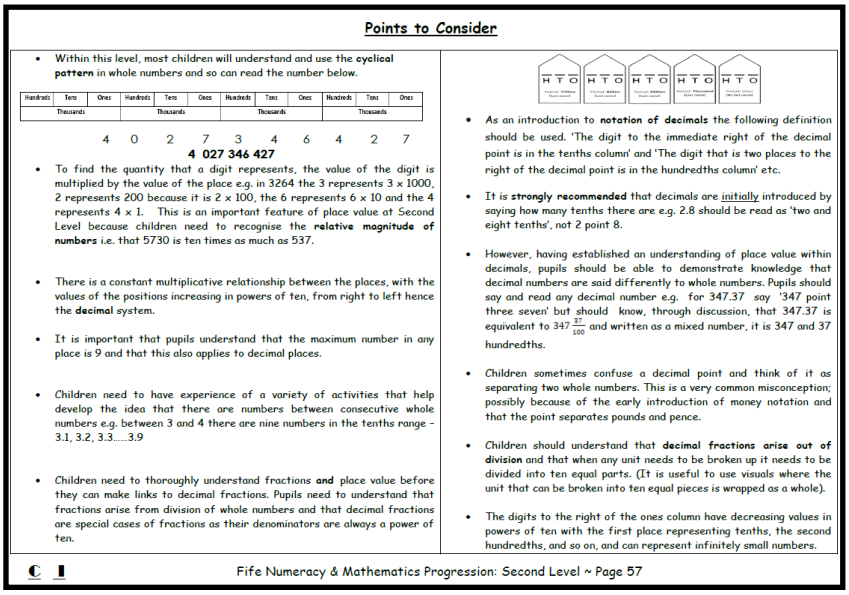
The Split Strategy
Pupils have an understanding of the value of each digit and split them before adding the tens together and then the ones, and then recombine to get the answer.
e.g. 36 + 8

There is the danger when recording on squared paper that children focus too heavily on the digits and not the number as a whole, it is recommended that children do not use squared paper at this level.

As children develop mathematically it is important they record and use mathematical vocabulary, terminology and symbolism.

Informal written recording should take place regularly.

Fife Numeracy & Mathematics Progression: First Level ~ Page 29



Points to Consider

- Within this level, most children will understand and use the **cyclical pattern** in whole numbers and so can read the number below.

Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
Thousands			Thousands			Thousands			Thousands		
4	0	2	7	3	4	6	4	2	7		

4 027 346 427

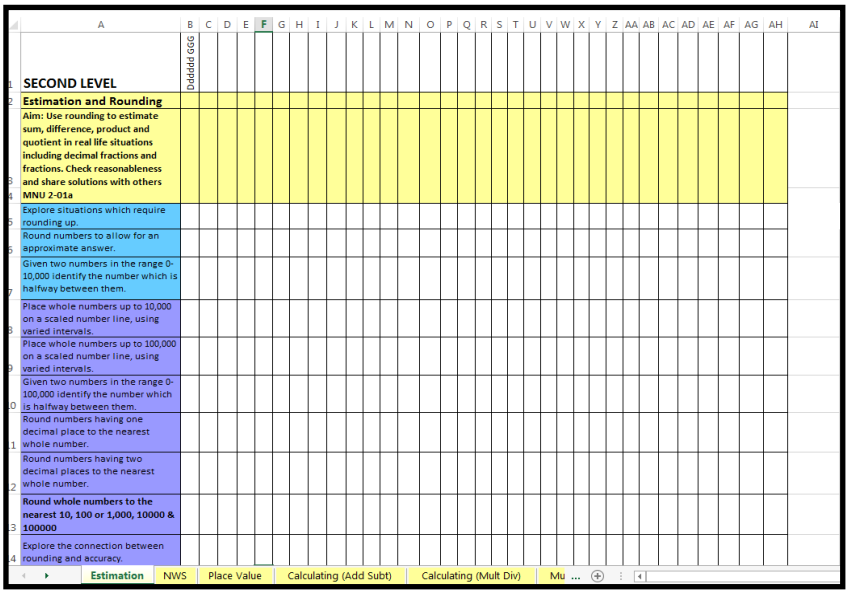
- To find the quantity that a digit represents, the value of the digit is multiplied by the value of the place e.g. in 3264 the 3 represents 3 x 1000, 2 represents 200 because it is 2 x 100, the 6 represents 6 x 10 and the 4 represents 4 x 1. This is an important feature of place value at Second Level because children need to recognise the **relative magnitude of numbers** i.e. that 5730 is ten times as much as 537.
- There is a constant multiplicative relationship between the places, with the values of the positions increasing in powers of ten, from right to left hence the **decimal** system.
- It is important that pupils understand that the maximum number in any place is 9 and that this also applies to decimal places.
- Children need to have experience of a variety of activities that help develop the idea that there are numbers between consecutive whole numbers e.g. between 3 and 4 there are nine numbers in the tenths range - 3.1, 3.2, 3.3.....3.9
- Children need to thoroughly understand fractions **and** place value before they can make links to decimal fractions. Pupils need to understand that fractions arise from division of whole numbers and that decimal fractions are special cases of fractions as their denominators are always a power of ten.

- As an introduction to **notation of decimals** the following definition should be used. 'The digit to the immediate right of the decimal point is in the tenths column' and 'The digit that is two places to the right of the decimal point is in the hundredths column' etc.
- It is **strongly recommended** that decimals are initially introduced by saying how many tenths there are e.g. 2.8 should be read as 'two and eight tenths', not 2 point 8.
- However, having established an understanding of place value within decimals, pupils should be able to demonstrate knowledge that decimal numbers are said differently to whole numbers. Pupils should say and read any decimal number e.g. for 347.37 say '347 point three seven' but should know, through discussion, that 347.37 is equivalent to 347 $\frac{37}{100}$ and written as a mixed number, it is 347 and 37 hundredths.
- Children sometimes confuse a decimal point and think of it as separating two whole numbers. This is a very common misconception; possibly because of the early introduction of money notation and that the point separates pounds and pence.
- Children should understand that **decimal fractions arise out of division** and that when any unit needs to be broken up it needs to be divided into ten equal parts. (It is useful to use visuals where the unit that can be broken into ten equal pieces is wrapped as a whole).
- The digits to the right of the ones column have decreasing values in powers of ten with the first place representing tenths, the second hundredths, and so on, and can represent infinitely small numbers.

Fife Numeracy & Mathematics Progression: Second Level ~ Page 57

Records of Understanding

This tool allows professionals to effectively track the developmental progression of individual learners. Each statement tile within the Progression Pathways is replicated within the Record of Understanding. As well as recording pupil progression, the document also acts as an effective tool to aid identification of any gaps in learning or misconceptions that exist. To ensure pace and challenge of learning is maintained between stages and sectors a consistent approach across schools (or even clusters) is recommended. This tracking and monitoring document also assists practitioners to justify declarations of achievement for their learners.



	A	B
1	SECOND LEVEL	Ddddd GGG
2	**Estimation and Rounding**	
3	**Aim: Use rounding to estimate sum, difference, product and quotient in real life situations including decimal fractions and fractions. Check reasonableness and share solutions with others MNU 2-01a**	
4		
5	Explore situations which require rounding up.	
6	Round numbers to allow for an approximate answer.	
7	Given two numbers in the range 0-10,000 identify the number which is halfway between them.	
8	Place whole numbers up to 10,000 on a scaled number line, using varied intervals.	
9	Place whole numbers up to 100,000 on a scaled number line, using varied intervals.	
10	Given two numbers in the range 0-100,000 identify the number which is halfway between them.	
11	Round numbers having one decimal place to the nearest whole number.	
12	Round numbers having two decimal places to the nearest whole number.	
13	**Round whole numbers to the nearest 10, 100 or 1,000, 10000 & 100000**	
14	Explore the connection between rounding and accuracy.	

Estimation | NWS | Place Value | Calculating (Add Subt) | Calculating (Mult Div) | Mu ...

Assessment

Assessment of a learner's progress and achievement should always be based upon a teacher's assessment of their knowledge, understanding and skills in curriculum areas. Teachers assess learning in mathematics and numeracy using a variety of approaches and a wide range of evidence.

Baseline and diagnostic assessment methods are highly effective ways of monitoring and tracking pupil understanding and progress. Using these types of assessments allow practitioners to identify, plan, implement and judge the effectiveness of the learning experiences they are providing for their learners. The *First Steps in Mathematics* series of resources, developed by Western Australia's Department of Education, provide many excellent examples of diagnostic assessments that can be used with learners. Further examples of baseline assessment are available to access on the Pedagogy Team's Glow site.

Flexible & Fluid Groupings

Flexible and fluid groupings are an integral part of a differentiated classroom. As the name suggests, groups should be dynamic and changing depending upon the learning experiences as opposed to fixed or static. For example, although learners may be grouped by ability using information from baseline assessments, these ability groups may change depending upon topics and/or progress made by learners. Similarly, groupings may be mixed as learners work collaboratively on tasks. Differentiation is the key to meeting all learners' needs.

Maths Talk

Learners should be encouraged to discuss their mathematical thinking on a daily basis. This not only builds confidence, but allows others to hear methods and strategies that they may not have considered. It can also assist practitioners to highlight and address any misconceptions before they become embedded. Opportunities to link learning experiences to real life situations should be embraced wherever possible, allowing learners to make connections to the World of Work and Employability.

"The most effective grouping strategy is one that is flexible and balanced, and that allows for a moderate but not extreme range of skill levels"

Hattie (2017)

"Describing one's methods to another person can clarify one's own thinking as well as clarify the matter for others."

Fuson (2015)

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