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ACKNOWLEDGEMENT

This programme is extensively based on proven approaches and strategies defined within the First Steps in Mathematics resources, Maths Recovery and New Zealand programmes.

Unlike many resources that present mathematical concepts that have been logically ordered and prioritized by mathematicians or educators, First Steps in Mathematics follows a sequence derived from the mathematical development of real children. It is based on five years of research by a team of teachers from the Western Australia Department of Education and Training, and tertiary consultants led by Professor Sue Willis at Murdoch University. The First Steps in Mathematics project team conducted an extensive review of international research literature, which revealed gaps in the field of knowledge about students' learning in mathematics. Using tasks designed to replicate those in the research literature, team members interviewed hundreds of elementary school children in diverse locations. Analysis of the data obtained from these interviews identified characteristic phases in the development of students' thinking about mathematical concepts.

Maths Recovery, founded on years of research by Dr Robert Wright, Professor in Maths Education within Southern Cross University in Australia and colleagues, is now internationally renowned in responding to problems of children's failure in early numeracy and primary mathematics and has been used extensively by many nations, including New Zealand, to develop their own standards and teaching approaches in mathematics.

We are also very grateful to the work done by both Angus and Highland Council. Their input has been key in the development of this progression



GUIDANCE

The Fife Numeracy and Mathematics Progression (Number, Money and Measure) sets out a clear set of learning experiences and outcomes from the following Curriculum for Excellence Numeracy and Mathematics strands: Estimation and Rounding Number and Number Processes Multiples, Factors and Primes Powers and Roots Fractions, Decimals and Percentages Money Time Measurement Mathematics - its impact on the world, past, present and future Patterns and Relationships Expressions and Equations

The purpose of this document is to provide a continuum of learning both within a level and through the Early, First, Second and Third Levels. The developmental stages of learning in numeracy are clearly documented and this will support teachers when identifying starting points for learners. The progression is intended to assist teachers as they plan their numeracy and mathematics curriculum. Links are made throughout the document to the Benchmarks. However, it should be noted that, as the Benchmarks are the minimum requirement for a level, the language of "at least" has been used in order that ceilings are not placed on learners' experiences.

The 'Points to Consider' section provides detailed descriptions of key mathematical ideas or concepts and provides clear and concise guidance for teachers.

Each strand is shown as a pyramid to show how learning and teaching progress within this. The skills at the base of the pyramids are required to be understood for further learning to be built upon and are not aligned to any particular year group (at First Level, statements in blue do *not* equate to Primary 2, for example). Pupils will progress through the pyramids as and when they are ready and able to do so.

The Fife Numeracy and Mathematics Progression focuses on developing increasingly sophisticated and refined mathematical understanding, fluency, logical reasoning, analytical thought and problem solving skills which can be applied in unfamiliar situations.

Early Level Experiences and Outcomes

* The numeracy learner statements that are the responsibility of all are shown in bold italics.

I am developing a sense of size and amount by observing, exploring, using and communicating with others about things in the world around me. MNU 0-01a Pg 10 I have explored numbers, understanding that they represent quantities, and I can use them to count, create sequences and describe order. MNU 0-02a Pg's 11-13 I use practical materials and can 'count on and back' to help me to understand addition and subtraction, recording my ideas and solutions in different ways. MNU 0-03a Pg 14 I can share out a group of items by making smaller groups and can split a whole object into smaller parts. MNU 0-07a Pg 15 I am developing my awareness of how money is used and can recognise and use a range of coins. MNU 0-09a Pg 16 I am developing my awareness of how money and can split a whole object into smaller parts. MNU 0-09a Pg 16

I am aware of how routines and events in my world link with times and seasons, and have explored ways to record and display these using clocks, calendars and other methods.

MNU 0-10a Pg 17

I have experimented with everyday items as units of measure to investigate and compare sizes and amounts in my environment, sharing my findings with others.

MNU 0-11a Pg 18

I have spotted and explored patterns in my own and the wider environment and can copy and continue these and create my own patterns.

MTH 0-13a Pg 19

Points to Consider

- The key focus in **pre-counting** is an understanding of the concept **more**, **less** and **the same** and an appreciation of how these are related. Children at the early stages of Early level develop these concepts by comparison and no counting is involved.
- Children often have some concept of 'more'; this needs to be extended and refined. 'Less' is a more difficult concept and understanding can be developed by pairing the terms less and more to help develop an understanding of the relationship between the two.
- The **number ranges** e.g. 1 5 are guides only and should be adapted to suit the needs/ability levels of the pupils.
- **Representing numbers** in a variety of ways is essential for developing number sense e.g.



- **Counting** is an important component of number and the early learning of operations. There is a distinction between counting by rote (not necessarily having a **one-to-one correspondence**) and counting with understanding.
- Teaching the Counting Principles is essential for success.

Order-irrelevance principle: When counting the number of objects in a set, the order they are counted in is irrelevant, as long as each object is counted and pupils know and understand that the last number is 'the count'. Sometimes an item is counted more than once by pupils or two items are counted for one number, e.g.... se-ven so lots of practice is required.



Can start at any animal - the count stays the same.

- Once children become very familiar with counting collections they realise that numbers can be used to count **anything**.
- The **teen numbers** are often the most difficult for pupils. The oral language pattern of teen numbers is the reverse of the usual pattern of 'tens first and **then** ones', e.g....13 is 'thir' 'teen' and 31 is 'thirty' 'one'. There is often difficulty distinguishing between 12 and 21. Confusion can also arise when saying ty and teen endings.
- Children can and should become skilful at associating number words with numerals in the **teens** long before they understand that the left hand digit ('1') stands for the number 'ten'.
- Subitising involves immediately recognising the number of objects in a small collection without having to count the objects e.g. the dots on a dice. The pattern should be flashed to avoid the temptation to count. Six is generally the most anyone can individually recognise in one pattern.

• Collections should be presented in a variety of ways.



- Addition and Subtraction needs to move from counting and combining concrete materials to using abstract methods e.g. hiding materials behind screens and in boxes to develop the pupils' ability to visualise and internalise before introducing written number sentences.
- Numeral tracks, numeral rolls and hundred squares are all excellent tools to support children's understanding of **number sequences**, forwards and backwards.
- Addition and subtraction problems should be related to **real-life** experiences and stories.
- Wherever possible, addition and subtraction should be taught simultaneously to reinforce the concept of the **inverse operation**.

- Subtraction typically covers three different situations:
 - > 'Taking away' from a group (need to be confident at this before being introduced to the next two).
 - 'Comparing' two groups find the difference to a numeric value not any other attributes - see example of towers below.
 - > Finding 'how many more or less' are in a group.
- Children may have difficulty linking their ideas about addition and subtraction to situations involving the comparison of collections e.g. how much bigger is the tallest tower?



- All activities should involve children manipulating concrete materials to begin with and then gradually removing these by flashing (a quick look) the materials/tool and then screening (hiding). The fact that the materials/tools are still there can support children's imagery, through visualisation and eventually help embed the basic facts or strategies.
- Division there are two forms of division: sharing and grouping.
- When sharing a collection of objects fairly, children should understand that their share is the same as everyone else's share i.e. that all **fair shares are equal**.
- Children should also have an understanding that within real life situations some shares may have some items 'left over' and that they then need to think about how they are going to deal with this.

- After children have shared objects equally, the process can be INVERSED to begin to develop the link between division and multiplication. This can be done by pupils first sharing a group of objects and then putting back together all of the shares to form one collection.
- Children may be able to represent division-type situations by sharing out or **forming equal groups**, but become confused about what to count to solve the problem, often choosing to count all the items. Rich discussion and experiences, putting 'sharing problems' into context (listening to and making up their own stories), will support their development.
- Pupils may deal out an equal number of items or portions in order to share, but do not use up the whole quantity or attend to the equality of the size of the portions. This is perfectly acceptable and should be supported by discussion with the child if one share having more (or not using the whole up) is fair.
- Multiplication pupils will begin to use forward and backward number word sequences using the multiples of 2 and 5. They begin counting these using a rhythmic count 1, 2, 3, 4, 5, 6, 7, 8... and progress to skip counting 2, 4, 6, 8.

At this level children learn that building two collections by matching one to one leads to collections of equal size, and can 'fix' one collection to make it match another in size e.g. show a row of three horses and ask the child if they can make a second row of three horses to match.



Children may 'skip count' but do not realise it gives the same answer as counting by ones and, therefore do not trust it as a strategy to find out 'how many?' e.g. for above they may count in 2s - 2, 4, 6 but then not rely on this answer and so then count the cars individually.

- **Modelling** (using concrete materials to represent the numbers in the sum), drawing and writing mathematical problems should be encouraged at this level e.g. when told a story about black horses and white horses, they represent the horses with black and white counters or cubes.
- They may 'act out' or draw a picture to **represent a small number** of things e.g. they pretend to be the horses and act out the story.

- Children should also be asked to represent a story with objects or on a **think board** e.g. substitute counters for cows to solve the problem 'Three cows were put in a field with four other cows. How many cows were there in the field?'
- <u>EXAMPLES OF THINK BOARDS</u> Specific key areas are designated to 'show strategies used', 'show with a picture', 'show using a number sentence', and 'show using materials'.
 - Im Frame
 Potential

 Im Frame
 Image: Control of Cont

Mr Hoppy had 4 tortoises. Now he has 9 altogether. How many did he buy?



Miss Wakefield has 9 books. Yami has 6. How many do they have altogether?

When exploring measurement, although the term "weighing" is used it is important to be clear that it is the mass of an object that is being calculated not the weight. Weight is the force exerted by the gravity on objects, mass is the amount of material contained and is measured in kg, gm etc.

Suggested Written Recording

At Early Level children's written recording can be nurtured by the provision of a mathematically rich environment which helps them to explore the world of mathematics. A wide range of resources (pens, clipboards, post-its, notebooks) should be readily available to encourage children to record their own mathematical thinking. The following points are important when considering what, when and where they should be recording.

Children need to be free to choose how they will represent and communicate their own mathematical understanding. Emergent writing may also be visible at this point.

Between the ages of 3 and 4 years of age children will begin to attribute mathematical meanings to the marks they make on paper.

'Emergent' and 'individual' mathematical graphics and open ended discussions should be encouraged to help children make sense of the sometimes confusing symbols and abstract nature of mathematics.

Recording can be temporary, for example on a white/chalk board, scrap paper and post-it notes or stored more permanently in a designated maths jotter/folder. Photos may be taken of this as and when appropriate.



'exploring telephone numbers'

Children are encouraged to give their own meanings to their graphics (drawings, writing, symbols, and marks).

0000 00 000000





Fife Numeracy & Mathematics Progression: Early Level ~ Page 8

At Early Level children's written recording can be nurtured by the provision of a mathematically rich environment which helps them to explore the world of mathematics. A wide range of resources (pens, clipboards, post-its, notebooks) should be readily available to encourage children to record their own mathematical thinking. The following points are important when considering what, when and where they should be recording.



Once children represent quantities that are counted, they begin to explore calculations. Supportive and encouraging adults can support children's own ways of exploring their mathematical thinking so that they make strong connections with their own understanding.



Children do not need to record their mathematics if they can work something out mentally, neither do they need to record something they have already worked out in a practical context with resources.



"8 more sleeps until mummy comes home." This pupil has recorded 8 tally marks to represent this and recognises the numeral 8. Children will integrate standard symbols and written methods as they develop their understanding. Numerals will be written as labels for collections, both to represent quantities that have not been counted as well as those that have been counted.



2+4=6 1 × 5 = 6 2+3=6

Young children are very good at making sense of their solutions through their own marks and symbols. Children need to make sense of maths in their own ways rather than being required to 'colour-in' or complete worksheets designed by teachers.

Un-ruled jotters have a distinctive advantage over lined/squared books in that pupils are not restricted by lines and have the space to choose whether to use pictures, numbers, words or a combination of these to record their thinking.



Can 12 be shared equally between 2?

Fife Numeracy & Mathematics Progression: Early Level ~ Page 9



MNU 0-02a



COUNTING AIM: In real life and everyday situations, demonstrate the principles of counting, subitise and use number words and numerals to represent quantities. MNU 0-02a



COUNTING PRINCIPLE ~ Conservation: Understand that the count for a set of objects stays the same no matter whether they are spread out or close together.

> Understand that by building two collections, matching one to one, leads to equal sized collections.

Subitise small collections e.g. 0-6, including irregular patterns using concrete materials such as five frames, ten frames, dice and arrays.

Be able to see at a glance how many there are in a small collection (0-3) and attach number names yet may not be able to say the number names in order. This is called **SUBITISING**.



Show an awareness of counting a collection using some number names, but not always reliably. (Staff model touching and moving an object to count it.)

STRUCTURE OF NUMBER AIM: Understand that numbers to 10 and beyond can be constructed in multiple ways, for example by combining and partitioning. MNU 0-02a



notice ladybird has two dots on both wings.

C I

Connect the differences they see between collections. *e.g. they see one, two and three* things and connect this with the spoken number string one, two, three.

CALCULATING AIM: Use a range of strategies to add some, take some away, or combine two amounts mentally to 10 in a variety of contexts. MNU 0-03a



Use the relationship between addition and subtraction to solve problems. *e.g. Solve* 'There are 7 children in a group, 2 leave, how many are still in class? By understanding that 2 + 5 = *T*.

> Can count items when both collections are screened (hidden) but will count all the items starting from one.

Record addition and subtraction informally using role play, concrete materials, subitising, singing, drawings, numerals and words.

Any algorithms should be horizontal rather than vertical.

Solve a variety of simple stories (change unknown, start unknown, result unknown) by subitising, acting out, using concrete materials or by drawing a picture. e.g. 'Kate has two cows and Jordan has three cows. How many cows are there altogether?' can be acted out.

Informally record addition and subtraction using role play, singing songs, drawings, marks and possibly numerals and letters to represent answers.



MONEY AIM: Recognise that there are different ways to make exact amounts of money.

MNU 0-09a



Associated Knowledge

- understanding of whole numbers and their relative
- Know and understand whole number addition and

TIME AIM: Use appropriate language when discussing time. Recognise, talk about and use simple visual calendars, timers and clocks to record and find information. MNU 0-10a



Name the word sequence of the months of the year and understand the cyclical pattern.

Order everyday events by duration. e.g. the journey to Granny's is longer than the

Use clapping to compare duration of two 'quick' events. 'Do we clap for longer watching Jo walk to the door or Mark go to the cloakroom?'

Talk about important events in own lives.

e.g. birthdays, tooth fairy at night.

MEASUREMENT AIM: In real life contexts use non-standard units and appropriate language to compare the length, height, mass and capacity MNU 0-11a of objects and justify thinking. Describe common objects using the appropriate measurement language.



actions. MTH 0-13a



Key Aspect	Experiences and Outcomes	National Benchmarks		
Estimating and Rounding Pg 10	I am developing a sense of size and amount by observing, exploring, using and communicating with others about things in the world around me. MNU 0-01a	 Recognises the number of objects in a group, without counting (subitising) and uses this information to estimate the number of objects in other groups. Checks estimates by counting. Demonstrates skills of estimation in the contexts of number and measure using relevant vocabulary, including less than, longer than, more than and the same. 		
Number Word Sequences	I have explored numbers, understanding that they represent quantities, and I can use them to	 Recalls the number sequence forwards within the range 0 - 30, from any given number. Recalls the number sequence backwards from 20. Identifies and necessities numbers from 0 to 20. 		
Pg 11	count, create sequences and describe order. MNU 0-02a	 Identifies and recognises numbers from 0 to 20. Orders all numbers forwards and backwards within the range 0 - 20. Identifies the number before, the number after and missing numbers in a sequence within 20. Uses ordinal numbers in real life contexts, for example, 'I am third in the line'. Uses the language of before, after and in-between. 		
Counting		• Explains that zero means there is none of a particular quantity and is represented by the numeral O.		
Pg 12		 Uses one-to-one correspondence to count a given number of objects to 20. Identifies 'how many?' in regular dot patterns, for example, arrays, five frames, ten frames, dice and irregular dot patterns, without having to count (subitising). When counting objects, understands that the number name of the last object counted is the name given to the total number of objects in the group. 		

Early Level Overview * The numeracy learner statements that are the responsibility of all are shown in italics.

Key Aspect	Experiences and Outcomes	National Benchmarks
Structure of	I have explored numbers,	• Doubles numbers to a total of 10 mentally.
Number	understanding that they represent	• Groups items recognising that the appearance of the group has no
	quantities, and I can use them to	effect on the overall total (conservation of number).
Pg 13	count, create sequences and	• Partitions quantities to 10 into two or more parts and recognises
	describe order.	that this does not affect the total.
	MNU 0-02a	
Calculating:	I use practical materials and can	 Counts on and back in ones to add and subtract.
Addition and	'count on and back' to help me to	 Adds and subtracts mentally to 10.
Subtraction	understand addition and	 Uses appropriately the mathematical symbols +, - and =.
	subtraction, recording my ideas and	 Solves simple missing number problems.
Pg 14	solutions in different ways.	
	MNU 0-03a	
Fractions	I can share out a group of items by	• Splits a whole into smaller parts and explains that equal parts are
	making smaller groups and can split	the same size.
Pg 15	a whole object into smaller parts.	 Uses appropriate vocabulary to describe halves.
	MNU 0-07a	• Shares out a group of items equally into smaller groups.
Money	I am developing my awareness of	• Identifies all coins to £2.
	how money is used and can	• Applies addition and subtraction skills and uses 1p, 2p, 5p and 10p
Pg 16	recognise and use a range of coins.	coins to pay the exact value for items to 10p.
	MNU 0-09a	
Time	I am aware of how routines and	 Links daily routines and personal events to time sequences.
	events in my world link with times	• Names the days of the week in sequence, knows the months of the
Pg 17	and seasons, and have explored	year and talks about features of the four seasons in relevant
	ways to record and display these	contexts.
	using clocks, calendars and other	 Recognises, talks about and, where appropriate, engages with
	methods.	everyday devices used to measure or display time, including clocks,
	MNU 0-10a	calendars, sand timers and visual timetables.
		• Reads analogue and digital o'clock times (12 hour only) and
		represents this on a digital display or clock face.

Key Aspect	Experiences and Outcomes	National Benchmarks		
		 Uses appropriate language when discussing time, including before, after, o'clock, hour hand and minute hand. 		
Measurement	I have experimented with everyday items as units of measure to	 Shares relevant experiences in which measurements of lengths, heights, mass and capacities are used, for example, in baking. 		
Pg 18	investigate and compare sizes and amounts in my environment, sharing	 Describes common objects using appropriate measurement language, 		
	my findings with others.	 including tall, heavy and empty. 		
	MNU 0-11a	• Compares and describes lengths, heights, mass and capacities		
		using everyday language, including longer, shorter, taller, heavier, lighter, more and less.		
		• Estimates, then measures, the length, height, mass and capacity		
		of familiar objects using a range of appropriate non-standard		
		units.		
Patterns and	I have spotted and explored patterns	 Copies, continues and creates simple patterns involving objects, shapes 		
Relationships	in my own and the wider environment	and numbers.		
	and can copy and continue these and	 Explores, recognises and continues simple number patterns. 		
Pg 19	create my own patterns.	 Finds missing numbers on a number line within the range 0 - 20. 		
	MTH 0-13a			

First Level Experiences and Outcomes

* The numeracy learner statements that are the responsibility of all are shown in italics.

I can share ideas with others to develop ways of estimating the answer to a calculation or problem, work out the actual answer, then check my solution by comparing it with the estimate. MNU 1-01a Pg 31

I have investigated how whole numbers are constructed, can understand the importance of zero within the system and can use my knowledge to explain the link between a digit, its place and its value. MNU 1-02a Pa's 32-36

I can use addition, subtraction, multiplication and division when solving problems, making best use of the mental strategies and written skills I have developed. MNU 1-03a Pg's 37-39

Having explored fractions by taking part in practical activities, I can show my understanding of:

how a single item can be shared equally
the notation and vocabulary associated with fractions
where simple fractions lie on the number line.

MNU 1-07a Pg 40

Through exploring how groups of items can be shared equally, I can find a fraction of an amount by applying my knowledge of division. MNU 1-07b Pg 40

Through taking part in practical activities including use of pictorial representations, I can demonstrate my understanding of simple fractions which are equivalent.

MTH 1-07c Pg 40

I can use money to pay for items and can work out how much change I should receive. MNU 1-09a Pg 41

I have investigated how different combinations of coins and notes can be used to pay for goods or be given in change. MNU 1-09b Pg 41

I can tell the time using 12 hour clocks, realising there is a link with 24 hour notation, explain how it impacts on my daily routine and ensure that I am organised and ready for events throughout my

> day. MNU 1-10a Pg 42

I can use a calendar to plan and be organised for key events for myself and my class throughout the year. MNU 1-10b Pg 42

I have begun to develop a sense of how long tasks take by measuring the time taken to complete a range of activities using a variety of timers. MNU 1-10c Pa 42

I can estimate how long or heavy an object is, or what amount it holds, using everyday things as a guide, then measure or weigh it using appropriate instruments and units. MNU 1-11a Pg's 43 & 44

I can estimate the area of a shape by counting squares or other methods. MNU 1-11b Pg's 43 & 44

Fife Numeracy & Mathematics Progression: First Level ~ Page 23



Points to Consider

- Saying one, two or three number words forwards forms an important basis for counting on strategies (counting up from and counting up to). Children may choose to use their fingers to keep track. They typically will have more difficulty when crossing a decade e.g. three numbers after 68.
- The use of tools such as numeral rolls, numeral tracks and hundred squares are very beneficial for promoting visualisation and help to support children when internalising the number sequences.
- Scaled number lines are an invaluable tool for children to show their understanding of number sequences and the relation numbers have with each other. They can have increments marked regularly or just at the beginning and end.



(Number lines with marked intervals will be easier to use. Only some children at First Level will be comfortable using the last two examples).

- Reversing 2 digit numbers can still be a problem for children who are using the sound of the number e.g. 16 and 61, both start with the six sound.
- Children need to be taught that 'ty' words mean 'tens' (e.g. sixty means six tens) and decoding ones and tens notation (e.g. fifty eight means five tens and eight ones).

- Children can have difficulty with 3 digit numbers with a zero in the tens column especially whilst their knowledge of place value is not wholly secure.
- Zero is used as a place holder. It indicates that there is none of a particular quantity and holds the other digits in their place.
- It is important that children are able to see numbers being represented on a tens frame using both the five-wise and pair pattern. This supports their understanding of **5 and 10 as an anchor** to making larger numbers and supports their doubling capabilities.
- Other tools like the Rekenrek, bead strings, dot cards and counters all help to support children's number sense moving from concrete, to visualisation and then into understanding the abstract.



- Traditionally, the first column on the right has been called the 'units' column. The term 'ones' can be easier to understand as ten ones equate to one unit of ten.
- Developing children's early knowledge of tens and ones is a forerunner to the development of place value knowledge. It is very important that they have plenty of experience of counting large collections (within 100) and grouping these into 10s.
- Materials such as straws organised into bundles of ten and then hundreds support early knowledge of place value rather than the use of traditional Dienes apparatus. This is because children benefit significantly from creating the units of ten themselves.
- Children should be able to split (partition) 2 and 3 digit numbers into standard form (e.g. 369 = 300 + 60 + 9) without reference to actual quantities (concrete materials).
- It is also vital that children are able to split 2 and 3 digit numbers into non-standard form. They should understand that the number 237 has 23 tens and 7 ones and 237 ones. This will support their mental agility enormously in Second Level e.g. the sum 237 176 becomes 23 tens 17 tens and 7-6, which leaves 6 tens and 1 one which equals 61. They will still need support materials at this stage.
- Children need to use the strategy of starting with the larger number when combining two or more collections and understand this is a more efficient use of time. They also need to keep track of the number of objects in the second, smaller collection.
- It isn't vital that children learn the word 'commutative' but it is important that they learn how to use the strategy i.e. 2+3 = 3+2.

- Part, Part, Whole (PPW) involves seeing numbers as being made up of two or more parts. This is a major conceptual achievement at the Early and First Level. A strong understanding of PPW has been shown to increase understanding of subsequent work with place value, number concepts and word problems.
- Children need to learn to use equal groupings or parts to help count collections. Simply learning how to skip count by reciting every second or every third number or by jumping along a number line may not help them to realise that they are in fact 'counting all.'
- The sequence for how children learn to say multiple counts starts with children counting rhythmically and identifies number patterns, e.g. 1, 2, 3, 4, 5, 6, 7, 8, 9... This in turn enables them to use skip counts and counting by ones e.g. 3, 6, 9, 12, 13, 14, 15, 16, 17, 18. With practice they move onto skip counting in multiples, e.g. 3, 6, 9, 12, 15, 18, 21.
- Children need to be able to keep a double count in multiplicative situations by representing one group (e.g. by holding up four fingers).
- Children should keep track of their count using fingers or a temporal count (sequence of sounds) e.g. they hear themselves make three counts eleven, twelve, thirteen.
- Children should be familiar with everyday vocabulary, such as 'groups of', 'lines of', 'bags of', 'boxes of' 'packets of', 'sets of' alongside the mathematical terminology. This vocabulary should support children when working out if the problem is a multiplication or division problem.



• In the operation 5×3 , 5 is the multiplier (how many of) and 3 is

A word of warning!

- Writing number stories helps children visualise mathematical facts and integrate mathematics being learned into their own lives and experiences. When children write a number story, they are devising their own images for the numbers in an abstract number problem which can be presented in a variety of ways.
- Children should be actively encouraged to listen attentively to maths stories and then be asked to draw solutions or act out the solution e.g. 13 grey seals are on the beach and are soon joined by 12 more. How many seals are now on the beach?
- Children should also be given a picture of the solution and asked to draw, tell or act out the number story. The answer is five cows, what's the story? Children should share different responses from group/class.
- Written number sentences which include symbols and numerals could be included once they are proficient with using models to represent their stories.
- Children also need to experience number problems (change unknown, start unknown, result unknown) where the answer is not always on the right hand side e.g. = 3 + 8
- Children do not need to know these terms, but need to experience number problems where they are finding the missing augend (__ + 3 = 9), addend (6 + __ = 9), minuend (__ 6 = 3) and subtrahend (9 __ = 3).

- The 'equal sign' can initially be interpreted as 'find the answer'. For example, 2 dogs plus 3 dogs equals how many dogs ~ 2 + 3 =? However, during First Level children need a broader definition of 'equals'. To work out 8 + 7 using a 'jump to the next decade' strategy, children need to understand that 8 + 7 = 8 + 2 + 5 = 10 + 5 = 15 (associative law). Here, 'equals' means 'is the same as'. Both sides balance.
- Children should also be asked to represent a story with objects or on a **think board** e.g. use tally marks for cows to solve the problem 'Twenty three cows were put in a field with fourteen other cows. How many cows were there in the field?'
- <u>EXAMPLES OF THINK BOARDS</u> Specific key areas can be targeted to suit the learning e.g.



Suggested Written Recording



Suggested Written Recording

Whilst the main emphasis in this progression is on using mental strategies to solve numerical problems, it is important that written recording is recognised as an important part of mathematics.					
The follo	owing points are important w	hen considering what, when	and where pupils should be re	ecording.	
Pupils use of standard symbols and conventions, for example numbers 0-9, equals sign and operation signs, will become more precise when recording their own explanations about a calculation.	Using paper and pencil for informal jottings can help keep a track of calculations if the numbers are large.	TO + TO possibly moving into HTO + TO or HTO + HTO Example of horizontal expanded method, using splitting. 612 + 135 = 600 + 10 + 2 + 100 + 30 + 5 H T 0 600 + 100 + 10 + 30 + 2 + 5 700 + 40 + 7 = 747		A jotting area can be created by dividing the page in the maths jotter rather than using a separate book. This helps the pupil and teacher understand how the calculation was undertaken.	
For have 50p in ten pence pieces, how many friends can you each give 10p? $\underbrace{\begin{array}{c} \bullet\\ \bullet\\ \bullet\end{array}}$		Recording their own ideas on paper with words, numbers, symbols, diagrams and pictures is part of pupils' exploration of number. See Think Boards in 'Points to Consider'	It is important for children to establish connections between practical experiences, symbols, language and patterns.		
Introducing standard algorithms before pupils have fully established partitioning strategies can slow down the development of mental problem solving skills and number sense.	s 3 + 3 + 3 = 9 3 × 3 = 9		Children at this level begin to understand that diagrams and equations often provide a more efficient means of communicating ideas than textual explanations.		

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ESTIMATION AND ROUNDING AIM: Prior to calculating, use different strategies to make a reasonable estimate of the answer, for example by rounding or doubling. MNU 1-01a



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e.g. give me a number which comes between 67 and 70? Can

Say backward number multiples of 2, 5, 10 e.g. 66, 64, 62, 60... COUNTING AIM: Count forwards and backwards in at least 10s and 100s using numerals in multiplicative situations. Select counting as a strategy when MNU 1-02a appropriate.



Understand that when you add or subtract zero to any number it does not change the value of the set.
STRUCTURE OF NUMBER AIM: Understand the part-part-whole relationship of numbers up to 100. Quickly recall number facts within 20 and use this knowledge to support working with larger numbers. MNU 1-02a



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PLACE VALUE AIM: Recognise zero as a placeholder in whole numbers and appreciate when it is necessary to include it in whole numbers to at least 1000. Understand that the position of a digit tells us the quantity it represents (up to at least three digits). Apply an understanding of place value to solve problems by mentally multiplying and dividing by 10 and 100 (where the answer is a whole number). Note: This pyramid appears across two pages. MNU 1-02a



603 Systematically record all the possible digit combinations for 2 and 3 single digit numbers, starting with the smallest number. e.q. 739= 379, 397, 739,793, 937, 973

Efficiently make and draw 2 digit numbers in non-standard form and convert tens into ones and vice versa. e.g. 36 ones can look like this

Apply an understanding of place value and the role of zero to identify, say and represent numbers up to 1000 and beyond, e.g. know that the value of the digits in 20 are worth

2 bundles of 10 and 16 ones

2 tens or twenty and zero ones.



Show familiarity with grouping and counting in tens and explain why it is a more efficient way to count a larger collection. Connect these into words and symbols.



Count the tens first and add the ones 10, 20, 30, 40, 50, 51, 52, 53, 54, 54, 55, 56 instead of counting in ones, 10, 11, 12, 13, 14, 15, 16, 17, 18.

Increase and decrease by hundred off the century. e.g. 100 square and 2 ten dot strips. Add multiples of 100 squares and keep count, 120, 220, 320...

Begin to make and draw collections of a

given size to 100s using tens and ones.

Convert into words and symbols. e.g.

Using materials such as bundles of

sticks - see 'Points to Consider'.

6 bundles of ten and 7ones = 67

9 bundles of ten

and 7 ones = 97

Increase by tens and ones e.g. 34 24

Make and draw 2 and 3 digit numbers in standard hundred, tens and ones form and connect these

into words and symbols. e.g. using materials such as bundles of sticks or dot strips in ones and

bundles/strips of ten, convert into words and

symbols.

2 hundreds, 3 tens

and 3 ones = 233

Increase and decrease by 100s on the

century to 1 000. e.g. 6 hundred, 7

hundred, etc.

Identify the patterns within numbers and

the cyclical nature of the numbers 0-9.

37



Incre	ase and c	lecre	ase by to	ens o		
decade (within 100)						
##	+1 bundle	Ø	+2 bundles	Ø		
20		30		50		
2 tens		3 tens		5 ten		

Fife Numeracy & Mathematics Progression Pyramid: First Level ~ Page 36

CALCULATING (ADDITION AND SUBTRACTION) AIM: Select an appropriate operation to solve different addition and subtraction situations by carefully interpreting a multi-step problem. Apply a range of strategies, justifying choice. MNU 1-03a

Find the difference between quantities by adding or subtracting. e.g. Ann has read 25 pages of a book which has 80 pages. How many pages will she need to read to finish the book? Can be written as 25 + [] = 80 and 80 - 25 = []

Solve subtraction problems by subtracting ones and tens, using a split strategy. e.g. 433 - 21= 433 - 20 - 1

Understand that subtraction is the inverse of addition in written and mental calculations. Change subtraction to addition. e.g. 13 - 6. What do I add to 6 to make 13? 6 +? = 13I had some toy cars and gave thirteen to my friend and had two left. How many cars did I have to start with?' may be thought of as \square -13 = 2 but can be worked out using 13 + 2 = \square



Say or write addition and subtraction number sentences to match the meaning of the problem. e.q. for 'I had some strawberries for lunch and ate three. There were five left. How many strawberries were there to start with?' Say or write $\Box - 3 = 5$

Record written number sentences, including the symbols (+, -) to represent the operations of addition and subtraction using appropriate mathematical vocabulary. e.g. add, subtract

> Model, draw and represent number problems; record addition and subtraction informally but begin to include symbols and numerals.

Represent a story with objects or on a think board. e.g. Substitute counters for cows to solve the problem -Three cows were put in a field with fourteen other cows. How many cows were in the field altogether?

Write sensible addition and subtraction story problems to match given number sentences. e.g. for 'Sue invited 68 friends to her party Use the inverse to routinely but only 45 went. How many of her friends did not go?' check answers. Write 68 - ? = 45 Solve addition problems by adding Use 'part-part-whole' diagrams to show the link between addition/subtraction ones and tens using a split word problems with a 'transformed' number sentence. e.g. I had some strawberries and ate fourteen. There were thirty five left. How many strategy. e.g. 234 + 26 = 234 + 20 + 6 strawberries were there to start with?' may be thought of as $\square - 35 = 14$ but can be worked out as 35 + 14 = 1) 35 14 Solve subtraction problems by subtracting ones and tens, using a split strategy. Identify the whole amount and the parts within addition and Solve addition problems by adding ones and tens using subtraction sums to 100 and write appropriate number a split strategy (answers within 99 - where the ones sentences. (Should only cross one decade) e.g. 48 pages from the class story were read in one week. There are 56 pages in the entire column does not add up to ten or more). book so how many pages are left? 48 + []=56. *e.q.* 34 + 55 = 30 + 50 + 4 + 5 = 89 Use known simple facts to discover an unknown Solve problems by using the Use a known fact to work out a new one. fact beyond 100. e.g. I know 160 + 42 is 202 because I commutative property to: Use knowledge of doubles to 20 to discover near know 160 + 40 is 200 * add three or more numbers, doubles. by making pairs of ten. e.g. Can add or subtract multiples of ten to/from 2+6+8. 4+7+6 Can add or subtract multiples of ten to/from any two any multiple of ten, within 100. e.g. • Put largest number first. digit number, within 100, using a split strategy. 20+30 30+40 30-10 60-30 60-50 80-50 e.q. 6 + 13 = 13 + 6*e.g.* 12 + 30 = 30 + 10 + 2 = 40 + 2 20 + 30 = 2 tens and 3 tens = 5 tens = 50 Connect addition and subtraction problems and Use a known fact to work out a new one. Use known simple facts to discover an unknown fact the symbolic representations of them. Use knowledge of doubles to 10 to discover e.g. use a think board (see 'Points to Consider') to within 100. near doubles within ten. e.g. I know 20 + 4 = 24 so 21 + 4 = 25. show the story, materials, picture, diagram and the *e.g.* 4 + 5 is same as double 4 + 1 number sentence for the calculation. 3 + 5 is same as double 3 + 2Identify the whole amount and the parts within addition and subtraction sums and write number Understand that subtraction is not commutative. sentences. (One part needs to be 5 or less). e.g. Solve simple addition/subtraction sums by adding e.g. We can take 4 away from 6 but when we take 6 5 girls and 3 boys went ice-skating. There were 8 away from 4 we get a different answer...negative on/counting back from the largest number in jumps children altogether. 5 + 3 = 8 of one. numbers are for later! 8 children went ice-skating. 5 were girls. How many were boys? 8 - 5 = ? e.g.... 9+4 by 910, 11, 12, 13 1 2 3 4 - track of count Understand that addition is commutative:

Understand that addition and subtraction can be described

using everyday language. e.g. take away, from, difference

between, sum, combine and join.

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The order of the numbers can be rearranged to

make counting on easier. e.g. 2+6 is the same as 6+2

Start at the largest number!



e.g.... 59 - 4 =, start at 59 and track number of counts starting at 58, count back 4 to 55. e.g.... 59 - 58 (1), 57 (2), 56 (3), 55 (4) (Largest number can be between 1-100 and number being subtracted is 5 or less).

CALCULATING (MULTIPLICATION AND DIVISION) AIM: Select an appropriate operation to solve different multiplication and division situations within the number range 0-1000 by carefully interpreting a two-step problem. Apply a range of strategies to determine multiplication and division facts. Use the relationship between multiplication and division to complete mental and written calculations. Note: This pyramid appears across two pages. MNU 1-03a



Use the inverse operation to routinely check answers.

Use known facts to solve an unknown fact.

e.g. solve multiplication facts about the two times table from their knowledge of doubling.

Begin to understand the connection between repeated subtraction and division by grouping. *e.g. Fiona has 15 large sunflower seeds that she puts into packets with three seeds in each. How many packets does she fill?*

15 - 3 - 3 - 3 - 3 - 3 = 0

15 ÷ 3 = 5

Begin to use an addition strategy to solve a division by grouping by imaging. e.g. The bakers have made a fresh batch of 18 rolls and place them in bags of 3. How many bags will they need? 3 + 3 + 3 + 3 + 3 = 18

Use materials, concrete or symbolic to form small equal-sized groups. e.g. 'Use blocks to make 5 towers with 3 blocks in each. How many blocks there are altogether?' Can the 15 blocks be rearranged in towers of equal size to again equal 15? (Consider having vertical and flat groups)

Begin to use symbols (x, \div) to represent the operations of multiplication and division.

Comment on the patterns that skip-counting sequences make on a hundreds board and predict other numbers in a sequence in a forwards or backwards direction, whilst looking at a partially filled in pattern.



e.g. in this sequence of 3s, will 96 be covered?

Record multiplication problems using the terms 'groups of', 'lots of', 'sets of', 'shared equally' and 'remainders'. Use the rows and columns in arrays to represent the number of groups and the number of objects in each group. *e.g.*



Six groups of two represents two rows of six

Begin to understand the connection between repeated addition and multiplication. e.g. 4 + 4 + 4 + 4 + 4 is the same as 5 x 4

•••• 5 rows of 4 = 5 x 4

3 scuba divers catch 2 fish eac. How many fish do they catch altogether? 2 + 2 + 2 = 6. They also can record this as '3 groups of 2 is 6'.

Begin to solve a multiplication problem by using skip counting and understand that each successive count relates to putting sets of the same number together. *e.g.* 5, 10, 15 is 3 groups of 5 and another group of 5 will make 20. Use arrays to show equal-sized groups that make the same quantity and note the commutative property. e.g. 'Group 12 blocks into equal sized rows. How many ways can you do it? Record the number combinations:



Use the symbols (x, ÷) to represent the operations of multiplication and division and appropriate mathematical vocabulary. *e.g. multiply, divide, product, shared equally.*

Solve simple division problems by grouping collections into sets of particular size.

Divide 8 star biscuits onto plates with 4 on each plate. How many plates (groups) will we have?



Recognise equal-sized groups found in everyday situations.

e.g. Look for equal-sized groups such as stacks of class chairs. Count how many in each group and how many



4 stacks of chairs 10 chairs in each stack 4 stacks of 10 chairs 10, 20, 30, 40 - skip count

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Share 8 star biscuits shared amongst 4 children.



Use multiplication for situations involving repeating equal quantities.

e.g. draw four oranges and then another four oranges. This is the same as two lots of four oranges.



quantity can be represented by different fractions e.g $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$. MNU 1-07a, MNU 1-07b, MNU 1-07c



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MONEY AIM: Identify and use all coins and notes up to at least £20 and explore different ways of making the same total. Use a variety of coin and MNU 1-09a & MNU 1-09b note combinations, up to at least £20 to pay for items and given change.



Associated Knowledge

- Use a range of strategies to be able to add, subtract, multiply and divide using whole numbers.
- Demonstrate that I can use a range of methods to calculate total amounts and change.

Count and order small collections of coins and notes according to their value. TIME AIM: Use and interpret a variety of calendars and 12 hour timetables to plan key events and calculate durations. Use measures of time to assist problem solving in real life including situations involving more than one unit of time. Estimate and calculate durations.

MNU 1-10a, MNU 1-10b & MNU 1-10c





MNU 1-11a & MNU 1-11b

Note: This pyramid appears over two pages.



Read a given calibrated scale to the nearest labelled graduation using knowledge of fractions. e.g. 250ml, which is one quarter of a litre.

Use knowledge of relationships between units of measure to make simple conversions. e.g. 1m 58cm = 158cm



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MATHEMATICS - IT'S IMPACT ON THE WORLD, PAST, PRESENT, FUTURE AIM: Compare different number systems, decide which is most MTH 1-12a useful and justify thinking. Explain opportunities to use mathematics in everyday life.



This Experience and Outcome should be explored throughout all other experiences and outcomes and at appropriate points in other curricular areas. There is a great deal of opportunity for learner choice and the suggestions above should not be seen as definitive.

Identify evidence of historical number systems in everyday life.



Identify and explore numeracy and mathematics in the world of work.



e.g. nurses and lorry drivers

PATTERNS AND RELATIONSHIPS AIM: Describe, extend and generate simple number patterns. Describe, create and explain the rules for patterns and MTH 1-13a & MTH 1-13b sequences involving shapes, pictures, symbols and movements and be able to anticipate further terms.



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EXPRESSIONS AND EQUATIONS AIM: Explain number relationships and comparisons using appropriate vocabulary and symbols. Find the value of a missing MTH 1-15a & MTH 1-15b number when it is replaced by a picture or a symbol by using understanding of the equals sign as a balance.



First Level Overview

* The numerac	y learner statements	that are the resp	ponsibility of all are sho	wn in italics.
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Key Aspect	Experiences and Outcomes	National Benchmarks
Estimation and Rounding Pg 31	I can share ideas with others to develop ways of estimating the answer to a calculation or problem, work out the actual answer, and then check my solution by comparing it with the estimate. MNU 1-01a	 Uses strategies to estimate an answer to a calculation or problem, for example, doubling and rounding. Rounds whole numbers to the nearest 10 and 100 and uses this routinely to estimate and check the reasonableness of a solution.
Number Word Sequences Pg 32	I have investigated how whole numbers are constructed, can understand the importance of zero within the system and can use my knowledge to explain the link between a digit its place and	• Reads, writes, orders and recites whole numbers to 1000, starting from any number in the sequence.
Counting Pg 33	its value. MNU 1-02a	• Counts forwards and backwards in 2s, 5s, 10s and 100s.
Structure of Number		• Identifies the value of each digit in a whole number with three digits, for example, 867 = 800 + 60 + 7.
rg 34		

Key Aspect	Experiences and Outcomes	National Benchmarks
Place Value		• Demonstrates understanding of zero as a placeholder in whole numbers to 1000.
Pg's 35 & 36		 Identifies the value of each digit in a whole number with three digits, for example, 867 = 800 + 60 + 7.
		 Adds and subtracts multiples of 10 or 100 to or from any whole number to 1000.
		 Multiplies and divides whole numbers by 10 and 100 (whole number answers only).
Calculating (Addition and	I can use addition, subtraction, multiplication and division when solving	 Demonstrates understanding of the commutative law, for example, 6 + 3 = 3 + 6 or 2 × 4 = 4 × 2.
Subtraction)	problems, making best use of the	 Solves addition and subtraction problems with three digit whole
Pg 37	have developed. MNU 1-03a	 Applies knowledge of inverse operations (addition and subtraction; multiplication and division).
		 Uses correct mathematical vocabulary when discussing the four operations including, subtract, add, sum of, total, multiply, product, divide and shared equally.
		 Solves two step problems.
Calculating (Multiplication and Division)	I can use addition, subtraction, multiplication and division when solving problems, making best use of the	 Applies strategies to determine multiplication facts, for example, repeated addition, grouping, arrays and multiplication facts. Applies knowledge of inverse operations (addition and subtraction;
	mental strategies and written skills I	multiplication and division).
Pgs 38 & 39	have developed. MNU 1-03a	 Demonstrates understanding of the commutative law, for example, 6 + 3 = 3 + 6 or 2 × 4 = 4 × 2.
		 Applies strategies to determine division facts, for example, repeated subtraction, equal groups, sharing equally, arrays and multiplication facts.
		• Uses multiplication and division facts to solve problems within the number range 0 to 1000.

Key Aspect	Experiences and Outcomes	National Benchmarks
Calculating (Multiplication and Division – cont'd) Pg's 38 & 39		 Multiplies and divides whole numbers by 10 and 100 (whole number answers only). Solves two step problems. Uses correct mathematical vocabulary when discussing the four operations including, subtract, add, sum of, total, multiply, product, divide and shared equally.
Fractions Pg 40	 Having explored fractions by taking part in practical activities, I can show my understanding of: how a single item can be shared equally the notation and vocabulary associated with fractions where simple fractions lie on the number line. MNU 1-07a Through exploring how groups of items can be shared equally, I can find a fraction of an amount by applying my knowledge of division. MNU 1-07b Through taking part in practical activities including use of pictorial representations, I can demonstrate my understanding of simple fractions which are equivalent. MTH 1-07c 	 Explains what a fraction is using concrete materials, pictorial representations and appropriate mathematical vocabulary. Demonstrates understanding that the greater the number of equal parts, the smaller the size of each share. Uses the correct notation for common fractions to tenths, for example, ¹/₂, ²/₃ and ⁵/₈. Compares the size of fractions and places simple fractions in order on a number line. Uses pictorial representations and other models to demonstrate understanding of simple equivalent fractions, for example, ¹/₂ = ²/₄ = ³/₆. Explains the role of the numerator and denominator. Uses known multiplication and division facts and other strategies to find unit fractions of whole numbers, for example, ¹/₂ or ¹/₄.

Key Aspect	Experiences and Outcomes	National Benchmarks
Money	I can use money to pay for items and can work out how	• Identifies and uses all coins and notes to £20 and explores different ways of making the same total.
Pg 41	much change I should receive. MNU 1-09a	 Records amounts accurately in different ways using the correct notation, for example, 149p = £1.49 and 7p = £0.07. Uses a variety of coin and note combinations, to pay for items and give change within £10.
	I have investigated how different combinations of	• Applies mental agility number skills to calculate the total spent in a shopping situation and is able to calculate change.
	coins and notes can be used to pay for goods or be given in change. MNU 1-09b	 Demonstrates awareness of how goods can be paid for using cards and digital technology.
Time Pg 42	I can tell the time using 12 hour clocks, realising there is a link with 24 hour notation, explain how it impacts on my daily routine and ensure that I am organised and ready for events throughout my day. MNU 1-10a	 Tells the time using half past, quarter past and quarter to using analogue and digital 12 hour clocks. Records 12 hour times using am and pm and is able to identify 24 hour notation, for example, on a mobile phone or computer. Records the date in a variety of ways, using words and numbers. Uses and interprets a variety of calendars and 12 hour timetables to plan key events. Knows the number of seconds in a minute, minutes in an hour, hours in a day, days in each month, weeks and days in a year. Orders the months of the year and relates these to the appropriate seasons. Selects and uses appropriate timers for specific purposes.

Key Aspect	Experiences and Outcomes	National Benchmarks
Time (continued) Pg 42	I can use a calendar to plan and be organised for key events for myself and my class throughout the year. MNU 1-10b	
	I have begun to develop a sense of how long tasks take by measuring the time taken to complete a range of activities using a variety of timers. MNU 1-10c	
Measurement Pg 43 & 44	I can estimate how long or heavy an object is, or what amount it holds, using everyday things as a guide, then measure or weigh it using appropriate instruments and units. MNU 1-11a	 Uses knowledge of everyday objects to provide reasonable estimates of length, height, mass and capacity. Makes accurate use of a range of instruments including rulers, metre sticks, digital scales and measuring jugs when measuring lengths, heights, mass and capacities using the most appropriate instrument for the task. Records measurements of length, height, mass and capacity to the nearest standard unit, for example, millimetres (mm), centimetres (cm), grams (g), kilograms (kg), millilitres (ml), litres (l).
	I can estimate the area of a shape by counting squares or other methods. MNU 1-11b	 Compares measures with estimates. Uses knowledge of relationships between units of measure to make simple conversions, for example, 1 m 58 cm = 158 cm. Reads a variety of scales on measuring devices including those with simple fractions, for example, litre. Uses square grids to estimate then measure the areas of a variety of

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Key Aspect	Experiences and Outcomes	National Benchmarks		
		 simple 2D shapes to the nearest half square. Creates shapes with a given area to the nearest half square using square tiles or grids. Recognises that different shapes can have the same area (conservation of area). 		
Mathematics - its impact on the world, past, present and future Pg 45	I have discussed the important part that numbers play in the world and explored a variety of systems that have been used by civilisations throughout history to record numbers. MTH 1-12a	 Investigates and shares understanding of the importance of numbers in learning, life and work. Investigates and shares understanding of a variety of number systems used throughout history. 		
Patterns and Relationships Pg 46	I can continue and devise more involved repeating patterns or designs, using a variety of media. MTH 1-13a Through exploring number patterns, I can recognise and continue simple number sequences and can explain the rule I have applied. MTH 1-13b	 Counts forwards and backwards in 2s, 5s and 10s from any whole number up to 1000. Describes patterns in number, for example, in the multiplication tables and hundred square. Continues and creates repeating patterns involving shapes, pictures and symbols. Describes, continues and creates number patterns using addition, subtraction, doubling, halving, counting in jumps (skip counting) and known multiples. 		

Key Aspect	Experiences and Outcomes	National Benchmarks
Expressions and Equations Pg 47	I can compare, describe and show number relationships, using appropriate vocabulary and the symbols for equals, not equal to, less than and greater than. MTH 1-15a When a picture or symbol is used to replace a number in a number statement, I can find its value using my knowledge of number facts and explain my thinking to others. MTH 1-15b	 Understands and accurately uses the terms 'equal to', 'not equal to', 'less than', 'greater than', and the related symbols (=, ≠, <, >) when comparing quantities. Applies understanding of the equals sign as a balance, and knowledge of number facts, to solve simple algebraic problems where a picture or symbol is used to represent a number, for example, [] + 17 = 30 and [] × 6 = 30.

Second Level Experiences and Outcomes

* The numeracy learner statements that are the responsibility of all are shown in bold italics.

I can use my knowledge of rounding to routinely estimate the answer to a problem then, after calculating, decide if my answer is reasonable, sharing my solution with others. MNU 2-01a Pg 63

I have extended the range of whole numbers I can work with and having explored how decimal fractions are constructed, can explain the link between a digit, its place and its value. MNU 2-02a Pg's 64 & 65

Having determined which calculations are needed, I can solve problems involving whole numbers using a range of methods, sharing my approaches and solutions with others. MNU 2-03a Pg's 67-74

I have explored the contexts in which problems involving decimal fractions occur and can solve related problems using a variety of methods. MNU 2-3b Pa 67-78

Having explored the need for rules for the order of operations in number calculations, I can apply them correctly when solving simple

problems.

MTH 2-03c Pg's 64, 67-74

I can show my understanding of how the number line extends to include numbers less than zero and have investigated how these numbers occur and are used. MNU 2-04a Pg 64 Having explored the patterns and relationships in multiplication and division, I can investigate and identify the multiples and factors of numbers. MTH 2-05a Pg 75

I have investigated the everyday contexts in which simple fractions, percentages or decimal fractions are used and can carry out the necessary calculations to solve related problems. MNU 2-07a Pg's 76-78

I can show the equivalent forms of simple fractions, decimal fractions and percentages and can choose my preferred form when solving a problem, explaining my choice of method. MNU 2-07b Pg's 76-78

I have investigated how a set of equivalent fractions can be created, understanding the meaning of simplest form, and can apply my knowledge to compare and order the most commonly used fractions. MTH 2-07c Pg's 76-78

I can manage money, compare costs from different retailers, and determine what I can afford to buy. MNU 2-09a Pg 79

I understand the costs, benefits and risks of using bank cards to purchase goods or obtain cash and realise that budgeting is important. MNU 2-09b Pg 79

Second Level Exper	iences and Outcomes				
* The numeracy learner statements that are t	he responsibility of all are shown in bold italics.				
I can use the terms profit and loss in buying and selling activities and can make simple calculations for this. MNU 2-09c Pg 79	I can explain how different methods can be used to find the perimeter and area of a simple 2D shape or volume of a simple 3D object.				
I can use and interpret electronic and paper-based timetables and schedules to plan events and activities, and make time calculations as part of my planning. MNU 2-10a Pg 80 I can carry out practical tasks and investigations involving timed events and can explain which unit of time would be most appropriate to use. MNU 2-10b Pg 80 Using simple time periods, I can give a good estimate of how long a journey should take, based on my knowledge of the link between time, speed and distance. MNU 2-10c Pg 80 I can use my knowledge of the sizes of familiar objects or places to assist me when making an estimate of measure. MNU 2-11a Pg's 81 & 82 I can use the common units of measure, convert between related units of the metric system and carry out calculations when solving problems. MNU 2-11b Pg's 81 & 82	MNU 2-11c Pg \$ 81 & 82 I have worked with others to explore and present our findings on how mathematics impacts on the world and the important part it has played on advances and inventions. MTH 2-12a Pg 83 Having explored more complex number sequences, including well-known named number patterns, I can explain the rule used to generate the sequence, and apply it to extend the pattern. MTH 2-13a Pg 84 I can apply my knowledge of number facts to solve problems where an unknown value is represented by a symbol or letter. MTH 2-15a Pg 85				

Points to Consider

7

• Within this level, most children will understand and use the **cyclical pattern** in whole numbers and so can read the number below.

Hundreds	Tens	Ones									
	Thousands			Thousands			Thousands			Thousands	

4 0 2 7 3 4 6 4 2 **4 027 346 427**

- To find the quantity that a digit represents, the value of the digit is multiplied by the value of the place e.g. in 3264 the 3 represents 3 × 1000, 2 represents 200 because it is 2 × 100, the 6 represents 6 × 10 and the 4 represents 4 × 1. This is an important feature of place value at Second Level because children need to recognise the **relative magnitude of numbers** i.e. that 5730 is ten times as much as 537.
- There is a constant multiplicative relationship between the places, with the values of the positions increasing in powers of ten, from right to left hence the **decimal** system.
- It is important that pupils understand that the maximum number in any place is 9 and that this also applies to decimal places.
- Children need to have experience of a variety of activities that help develop the idea that there are numbers between consecutive whole numbers e.g. between 3 and 4 there are nine numbers in the tenths range 3.1, 3.2, 3.3.....3.9
- Children need to thoroughly understand fractions **and** place value before they can make links to decimal fractions. Pupils need to understand that fractions arise from division of whole numbers and that decimal fractions are special cases of fractions as their denominators are always a power of ten.



- As an introduction to **notation of decimals** the following definition should be used. 'The digit to the immediate right of the decimal point is in the tenths column' and 'The digit that is two places to the right of the decimal point is in the hundredths column' etc.
- It is strongly recommended that decimals are <u>initially</u> introduced by saying how many tenths there are e.g. 2.8 should be read as 'two and eight tenths', not 2 point 8.
- However, having established an understanding of place value within decimals, pupils should be able to demonstrate knowledge that decimal numbers are said differently to whole numbers. Pupils should say and read any decimal number e.g. for 347.37 say '347 point three seven' but should know, through discussion, that 347.37 is equivalent to $347\frac{37}{100}$ and written as a mixed number, it is 347 and 37 hundredths.
- Children sometimes confuse a decimal point and think of it as separating two whole numbers. This is a very common misconception; possibly because of the early introduction of money notation and that the point separates pounds and pence.
- Children should understand that **decimal fractions arise out of division** and that when any unit needs to be broken up it needs to be divided into ten equal parts. (It is useful to use visuals where the unit that can be broken into ten equal pieces is wrapped as a whole).
- The digits to the right of the ones column have decreasing values in powers of ten with the first place representing tenths, the second hundredths, and so on, and can represent infinitely small numbers.

- During this level children will need to be given ample opportunity to solve problems and develop a wide range of strategies to support the necessary mental calculations needed to solve them. Children must be able to discuss the strategies they have used to solve problems.
- When setting problems, set them out in a **horizontal form**. This will encourage a more flexible response. One drawback to textbook questions is that they are often set out in vertical form and this encourages children to follow a more procedural approach rather than an intuitive one.
- Writing number stories and representing what a story will look like using diagrams and number sentences is very important for children to be able to contextualise the numbers. This is just as relevant at Second Level as it is in Early and First Levels.
- When asking children to write their own stories, it is useful to reduce the level of difficulty in a question so pupils can concentrate on the language that needs to be included in the story to match the number sentence that is written. For example, $13 \times 6 = ?$ and '208 people went to a football match and sat in the first 6 rows, how many people were in each row?' do not match. The number sentence should be $208 \div 6 =$
- It is very important that children **estimate answers** before attempting to solve a problem by calculating. They should then use their estimate to assess if their final result was reasonable.
- Some children will find it difficult to use the **inverse relationship** between addition and subtraction to choose the more efficient strategy between counting on or counting back for solving particular problems. They need to be able to re-interpret 47 26 as, 'what do you have to add to 26 to get to 47' and so count by tens and ones.

- Another useful strategy is to change them by adding or subtracting an amount to make the numbers more manageable. This '**transformation' strategy** (or 'Same Difference' strategy for subtraction) works as long as you:
 - \circ add/subtract the same amount to both sides for a subtraction
 - $\circ\;$ add to one side and take exactly the same amount off the other side for an addition.
- We know that children have to go through many counting experiences before they **trust** that the order in which they **count** does not affect the final amount and so learn that starting at the largest number is the most efficient. The same idea underpins why facts for subtraction, multiplication and division can be relied upon. If offered a variety of rich experiences to construct number problems using concrete materials they will begin to trust the associations between the four operations.
- This trust enables children to recall a 'related facts' from just one calculation. For example, if they have worked out that 18 x 6 is (10 x 6) + (8 x 6) = 60 + 48 = 108, then they will also know that 6 x 18 = 108 108 ÷ 18 = 6 and 108 ÷ 6 = 18.
- A prime number can only be divided by one and itself i.e, has two factors. One is not a prime number because it has only one factor i.e. itself. Prime numbers are formally introduced at Third Level.
- For children to understand negative numbers they need to understand that there are a set of <u>whole</u> numbers called integers. The set of integers consists of the 'Natural Numbers' {0, 1, 2, 3 ...} and their non-zero negatives (-1, -2, -3 ...). Zero is an integer, 2.6 and 8¹/₂ are not.

• Children will need to record their calculations using a range of symbols and numbers. They will also use a wide range of jottings including empty number lines and arrow notation to show jump strategies and drop down notation for split strategies. Definitions of the seven most common strategies are given below:

1. Jump to the next decade

Begin from one number, jump to the nearest decade, jump tens, and then jump the remaining ones.

2. Jump

Begin from one number, jump tens then jump ones (or ones and tens).

3. Split

Split tens and ones, add/subtract them separately, and then recombine.

4. Split Jump

Split tens and ones, add/subtract tens first, then add first lot of ones and then second ones. E.g. 48 + 94 = 90 + 40 \rightarrow 130 \rightarrow 138 \rightarrow 142

5. Over jump

Begin from one number, overshoot the jump, and then compensate

6. Compensation

Change one or both numbers, add/subtract, and then compensate.

7. Transformation

Change both numbers while preserving the result, and then add/subtract.

• The aim at this level is for children to have a flexible approach to the use of different strategies and be able to consider the efficiency. Written strategies should be used when the numbers get too big to calculate mentally but at all times, children should be able to explain their thinking. • As children become fluent with empty number line notation and split notation, more formal arrow notation can be used. It is more compact and moves closer to more conventional notations, such as the semiformal column sum and the standard written algorithm. These need to be used with more precision especially when considering the meaning of the equals sign.

Eg. We can write $37 + 25 \rightarrow 37 + 20 \rightarrow 57 + 5 \rightarrow 62$

But not 37 + 25 = 37 + 20 = 57 + 5 = 62

- Working with equations such as $4 \times 5 = 10 + 6 + 4$ helps develop the understanding that the same quantity is shown on both sides of the equals sign, rather than one side showing an instruction for an operation and the other the answer.
- It would be useful if teachers were to show pupils that 'balance' is preserved provided we carry out the same operation to both sides of the equation. This then paves the way for a more formal treatment of equation solving when pupils move into Third Level.

Informal Jottings

- Children should be encouraged to record jottings. This is where a mental strategy has been used but some initial calculations are jotted down to avoid having to remember them. Keeping track of all the necessary parts allows space in the brain to apply to solving the problem.
- Problems should be chosen which help children to apply combinations of mental strategies and which link with other areas of mathematics e.g. John has dug up his old lawn and wants to reseed it. His lawn is rectangular and measures 11.7 metres by 4.8 metres. He needs 70 grams of grass seed for every square metre of lawn. Grass seed is sold in 1 kg boxes. How many boxes should John buy?

Written strategies should be used when the numbers get too big to calculate mentally i.e. when numbers have more than 3-digits.

Semi-formal Strategies for Addition and Subtraction

These are well-organised, standardised, written strategies. This strategy requires reasoning with whole numbers. For example, the pupil calculates 500 + 200 = 700, not 5 + 2 = 7. They are generally set out in columns and still involve children doing multi-digit calculations mentally, but the writing systematically records intermediate results, and keeps them organised. It is essentially a split method where the hundreds, tens and ones part of each number are added separately, and then these sub-totals are added. The writing helps to keep track of the calculations, while actually adding the numbers together remains a mental task. It is not crucial where the subtotals are placed as they are added together as whole numbers.

267	267	446 - 178 =	(decomposing hundreds)
+ 4 50	13	400 + 40 + 6	300 + 140 + 6
110	110	100 + 70 + 8	- 100 + 70 + 8
3	600		(decomposing tens) 300 + 130 + 16
723	723		- 100 + 70 + 8
Significant	Least Significant Numbers Are		200 + 60 + 8
Added First	Added First		= 268

This practice with decomposition supports the 'contracted form' to the right.

Formal Algorithms for Calculations

Formal algorithms should ideally be introduced once children have a clear understanding of place values within calculations through using a 'nested view' of numbers i.e. understand that 'nested' within thousands are hundreds, tens and ones.

For example, how many tens are there in 347? If a child recognises that there are 34 tens or 2 hundreds and 14 tens then they can tackle the problem through use of a formal algorithm. If they do not understand this core concept, they should not be using a column layout.

Formal algorithms are probably the most familiar written computational methods. They are sometimes referred to as standard or traditional algorithms. Algorithm means a step-by-step procedure for computing a standard task. In the formal algorithms, mental calculations mainly involve 1-digit numbers. Using this method requires pupils to follow a precise procedure, working from right to left, and so on. It also involves a precise layout of the writing, aligning the digits in columns.

Semi-formal Strategies for Multiplication

These are well- organised, standardised written strategies.

Children can begin to use the '**empty number lines'** efficiently by making bigger jumps.



Or use the **grid method** (provide children with opportunities to create arrays for two-digit x one-digit multiplication as this leads them into recording the 'grid method' effectively). E.g. $14 \times 4 =$



Semi-formal Strategies for Division

Chunking - pupil repeatedly takes away 'chunks' of the large number, where each 'chunk' is a known multiple of the divisor, until the large number has been reduced to zero or the remainder is less than the divisor. At the same time the pupil keeps track of the total amount of 'chunks' subtracted, which eventually becomes the final result of the sum.



132 ÷ 6 = 22

Ensure pupils have a clear layout and that they understand the process for chunking by first dividing a single-digit number before moving into dividing by a two-digit number.



Suggested Written Recording

Order of Operations

- 1. Where a calculation contains only addition and subtraction, the operations should be carried out in the order they appear.
- 2. Where a calculation contains only multiplication and division, the operations should be carried out in the order they appear.
- 3. Where a calculation contains a combination of the 4 operations, multiplication and division take priority over addition and subtraction.

Pupils should also set out their working correctly. When a calculation requires more than one operation, there are rules for ensuring that the operations are done in the correct order.



ESTIMATION AND ROUNDING AIM: Use rounding to estimate sum, difference, product and quotient in real life situations including decimal fractions and fractions. Check reasonableness and share solutions with others. MNU 2-01a

Round numbers to allow for an approximate answer, appropriate to a context, including decimals. e.g. If the length of park is 56.36m and the width is 37.79m, approximately what is the perimeter?

Round numbers to allow for an approximate answer, appropriate to a context, including fractions.

e.g. If I have 7/8 and 9/10, roughly how much do I have?

a) 1 b) 2 c) 16 d) 18

Round numbers having two decimal places to the nearest whole number. e.g. round 3.48 to 3, 16.83 to 17

Explore situations which require rounding up.

e.g. If 12 friends order a taxi to take them to the cinema and a taxi

can seat 5, how many do they need?

Round numbers having one decimal place to the nearest whole number. e.g. round 3.6 to 4 17.2 to 17

Explore the connection between rounding and accuracy. e.g. rounding to tenths gives a more accurate answer than rounding to the nearest ten.

Round whole numbers to the nearest 10, 100, 1 000, 10 000 or 100 000. e.g. 4 790 to 5 000 28 399 to 28 000

nearest hundredth, tenth or whole number. e.g. 5.471 rounded to nearest hundredth - 5.47 nearest whole - 5 e.g. 6.786 rounded to nearest hundredth - 6.79 nearest tenth - 6.8 Place whole numbers up to 1 000 000 on a scaled number line, using varied intervals:

Round numbers having up to two decimal places to the

1 000 000 If this is where zero goes, and this is where 1 000 000, where will you put 582 000?

Place whole numbers up to 100 000 on a scaled number line, using varied intervals:

100 000

If this is where zero goes, and this is where 100 000 goes, where will you put 32 000

Place whole numbers up to 10 000 on a scaled number line, using varied intervals:

10 000 If this is where zero goes, and this is where 10 000 goes, where will you put 2 500 ?

Given two numbers in the range 0-1 000 000 identify the number which is halfway between them. e.g. an interval of 400 000 (what's halfway between 225 000 and 625 000?) e.g. an interval of 30 000 (what's halfway between 130 600 and 160 600? e.g. an interval of 1 500 (what's halfway between 7 500 and 9 000 e.g. an interval of 120 (what's halfway between 84 560 and 84 680 e.g. an interval of 1.6 (what's halfway between 2 and 3.6?)

Given two numbers in the range 0-100 000 them. e.g. an interval of 60 000 (what's halfway between 20 000 & 80 000?) 7 246 and 15 246 7 500 and 9 000

identify the number which is halfway between e.q. an interval of 8 000 (what's halfway between e.g. an interval of 1 500 (what's halfway between e.g. an interval of 120 (what's halfway between

Given two numbers in the range 0-10 000 identify the number which is halfway between them. e.g. an interval of 4 000 (what's halfway between 3 000 & 7 000?) e.g. an interval of 50 (what's halfway between 250 & 300?) e.g. an interval of 20 (what's halfway between 4 520 & 4 540?)

Round numbers to allow for an approximate answer.

e.g. If I earn £37 per day, roughly how much money will I have after 4 days' work?

84 560 and 84 680?)

NUMBER WORD SEQUENCES AIM: Understand the cyclical pattern of the number system to count, read, write, compare and order integers and decimal fractions. MNU 2-02a, MNU 2-04a, MTH 2-03a



MNU 2-02a

Sequence decimal numbers up to three decimal places. e.g. understand that a book coded 360.341 under the Dewey system will come before a book coded 360.56

Recognise and represent exact partitions of numbers up to 1 000 000 and beyond, including decimals and record in a standard and non-standard place value including expanded notations.

STANDARD PLACE VALUE

4 753 = 4 thousands, 7 hundreds, 5 tens and 3 ones or 47 hundreds and 53 ones or 475 tens and 3 ones.

NON-STANDARD PLACE VALUE 4 753 = 46 hundreds, 14 tens and 13 ones for example

EXPANDED NOTATION 4753 = 4000 + 700 + 50 + 3

Use a graphic organiser to read and write any whole number up to a million.

Use place value to compare and order numbers beyond 10 000.

Model numbers to tens of thousands or beyond using the base 10 place value system.



Fife Numeracy & Mathematics Progression Pyramid: Second Level ~ Page 65

THE NEXT DOMAIN NEEDS TO BE ORGANISED IN A DIFFERENT WAY BECAUSE THE INTRODUCTION, DEVELOPMENT AND PERFECTION OF THE STRATEGIES INCLUDED WILL NEED AT LEAST TWO/THREE YEARS FOR MOST SECOND LEVEL PUPILS TO FULLY MASTER. It is advised that the

strategies in the Calculating: Addition and Subtraction and Calculating: Multiplication and Division domains are built up in more of a **linear progression**. Once the learner has proved competent in using a particular strategy, they may then move on to learning a new strategy. To be completely proficient with a strategy means that they have moved from understanding how the strategy works whilst using concrete materials to then being just as competent when visualising the materials or using pictorial clues. They then must make the important conceptual leap to being able to successfully answer problems given in the abstract form. This will involve using the strategies learned but applying these to numbers larger than they could possibly image in their heads.

The grey boxes along the bottom of the pyramids should be continued all the way through each pyramid and are a reminder how strategies can be introduced/developed.

New strategies introduced by using	Pupils will need to show they can successfully	Constant revision and practice will ensure that the
concrete materials	apply their knowledge/strategies to solve	knowledge and strategies are deeply embedded and
 visualising screening materials or using 	problems in familiar and unfamiliar	fully understood. This rich number sense will help to
pictorial clues	situations.	ensure that children can make informed decisions
• abstract problems - <i>e.g. 862 + 25</i>	FORMAL ALGORITHMS ARE INTRODUCED	and solve problems efficiently.
The strategies will then be practiced and refined	AT THIS STAGE - see 'Points to Consider	
anadually becoming confident with one strategy	- Part 1 and examples in 'Suggested	Calculations involving decimals and fractions are
before moving on until children slowly build up a	Written Recording'	introduced at this stage. Decipipes and dienes are
wide range of useful approaches to solving	A daily mix of calculations in the form of	excellent tools to support pupils with understanding
problems	equations and word problems should be given	what tenths and nundreaths look like.
F	to the children and ample time should be ring-	How much bigger is 0.6 than 0.58? Show using deci-pipes.
PLEASE LOOK CAREFULLY AT THIS DOMAIN	fenced to allow rich discussion about the	
IN FIRST LEVEL! A child will be unable to	most efficient strategies to use.	
proceed successfully if First Level learning	It is also very important that the type of	
intentions are not fully understood and deeply	calculations selected by the teacher provide	Children are beginning to use explore how to use
embedded.	plenty of opportunity to practice the range	multiplication to aid their addition and subtraction
	of strategies that have been taught.	calculations ('distributive law')
	As children progress, they should chose the	e.g. 21 + 42 + 14 are all multiples of 7 and so this can be
	strategy that best fits the problem;	changed to (3 x 7) + (6 x 7) + (2 x 7) = 11 x 7 = 77.
	teachers should not give a set of problems	
	and tell the children which strategy would be	
	the most efficient!	

CALCULATING AIM (JUMP STRATEGY): Use a range of mental and written strategies to solve multi-step problems, that involve a combination of MNU 2-03a, MNU 2-03b, MTH 2-03c addition, subtraction, multiplication and division with whole numbers ensuring the correct order of operations.



Investigate negative numbers in real life contexts and explore the associated vocabulary. e.g. Temperature - below freezing Bank balances, debt - overdraft, bank loan Height - above/below sea level Sport - in golf, explore the idea of 'above' or 'below' par

-6

67

Solve addition and subtraction problems by using a jump strategy across the decade.

Use a known fact to work out an unknown fact e.g. Number bonds to 10: $40 + 70 \rightarrow 40 + 60 = 100$ so 40 + 70 is ten more = 110.

Near doubles: e.g. 31 + 31 is double 3 tens add 2 69 + 69 is double 7 tens take 2 away

Use a known fact to work out an unknown fact e.g. 142 + 144 is *double 14 tens add 6 [280 + 6]*

CALCULATING AIM (SPLIT STRATEGY): Use a range of mental and written strategies to solve multi-step problems, that involve a combination of addition, subtraction, multiplication and division with whole numbers with whole numbers ensuring the correct order of operations.

MNU 2-03a, MNU 2-03b, MTH 2-03c



CALCULATING AIM (NUMBER STRATEGIES): Use a range of mental and written strategies to solve multi-step problems, that involve a combination of addition, subtraction, multiplication and division with whole numbers ensuring the correct order of operations. MNU 2-03a, MNU 2-03b, MTH 2-03c

USE FORMAL ALGORITHMS

Pupils should be taught how to use the standard written form to solve addition and subtraction calculations.

They should always mentally estimate the answer before using the written method.

(See 'Points to Consider ')

EFFICIENTLY SOLVE PROBLEMS

Choosing effective strategies is a skill that children will develop gradually.

Regular practice (daily) with a wide range of mental calculations will help support their ability to employ efficient strategies and make informed decisions that they can confidently discuss with others.

Children need to be provided with a wide mix of problems that should be presented as both equations and word problems.

Write their own stories to accompany a given number sentence or one of their own, including multistep problems.

Make connections between maths in school and everyday experiences.

e.g.

John has hired me to count his money. I counted £1 500 before stopping for coffee. After coffee I counted £5 more. John offered me £100 for the job or 10% of the money I had counted. Which choice will give me more money for my work?

Investigate negative numbers in real life contexts and explore the associated vocabulary. e.g. Temperature - below freezing Bank balances, debt - overdraft, bank loan Height - above/below seg level Sport - in golf, explore the idea of 'above' or 'below' par

Use the commutative property to add numbers by making decade numbers up to 100 e.g.

16 + 8 + 4 = 20 + 8 16 + 9 + 4 + 8 + 1 = 20 + 10 + 8 = 38 24 + 7 + 6 = 24 + 6 + 7 = 30 + 7 14 + 16 + 45 = 30 + 45 60 + 20 + 40 = 60 + 40 + 20 = 100 + 20

Solve problems that involve calculating ensuring start unknown, change unknown, result unknown:

Jamie has 150 fewer football cards than his cousin, Darren, If Darren has 378 how many does Jamie have?

Morag received some money for her birthday. When she added it to the £143 in her bank account she had £178. How much did she receive?

A double bed is 135 cm wide. Kiera's room is 3.5m wide. If she puts a bedside cabinet that is 45cm wide beside the bed, how much space will be left?

Use multiplication to solve addition and subtraction problems where common factors can be found.

e.q. 36 + 45 = (4 x 9) + (5 x9) = 9 x 9 = 81 63 - 27 = (7 x 9) - (3 x 9) = 4 x 9 = 36

29 + 30 + 28 + 33 + 32 + 27 + 31 + 30 = 8 x 30 = (8 x 3) x 10 = 24 x10 = 240

Solve addition and subtraction problems by using the 'Same Difference' Strategy (Transformation) Add - Take some from one number to give to the other e.g.

18 + 17 becomes 20 + 15 [by adding 2 to 18 and then subtracting 2 from 17]. 68 + 37 becomes 70 + 35 698 + 37 becomes 700 + 35 367 + 78 becomes 370 + 75 = 445

Subtract - change the numbers by adding or subtracting the same amount e.q.22 - 17 becomes 20 - 15 = 5 72 - 27 becomes 75 - 30 = 35

507 - 296 becomes 511 - 300 = 211

Start to use arrow notation to record thinking. 'Instead of 53 take

19, I did 54 take 20 $, 53 - 19 \rightarrow 54 - 20 \rightarrow 34$

53 - 19 = 54 - 20 = 34

Pupils should explore the effect of carrying out operations on an existing equation, e.g. if they know (and write down) 20+15=35 then what is the effect of (say) doubling both sides? What happens if we add 5 to one side but not the other? What happens if we carry out different operations to both sides of the equation?

Use non-standard place value splits to mentally calculate. e.g. 39 - 17 as 19 - 17 + 20

98 - 27 as 28- 27 + 70 = 71

Express understanding of a particular symbols pictures and real life situations.



Write number equations to match a given number sentence.

e.g. 24 -	- 13 =
76 - 🗌	= 35

97 = - 125

Solve addition and subtraction problems using a Rounding and Compensation strategy:

When one number is close to a multiple of ten. What is 39 + 66? Round 39 up to 40 and then add 66 = 106. Then subtract 1 to make 105

What is 51 - 35? Round 51 down to 50 and then subtract 35 = 15. Then add one more to make 16.

When one number is close to a multiple of ten or a hundred, What is £99 + £56? Round £99 up to £100 and then + \pounds 56 = \pounds 156. Then subtract \pounds 1 to make £155

What is £568 - £99? Round £99 up to £100. Then subtract £100 from £568 to make £468. Then add £1 to make £469.

When one number is close to any multiple of ten. What is £39 + £516? Round £39 to £40 and then add £516 to give £556 Then subtract £1 to make £555, The answer is £555

What is £768 - £59? Round £59 to £60. Then subtract £60 from £768 to make £708. Then add £1 to make £709. The answer is £709

> Use a known fact to work out an unknown fact e.g. Number bonds to 10: $40 + 70 \rightarrow 40 + 60 = 100$ so 40 + 70 is ten more = 110.

> Near doubles: e.g. 31 + 31 is double 3 tens add 2 69 + 69 is double 7 tens take 2 away Use a known fact to work out an unknown fact e.g. 142 + 144 is double 14 tens add 6 [280 + 6]
CALCULATING AIM (DECIMALS): Use a range of mental and written strategies to solve problems, that involve a combination of addition and subtraction of decimal fractions.

MNU 2-03a, MNU 2-03b, MTH 2-03c

Use the 'inverse rule' to add and subtract decimals to one decimal place.

0.6 + ? = 1.7 so reverse it 1.7 - 0.6 = e.g.

What do I need to add to 0.6 to make 1.7?

USE FORMAL ALGORITHMS

Pupils should be taught how to use the standard written form to solve addition and subtraction calculations.

They should always mentally estimate the answer before using the written method.

(See 'Points to Consider ')

EFFICIENTLY SOLVE PROBLEMS

Choosing effective strategies is a skill that children will develop gradually.

Regular practice (daily) with a wide range of mental calculations will help support their ability to employ efficient strategies and make informed decisions that they can confidently discuss with others.

Children need to be provided with a wide mix of problems that should be presented as both equations and word problems.

Write their own stories to accompany a given number sentence or one of their own, including multistep problems.

Make connections between maths in school and everyday experiences.

e.g.

John has hired me to count his money. I counted £1 500 before stopping for coffee. After coffee I counted £5 more. John offered me £100 for the job or 10% of the money I had counted. Which choice will give me more money for my work?

DECIMALS

Decimal addition and subtraction should be introduced when learners have a grasp of place value in relation to decimals and when strategies have been understood in the first place with whole numbers.

Solve problems that involve calculating ensuring start unknown, change unknown, result unknown:

Jamie has 150 fewer football cards than his cousin, Darren. If Darren has 378 how many does Jamie have?

Morag received some money for her birthday. When she added it to the £143 in her bank account she had £178. How much did she receive?

A double bed is 135cm wide. Kiera's room is 3.5m wide. If she puts a bedside cabinet that is 45cm wide beside the bed, how much space will be left?

Use a split strategy to add and subtract decimals to one decimal place using non-standard place value partitions.

e.g. $4.3 - 1.7 \rightarrow 4.3$ converts to 3 + 13 tenths, so 3 + 13 tenths -1 - 7 tenths 2 + 13 tenths -7 tenths 2 + 6 tenths = 2.6

 $4.3 - 1 = 3.3 \rightarrow 2 + 13$ tenths $\rightarrow 13$ tenths - 7 tenths = 6 tenths so 2 ones + 6 tenths $\rightarrow 2.6$

N.B. In subtraction examples, the digit in the tenths column being subtracted needs to be greater than the digit in the tenths column from which it is being subtracted.

e.g. $1.9 + 2.5 \rightarrow 1.9 + 2 \rightarrow 3.9 \rightarrow 3$ and 9 tenths $\rightarrow 9$ tenths add 5 tenths $\rightarrow 14$ tenths $\rightarrow 1$ one and 4 tenths $\rightarrow 1.4$ so 3 + 1.4 = 4.4

Can add and subtract decimal numbers by using a rounding and compensating method

e.g. 5.37 - 3.8
$$\rightarrow$$
 3.8 + 0.2 = 4 \rightarrow 5.37 - 4 \rightarrow

1.37 + 0.2 → 1.57

 $6.5 + 2.9 \rightarrow 6.5 + 3 \rightarrow 9.5 - 0.1 \rightarrow 9.4$

Add or subtract two decimal numbers to one place and explain what has happened to the decimal point (i.e. when the combined answer is a whole number and there are no parts of a whole in the answer!).

> e.g. 1.5 + 3.5 = 5 2.7 + 3.3 = 6 7.4 + 2.6 = 10 7.5 - 3.5 = 4 9.4 - 6.4 = 3



Write number equations to match a given number sentence.

e.q. 24 - 13 = $76 - \Box = 35$

97 = - 125

Use a split strategy to add and subtract decimals to one decimal place, using standard place value partitions.

Add 2 decimal numbers to one place by splitting the whole numbers and tenths,

e.q. 3.5 + 4.8 = 3 + 4 = 7

5 tenths + 8 tenths = 13 tenths \rightarrow 1 one and 3 tenths, so 7 + 1 + 0.3 = 8.3

97-53=9-5=4

7 tenths - 3 tenths = 4 tenths

4 + 0.4 = 4.4



CALCULATING AIM (MULTIPLICATION AND TIMES TABLES): Use a range of mental and written strategies to solve multi-step problems, that involve a combination of addition, subtraction, multiplication and division with whole numbers with whole numbers ensuring the correct order of operations.

MNU 2-03a, MNU 2-03b, MTH 2-03c

Note: This pyramid appears across two pages.



Fife Numeracy & Mathematics Progression Pyramid: Second Level ~ Page 71

Begin to solve multiplication or division problems involving a 3 - digit number by a 1-digit number by a short written algorithm - *see 'Suggested Written Recording' Advice.*

> Solve division problems by using standard place value. $e.g. \ 92 \div 4 = (80 \div 4) + (12 \div 4)$ $= 20 \div 3 = 23$ $186 \div 3 = (180 \div 3) + (6 \div 3)$

USE FORMAL ALGORITHMS

Pupils should be taught how to use the **standard written form** to solve multiplication and division calculations.

They should always mentally **estimate** the answer before using the written method.

EFFICIENTLY SOLVE PROBLEMS

Choosing effective strategies is a skill that children will develop gradually.

Regular practice (daily) with a wide range of mental calculations will help support their ability to employ **efficient strategies** and make informed decisions that they can **confidently discuss** with others.

Find solutions to multistep problems involving mixed operations and make up their own multistep word problems. e.g. 5 x 40 + 7 = 207 12 x 3 + 128 =

Write word problems to match a given number sentence. e.g. — 97 = 125 Work out the six, seven and eight times tables from their knowledge of the five times tables, using the distributive property of multiplication, e.g. $7 \times 6 = 5 \times 6 + 2 \times 6$

6 5 = 30 2 = 12	⁶ 7 = 42
6	

Multiplying by 6 is same as x 5 add one more set, 8 x 6 = (8 x 5) + (8 x 1) Multiplying by 7 is same as x 5 add 2 more sets, 9 x 7 = (9 x 5)

Solve division problems using known addition, subtraction and multiplication facts within the 2, 3, 4, 5 and 10 times table. *e.g.*





Solve problems that involve repeated equal quantities, rates etc. see *Points to Consider* Jamie has 150 fewer football cards than his cousin, Darren. If Darren has 378 how many does

Jamie have?

Morag received some money for her birthday. When she added it to the £143 in her bank account she had £178. How much did she receive?

A double bed is 135 cm wide. Kiera's room is 3.5m wide. If she puts a bedside cabinet that is 45 cm wide beside the bed, how much space will be left?

Represent multiplication/division stories using a variety of ways, including story, pictorial, concrete and abstract.

e.g. Tommy arranged his 15 football trophies evenly on three shelves. How many trophies were on each shelf? Fiona distributed 650 flyers for her new restaurant evenly amongst 13 streets. To how many houses did she deliver the pamphlets in each street? Use a known fact to derive a new fact

e.g. make the links between the 2 times table and the 3 times table; then the 6 and 9 times tables. Make the links between the 2, 4 and 8 times tables.

Make the links between the 2 times tables and the 3 times table and then the 6 and 9 times tables.



Use their ten times facts to work out the 9 times table.

8 x 9 is the same as 9 x 8. This is just 10 x 8 - 1 x 8 = 80 - 8 = 72





Continue to demonstrate an understanding of both the commutative law and the inverse rule in relation to multiplication and division.

Understand that multiplication is commutative and use this to solve a problem by changing the order of the factors.







Use a known fact to work out an unknown fact e.g. to find out 27 × 4, I know that 25 × 4 is 100, so 27 × 4 must be 2 × 4 more which is 108. CALCULATING AIM (MULTIPLICATION STRATEGIES): Use a range of mental and written strategies to solve multi-step problems, that involve a combination of addition, subtraction, multiplication and division with whole numbers with whole numbers ensuring the correct order of operations.

MNU 2-03a, MNU 2-03b, MTH 2-03c

Note: This pyramid appears across two pages.

USE FORMAL ALGORITHMS

Pupils should be taught how to use the **standard written form** to solve multiplication and division calculations.

They should always mentally **estimate** the answer before using the written method.

EFFICIENTLY SOLVE PROBLEMS

Choosing effective strategies is a skill that children will develop gradually.

Regular practice (daily) with a wide range of mental calculations will help support their ability to employ **efficient strategies** and make informed decisions that they can **confidently discuss** with others.

> Solve multi step problems by ensuring correct order of operations.

Solve a multiplication or division problem that involves decimal fractions and is beginning to express the remainder as a whole number, fraction or decimal depending on the context. e.q. 426 ÷ 8 = 426 ÷ 2 ÷ 2 ÷ 2 → $213 \div 2 \div 2 \rightarrow 106 \frac{1}{2} \div 2 \rightarrow 53 \frac{1}{4}, 53.25$ or 53 r2 $426 \div 4 = 100 + 6$ 100 + 6 26 2 100 + 6 100 + 6 $426 \div 4 = 106 r^2$ using the 'chunking down' method 8)426 53. 25 \rightarrow Begin to reason which answer would be most appropriate for the set word problem.

Multiply and divide whole numbers and decimal fractions with at least 2 decimal places mentally by 10, 100 and 1 000.



Use division for sharing or grouping a collection.

Remainders can be left or shared out depending on the context of the story e.g. There were 33 people at the family reunion dinner. The restaurant only had tables that would sit four people. How many tables did the family need to book?

Anya has bought 57 eggs from the farmer but they are all in one basket. She wants to pack them safely into egg boxes of 6 to stop them from breaking on her journey home. How many boxes will she need?



Multiply and divide whole numbers and decimal fractions with at least 3 decimal places by multiples of 10.

Begin to solve division problems by using a rounding and compensation method.

e.g. 238 ÷ 7

How many 7s are there in 280?

7 x 40 = 280

280 - 238 = 42

42 ÷7 = 6 so 40 sevens subtract 6 sevens equals 34 sevens.





Describe the rule for multiplication by 10 as 'the digits move one place to the left'. *6 x 10 = 60* 4 x 10 = 40 *24 x 10 = 240* 352 x 10 = 3,520 Describe the rule for division by 10 as 'the digits move one place to the right'. *50 ÷ 10 = 5* 490 ÷ 10 = 49

Describe the rule for multiplication by 100 as 'the digits move two places to the left'. *8 x 100 = 800* 2 x 10 0= 200 300 x 100 = 30 000

600 ÷ 10 = 60

Describe the rule for division by 100 as 'the digits move two places to the right'. *500 ÷ 100 = 5 4 900 ÷ 100 = 49 6 000 ÷ 100 = 60*

Use a known fact to work out an unknown fact e.g. to find out 27 x 4. I know that 25 x 4 is 100, so 27 x 4 must be 2 x 4 more which is 108

MTH 2-05a











MONEY AIM: Give sound reasons for my decisions when solving problems involving money. MNU 2-09a, MNU 2-09b, MNU 2-09c Compares costs and Create simple financial plans. Investigate 10%, 25% & 50% discounts for Investigate and calculate 'best buys' determines sale items. with and without digital technologies. Cost of a Recipe affordability within a given budget. e.g. buy one get one free v's 2 for f_x Number Price per Pack Price Per ngredients No. In Pack Needed In Total Cost Item Apples £1.50 25p 12 £3 6 Calculate profit and loss when working with a budget. Enter and read amounts correctly on a Use a range of strategies for the four Explain the possible calculator to the nearest pence. operations, with numbers to 2 decimal implications of debt. places, to solve problems. Explain the risks and e.g. using jump & split strategies to benefits of using bank solve £7.50 - £3.23 cards. 3 x £1.87 Understand how to access product prices from a variety of sources. Be able to work out what needs to e.g. be saved over a period of time internet, brochure, advert, price list etc. when planning a simple purchase. budgeting etc. Be able to explain Understand why the terms credit, ing Page Per Unit Total For This Item money uses decimal debit & debt. Explain why budgeting is important. notation. e.g. Spending Expenses Savings Money













PATTERNS AND RELATIONSHIPS AIM: Generate patterns and describe rules used to generate sequences using multiples, factors, square numbers and triangular numbers. MTH 2-13a



Fife Numeracy & Mathematics Progression Pyramid: Second Level ~ Page 84

Number of Vertices

12

Solve simple combination problems. e.g. record all the possible digit combinations for 4-digit numbers and order from smallest to largest: 1, 2, 3, 4 = 1 234, 1 243, 1 324, 1 342, 1 423, 1 432, 2 134, 2 143, 2 314, 2 341, 2 413, 2 431, 3 124, 3 142, 3 214, 3 241, 3 412, 3 421, 4 123, 4 132, 4 213, 4 231, 4 312, 4 321.

Identify a number pattern involving one operation (increasing/ decreasing) using fractions or decimals. The pattern 0.1, 0.15, 0.2, 0.25, 0.3 is increasing by 0.05

Describe the pattern of combinations formed when using 2, 3, 4 numbers and make a generalisation. e.g. in using 3 colours there will be 6 combinations RBG, RGB, BGR, BRG, GBR, GRB (1x2x3) e.g. we have 4 objects: A, B, C and D. How many possible combinations are there when arranging them on a shelf? e.g. ABCD, ABDC, DACB...

8 | 12 | 16 | 20 | 24 | 28

4

EXPRESSIONS AND EQUATIONS AIM: Solve problems that require finding the value of one unknown quantity MTH 2-15a or letter. Solve problems using the properties and relationships of the four operations.



represented by a symbol
ables. Nore
antity where there is one .
ematical statements. because 273 × 5 > 260 × 5 = 1300.
ematical statements (without finding er' to the actual calculations). $392 \times 15 = \square \times 392$ 7 □ 14 (Put in < or = or >.)

Second Level Overview

* The numeracy learner statements that are the responsibility of all are shown in italics.

Key Aspect	Experiences and Outcomes	National Benchmarks
Estimation and Rounding Pg 63	I can use my knowledge of rounding to routinely estimate the answer to a problem then, after calculating, decide if my answer is reasonable, sharing my solution with others. MNU 2-01a	 Rounds whole numbers to the nearest 1000, 10 000 and 100 000. Rounds decimal fractions to the nearest whole number, to one decimal place and two decimal places. Applies knowledge of rounding to give an estimate to a calculation appropriate to the context.
Number Word Sequences Pg's 64, 67- 74	I have extended the range of whole numbers I can work with and having explored how decimal fractions are constructed, can explain the link between a digit, its place and its value. MNU 2-02a Having explored the need for rules for the order of operations in number calculations, I can apply them correctly when solving simple problems. MTH 2-03c I can show my understanding of how the number line extends to include numbers less than zero and have investigated how these numbers occur and are used. MNU 2-04a	 Reads, writes and orders whole numbers to 1 000 000, starting from any number in the sequence. Adds and subtracts multiples of 10, 100 and 1000 to and from whole numbers and decimal fractions to two decimal places. Adds and subtracts whole numbers and decimal fractions to two decimal places, within the number range 0 to 1 000 000. Orders numbers less than zero and locates them on a number line.

Key Aspect	Experiences and Outcomes	National Benchmarks
Place Value Pg 65	I have extended the range of whole numbers I can work with and having explored how decimal fractions are constructed, can explain the link between a digit, its place and its value. MNU 2-02a	 Partitions a wide range of whole numbers and decimal fractions to three decimal places, for example, 3.6 = 3 ones and 6 tenths = 36 tenths. Explains the link between a digit, its place and its value for whole numbers to 1 000 000. Explains the link between a digit, its place and its value for numbers with at least 3 decimal places. Reads, writes and orders sets of decimal fractions to three decimal places.
Calculating: Addition and Subtraction: Pg's 67-70	Having determined which calculations are needed, I can solve problems involving whole numbers using a range of methods, sharing my approaches and solutions with others. MNU 2-03a I have explored the contexts in which problems involving decimal fractions occur and can solve related problems using a variety of methods. MNU 2-03b	 Adds and subtracts multiples of 10, 100 and 1000 to and from whole numbers and decimal fractions to two decimal places. Adds and subtracts whole numbers and decimal fractions to two decimal places, within the number range 0 to 1 000 000. Applies the correct order of operations in number calculations when solving multi-step problems. Identifies familiar contexts in which negative numbers are used.

Key Aspect	Experiences and Outcomes	National Benchmarks
Calculating: Multiplication and Division Pg's 71-74	Having determined which calculations are needed, I can solve problems involving whole numbers using a range of methods, sharing my approaches and solutions with others. MNU 2-03a I have explored the contexts in which problems involving decimal fractions occur and can solve related problems using a variety of methods. MNU 2-03b	 Uses multiplication and division facts to the 10th multiplication table. Multiplies and divides whole numbers by multiples of 10, 100 and 1000. Multiplies and divides decimal fractions to two decimal places by 10, 100 and 1000. Multiplies whole numbers by two digit numbers. Multiplies decimal fractions to two decimal places by a single digit. Divides whole numbers and decimal fractions to two decimal places, by a single digit, including answers expressed as decimal fractions, for example, 43 ÷ 5 = 8 · 6. Applies the correct order of operations in number calculations when solving multi-step problems.
Multiples, Factors and Primes Pg 75	Having explored the patterns and relationships in multiplication and division, I can investigate and identify the multiples and factors of numbers. MTH 2-05a	 Identifies multiples and factors of whole numbers and applies knowledge and understanding of these when solving relevant problems in number, money and measurement.

Key Aspect	Experiences and Outcomes	National Benchmarks
Fractions, Decimals and Percentages Pg's 76-78	I have investigated the everyday contexts in which simple fractions, percentages or decimal fractions are used and can carry out the necessary calculations to solve related problems. MNU 2-07a I can show the equivalent forms of simple fractions, decimal fractions and percentages and can choose my preferred form when solving a problem, explaining my choice of method. MNU 2-07b I have investigated how a set of equivalent fractions can be created, understanding the meaning of simplest form, and can apply my knowledge to compare and order the most commonly used fractions. MTH 2-07c	 Uses knowledge of equivalent forms of common fractions, decimal fractions and percentages, for example, ³/₄ = 0.75 = 75%, to solve problems. Calculates simple percentages of a quantity, and uses this knowledge to solve problems in everyday contexts, for example, calculates the sale price of an item with a discount of 15%. Calculates simple fractions of a quantity and uses this knowledge to solve problems, for example, find ³/₅ of 60. Creates equivalent fractions and uses this knowledge to put a set of most commonly used fractions in order. Expresses fractions in their simplest form.

Key Aspect	Experiences and Outcomes	National Benchmarks
Money Pg 79	I can manage money, compare costs from different retailers, and determine what I can afford to buy. MNU 2-09a I understand the costs, benefits and risks of using bank cards to purchase goods or obtain cash and realise that budgeting is important. MNU 2-09b I can use the terms profit and loss in buying and selling activities and can make simple calculations for this.	 Carries out money calculations involving the four operations. Compares costs and determines affordability within a given budget. Demonstrates understanding of the benefits and risks of using bank cards and digital technologies. Calculates profit and loss accurately, for example, when working with a budget or an enterprise activity.
Time Pg 80	I can use and interpret electronic and paper-based timetables and schedules to plan events and activities, and make time calculations as part of my planning. MNU 2-10a I can carry out practical tasks and investigations involving timed events and can explain which unit of time would be most appropriate to use. MNU 2-10b	 Reads and records time in both 12 hour and 24 hour notation and converts between the two. Knows the relationships between commonly used units of time and carries out simple conversion calculations, for example, changes 1 ³/₄ hours into minutes. Uses and interprets a range of electronic and paper-based timetables and calendars to plan events or activities and solve real life problems. Calculates durations of activities and events including situations bridging across several hours and parts of hours using both 12 hour clock and 24 hour notation. Estimates the duration of a journey based on knowledge of the link

Key Aspect	Experiences and Outcomes	National Benchmarks
	Using simple time periods, I can give a good estimate of how long a journey should take, based on my knowledge of the link between time, speed and distance. MNU 2-10c	 between speed, distance and time. Chooses the most appropriate timing device in practical situations and records using relevant units, including hundredths of a second. Selects the most appropriate unit of time for a given task and justifies choice.
Measurement Pg's 81 & 82	I can use my knowledge of the sizes of familiar objects or places to assist me when making an estimate of measure. MNU 2-11a I can use the common units of measure, convert between related units of the metric system and carry out calculations when solving problems. MNU 2-11b	 Uses the comparative size of familiar objects to make reasonable estimations of length, mass, area and capacity. Estimates to the nearest appropriate unit, then measures accurately: length, height and distance in millimetres (mm), centimetres (cm), metres (m) and kilometres (km); mass in grams (g) and kilograms (kg); and capacity in millilitres (ml) and litres (l). Calculates the perimeter of simple straight sided 2D shapes in millimetres (mm), centimetres (cm) and metres (m). Calculates the area of squares, rectangles and right-angled triangles in square millimetres (mm2), square centimetres (cm2) and square metres (m2). Calculates the volume of cubes and cuboids in cubic centimetres (cm3) and cubic metres (m3). Converts between common units of measurement using decimal notation, for example, 550 cm = 555 m; 2:000 ka = 3000 c

Key Aspect	Experiences and Outcomes	National Benchmarks
	I can explain how different methods can be used to find the perimeter and area of a simple 2D shape or volume of a simple 3D object. MNU 2-11c	 Chooses the most appropriate measuring device for a given task and carries out the required calculation, recording results in the correct unit. Reads a variety of scales accurately. Draws squares and rectangles accurately with a given perimeter or area. Demonstrates understanding of the conservation of measurement, for example, draw three different rectangles each with an area of 24 cm2. Shows awareness of imperial units used in everyday life, for example, miles or stones.
Patterns and Relationships Pg 84	Having explored more complex number sequences, including well- known named number patterns, I can explain the rule used to generate the sequence, and apply it to extend the pattern. MTH 2-13a	 Explains and uses a rule to extend well known number sequences including square numbers, triangular numbers and Fibonacci sequence. Applies knowledge of multiples, square numbers and triangular numbers to generate number patterns.

Key Aspect	Experiences and Outcomes	National Benchmarks
Mathematics - its impact on the world, past, present and future Pg 83	I have worked with others to explore, and present our findings on, how mathematics impacts on the world and the important part it has played in advances and inventions. MTH 2-12a	 Researches and presents examples of the impact mathematics has in the world of life and work. Contributes to discussions and activities on the role of mathematics in the creation of important inventions, now and in the past.
Expressions and Equations Pg 85	I can apply my knowledge of number facts to solve problems where an unknown value is represented by a symbol or letter. MTH 2-15a	 Solves simple algebraic equations with one variable, for example, a - 30 = 40 and 4b = 20.

Third Level Experie	ences and Outcomes
* The numeracy learner statements that are th	ne responsibility of all are shown in bold italics.
I can round a number using an appropriate degree of accuracy,	I can solve problems by carrying out calculations with a wide range
having taken into account the context of the problem.	of fractions, decimal fractions and percentages, using my answers
MNU 3-01a Pg 98	to make comparisons and informed choices for real-life situations. MNU 3-07a Pg's 103-106
I can use a variety of methods to solve number problems in familiar	
contexts, clearly communicating my processes and solutions.	By applying my knowledge of equivalent fractions and common
MNU 3-03a Pg 99	multiples, I can add and subtract commonly used fractions.
	МТН 3-07Ь Рд 103
I can continue to recall number facts quickly and use them	
accurately when making calculations.	Having used practical, pictorial and written methods to develop my
MNU 3-036 Pg's 99 & 100	understanding, I can convert between whole or mixed numbers and fractions.
I can use my understandina of numbers less than zero to solve	MTH 3-07c Pa 103
simple problems in context.	
MNU 3-04a Pg 100	I can show how quantities that are related can be increased or
These increasing the detector is a few identifying common multipleater d	aecreased proportionally and apply this to solve problems in
I have investigated strategies for identifying common multiples and	everyddy contexts.
common factors, explaining my laeas to others, and can apply my	MNU 3-08a Pg 107
understanding to solve related problems.	· · · · · · · · ·
MTH 3-05a Pg 101	When considering how to spend my money, I can source, compare and contrast different contracts and services, discuss their
I can apply my understanding of factors to investigate and identify when	advantages and disadvantages, and explain which offer best value
a number is prime.	to me.
MTH 3-05b Pg 101	MNU 3-09a <u>P</u> g 108
Having explored the notation and vocabulary associated with whole	I can budget effectively, making use of technology and other
number powers and the advantages of writing numbers in this form, I	methods, to manage money and plan for future expenses.
can evaluate powers of whole numbers mentally or using technology. MTH 3-06a Pg 102	MNU 3-096 Pg 108



Points to Consider

When working at Third Level teachers are advised to refer to 'Points to Consider' and 'Suggested Written Recording' at Second Level to ensure consistency.

Pupils should continue to build upon the language used in earlier • To support understanding of the relationship between fractions and decimal fractions it can be useful to display levels to describe numeracy concepts to ensure consistency e.g. place value charts that show tenths, hundredths e.g. include use partitioning instead of splitting, negative instead of minus, simplify instead of reduce, evaluate instead of solve, improper 10' 100 fractions instead of vulgar fractions. Like terms should be gathered together in equations and • Estimation should permeate through all subject areas before expressions. calculations are attempted. Opportunities should be provided for all learners to develop estimation skills as often as possible. The use of brackets within operations is critical and time must • Encouragement should be given to ensure that this is an automatic be taken to ensure pupils have had many opportunities to process for learners. explore their use. Pupils should **experience challenge** through applying their mathematical knowledge in a variety of contexts across the curriculum. • Pupils are continuing to develop their understanding of the relationships between fractions, decimal fractions and percentages knowledge. Every opportunity should be maximised to reinforce these links

Suggested Written Recording

Standard written recording that prepares pupils for the expectations of national examinations should be encouraged. When working at Third Level teachers are advised to refer to 'Points to Consider' and 'Suggested Written Recording' at Second Level to		
ensure cons	sistency.	
• When working with fractions answers should always be simplified e.g. $2/8 \times 2 = 4/8 = \frac{1}{2}$	 Answer should be given to the same number of decimal places as the other numbers in the calculations, unless prompted differently in the question. 	
 Answers should be presented in the same format as the question e.g. if there are mixed numbers in the questions, then the answer should be recorded as mixed numbers. 	 Pairs of factors should always be listed with the lowest factor first e.g. (1, 32) (2, 16) (4, 8). 	

ESTIMATION AIM: Anticipate whether an indicated change to a collection or quantity will make it bigger, smal same. MNU 3-01a

Develop use of language used from maximum & minimum to upper and lower bounds/limits.

e.g. $3 \le x \le 8$. So the upper bound is 8 and the lower bound is 3.

When completing a calculation, know what the maximum and minimum answers to expect might be.

e.g. Calculating the missing angle in a triangle, knows answer cannot be greater than 180 and using the other two given angle sizes know what the minimum size might be.

Show awareness of rounding too early leading to inaccuracies.

Rounding 100m sprint times to nearest 10 seconds would have multiple identical times.

Explore calculator interpretations of numbers in rounding situations. e.g. when to use brackets on a scientific calculator for 194 + 866

122 + 90

Round numbers to at least 3 decimal places.

> Demonstrate understanding of rounding rules (include multiple decimal places).

What is 1 485,531to the nearest 100? 1 485.531 is between 1 400 and 1 500 but is closer to 1500 so round up.

What is 1 485.531 to the nearest tenth? 1 485,531 is between 1 485.5 and 1 485.6 but is closer to 1485.5 so round down.

Show awareness of the danger of rounding through the stages of a calculation. *e.g.* know why when using 33 1/3 % of a number on a calculator, we ensure that a third is calculated rather than 0.3.

Round appropriately in context, justifying decision.

e.g. 99.5% in an exam gives a false impression if rounded up.

Finding the mean shoe size where the mean is calculated at 7.2 the average shoe size would actually be 7.

> Show awareness of accuracy with an answer (3.58 and 3.63 both round to 3.6)

e.q. when measuring a length, rounding 3.63 to 3.6 has consequences for what would fit into a space.

Use rounding strategies to calculate approximate prices.

e.g. 28 packets of pens at £7 each is 30 x 7 ≈ £210

Explore multiplication and division: whole numbers by decimals and fractions <1.

e.g. 5 x 0.3 and 5 x 1.3

Х

49.2m²

bigger, smaller or leave it the	
Know the expected reasonableness o answer.	f an
e.g. that a missing side on a rectain must be less than 13.3 when complet calculation including an area of 49.4	ngle ^r ing a 2m²
13.3m	

Link and reinforce relationship between fractions, decimals and percentages.

e.g. 50% = $\frac{1}{2}$ = 0.5

PLACE VALUE AIM: Demonstrate an understanding of numbers less than zero to solve simple problems in context. Use understanding of place value to solve mental problems involving the four operations. MNU 3-07a & MNU 3-04a



Relate multiplication of decimals to fractions.

e,q. 0.2 x 0.6 as 2/10 x 6/10.

Use knowledge of multiplicative relationships to mentally multiply and divide by multiples of 10, 100 and 1000.

Understand that with different rules and different starting points, there can be many possible different linear patterns of numbers.

e.g. 64, 32, 16, 8.....

4, 2, 2, 10.

CALCULATING AIM: Solve problems by identifying and applying efficient strategies and communicates both processes and MNU 3-03a, MNU 3-03b & MNU 3-04a solutions.









FRACTIONS AIM: Solve problems by flexibly using knowledge of fractions.





Understand that a decimal fraction can be shown in different ways.


FRACTIONS, DECIMAL FRACTIONS & PERCENTAGES AIM: Solve problems involving a wide range of decimal fractions, fractions and percentage calculations in real life situations. MNU 3-07a

Select the most effective strategy to calculate more complex percentages of a given amount.

e.g. Find 23% by finding 10% and 1% first and calculate using the answers from these.

Use fractions, decimal fractions and percentages interchangeably in real life situations.

e.g.

Shopping scenarios - 20% off, buy 1 get 1 half price, $\frac{1}{3}$ extra free

Bank scenarios – 10% interest

Be able to choose which representation is easiest to use in calculations. e.g.

$$40\% \text{ of } \pounds 5 \text{ or } \frac{2}{5} \text{ of } \pounds 5.00 \text{ or } 10\% \times 4$$

Investigate the relationship between fraction notation and decimal fraction notation using materials.



Understand the relationship between fractions, decimal fractions and percentages. e.g. $\frac{51}{100} = 0.51 = 51\%$ $\frac{27}{50} = 0.54 = 54\%$

problems.





MONEY AIM: Use understanding of effective budgeting, managing money and financial services to make financial decisions using technology if appropriate. MNU 3-09a & MNU 3-09b



TIME AIM: Use the relationships between the distance of a journey, the time taken and the speed, to calculate distances, times and average speeds. MNU 3-10a



In a problem where the distance, time & speed is given identify the correct formula to use.

> Given the distance, time & speed in a problem use a formula to show an average speed, time & distance.

> > е.g.

Distance = Speed x Time

Distance Time= Speed

Distance Speed= Time

Be able to provide an approximate estimate, before calculating, for an answer to a problem that makes sense.

e.g. that if a person walks 5 mph then it will not take them 7 hours to walk 2 miles. MNU 3-11a & MTH 3-11b to appropriate units, rounding as necessary.



AREA, PERIMETER AND VOLUME AIM: Select, apply and interpret appropriate formula to solve problems involving area, perimeter and volume. MNU 3-11a & MTH 3-11b





Explore the relationship between formula and known understanding of area, perimeter and volume.

MATHEMATICS - ITS IMPACT ON THE WORLD, PAST, PRESENT, FUTURE AIM: Create and deliver a presentation with others detailing the impact that mathematics has had or is having on the world. Explain the impact that mathematicians have had on the world. MTH 3-12a



There is a great deal of opportunity for learner choice and the suggestions above should not be seen as definitive.

Explore a mathematical area of personal interest.

e.g. Fuzzy logic

PATTERNS AND RELATIONSHIPS AIM: Identify and generate the rule for the 'nth' term.

MTH 3-13a





EXPRESSIONS AND EQUATIONS AIM: From information contained in a diagram, problem or statement create and evaluate a simple MTH 3-14a, MTH 3-15a & MTH 3-15b formula.



Construct an equation from a word problem.

e.g. I'm thinking of a number, I add 7 and I now have 11. What number did I start with? x + 7 = 11

Create a simple linear formula representing information contained in a diagram, problem or investigation.

From a word problem, create an appropriate table to investigate using different numbers to determine the rule.

e.g. if a pencil costs x pence what would 4 cost?

> Understand from a context that tables can be used to create a formula.

> > e.q.

Days	1	2	3	4	17
(d)					
Costs	20	40	60	80	?
©					

c = 20d

Third Level Overview

* The numeracy learner statements that are the responsibility of all are shown in italics.

Key Aspect	Experiences and Outcomes	National Benchmarks
Estimation and Rounding Pg 98	I can round a number using an appropriate degree of accuracy, having taken into account the context of the problem. MNU 3-01a	 Rounds decimal fractions to three decimal places. Uses rounding to routinely estimate the answers to calculations.
Place Value Pg 99	I can solve problems by carrying out calculations with a wide range of fractions, decimal fractions and percentages, using my answers to make comparisons and informed choices for real-life situations. MNU 3-07a	
Calculating Pg 100	I can use a variety of methods to solve number problems in familiar contexts, clearly communicating my processes and solutions. MNU 3-03a I can continue to recall number facts quickly and use them accurately when making calculations. MNU 3-03b	 Recalls quickly multiplication and division facts to the 10th multiplication table. Uses multiplication and division facts to the 12th multiplication table. Solves addition and subtraction problems working with whole numbers and decimal fractions to three decimal places. Solves addition and subtraction problems working with integers. Solves multiplication and division problems working with whole numbers and decimal fractions to three decimal places. Solves multiplication and division problems working with integers. Solves multiplication and division problems working with integers. Solves multiplication and division problems working with integers.

Key Aspect	Experiences and Outcomes	National Benchmarks
	I can use my understanding of numbers less than zero to solve simple problems in context. MNU 3-04a	
Multiples, Factors and Primes Pg 101	I have investigated strategies for identifying common multiples and common factors, explaining my ideas to others, and can apply my understanding to solve related problems. MTH 3-05a I can apply my understanding of factors to investigate and identify when a number is prime. MTH 3-05b	 Identifies common multiples, including the lowest common multiple for whole numbers and can explain method used. Identifies common factors, including the highest common factor for whole numbers and can explain method used. Identifies prime numbers to 100 and can explain method used. Solves problems using multiples and factors. Writes a given number as a product of its prime factors.
Powers and Roots Pg 102	Having explored the notation and vocabulary associated with whole number powers and the advantages of writing numbers in this form, I can evaluate powers of whole numbers mentally or using technology. MTH 3-06a	 Explains the notation and uses associated vocabulary appropriately, for example, index, exponent and power. Evaluates whole number powers, for example, 24 = 16. Expresses whole numbers as powers, for example, 27 = 33.

Key Aspect	Experiences and Outcomes	National Benchmarks
Fractions, Decimals and Percentages Pg's 103-106	I can solve problems by carrying out calculations with a wide range of fractions, decimal fractions and percentages, using my answers to make comparisons and informed choices for real-life situations. MNU 3-07a By applying my knowledge of equivalent fractions and common multiples, I can add and subtract commonly used fractions. MTH 3-07b Having used practical, pictorial and written methods to develop my understanding, I can convert between whole or mixed numbers and fractions.	 Converts fractions, decimal fractions or percentages into equivalent fractions, decimal fractions or percentages. Adds and subtracts whole numbers and fractions, including when changing a denominator. Converts between whole or mixed numbers, improper fractions and decimal fractions. Uses knowledge of fractions, decimal fractions and percentages to carry out calculations with and without a calculator.
Ratio & Proportion Pg 107	I can show how quantities that are related can be increased or decreased proportionally and apply this to solve problems in everyday contexts. MNU 3-08a	 Solves problems in which related quantities are increased or decreased proportionally. Expresses quantities as a ratio and where appropriate simplifies, for example, 'if there are 6 teachers and 60 children in a school find the ratio of the number of teachers to the total amount of teachers and children'.

Key Aspect	Experiences and Outcomes	National Benchmarks
Money Pg 108	When considering how to spend my money, I can source, compare and contrast different contracts and services, discuss their advantages 	 Demonstrates understanding of best value in relation to contracts and services when comparing products. Chooses the best value for their personal situation and justifies choices. Budgets effectively, using digital technology where appropriate, showing development of financial capability. Demonstrates knowledge of financial terms, for example, debit/credit, APR, pa, direct debit/standing order and interest rate. Converts between different currencies.
Time Pg 109	Using simple time periods, I can work out how long a journey will take, the speed travelled at or distance covered, using my knowledge of the link between time, speed and distance. MNU 3-10a	 Applies knowledge of the relationship between speed, distance and time to find each of the three variables. Calculates time durations across hours and days.

Key Aspect	Experiences and Outcomes	National Benchmarks
Measurement Pg's 110 & 111	I can solve practical problems by applying my knowledge of measure, choosing the appropriate units and degree of accuracy for the task and using a formula to calculate area or volume when required. MNU 3-11a	 Chooses appropriate units for length, area and volume when solving practical problems. Converts between standard units to three decimal places and applies this when solving calculations of length, capacity, volume and area. Calculates the area of a 2D shape where the units are inconsistent. Finds the area of compound 2D shapes constructed from squares, rectangles and triangles. Finds the volume of compound 3D objects constructed from cubes and cuboids.
Mathematics - its impact on the world, past, present and future Pg 112	I have worked with others to research a famous mathematician and the work they are known for, or investigated a mathematical topic, and prepared and delivered a short presentation. MTH 3-12a	 Researches and communicates using appropriate mathematical vocabulary and notation, the work of a famous mathematician or a mathematical topic and explains the relevance and impact they have on society.

Key Aspect	Experiences and Outcomes	National Benchmarks
Patterns and Relationships Pg 113	Having explored number sequences, I can establish the set of numbers generated by a given rule for a given sequence, expressing it using appropriate notation. MTH 3-13a	 Generates number sequences from a given rule, for example, T = 4n + 6. Extends a given pattern and describes the rule. Expresses sequence rules in algebraic notation, for example, the cost of hiring a car is £75 plus a charge of £0.05 per mile, 'm' driven, C = 0.05m + 75.
Expressions and Equations Pg 114	 Having discussed ways to express problems or statements using mathematical language, I can construct, and use appropriate methods to solve a range of simple equations. MTH 3-15a I can create and evaluate a simple formula representing information contained in a diagram, problem or statement. MTH 3-15b 	 Collects like terms, including squared terms, to simplify an algebraic expression. Evaluates expressions involving two variables using both positive and negative numbers. Solves linear equations, for example, ax ± b = c where a, b and c are integers. Creates a simple linear formula representing information contained in a diagram, problem or statement. Evaluates a simple formula, for example, C = 0.05m + 75.

Fourth Level Experiences and Outcomes

* The numeracy learner statements that are the responsibility of all are shown in bold italics.

Having investigated the practical impact of inaccuracy and error, I can use my knowledge of tolerance when choosing the required degree of accuracy to make real-life calculations. MNU 4-01a Pg 125

Having recognised similarities between new problems and problems I have solved before, I can carry out the necessary calculations to solve problems set in unfamiliar contexts.

MNU 4-03a Pg 126

I have investigated how introducing brackets to an expression can change the emphasis and can demonstrate my understanding by using the correct order of operations when carrying out calculations. MTH 4-03b Pg 126

I have developed my understanding of the relationship between powers and roots and can carry out calculations mentally or using technology to evaluate whole number powers and roots, of any appropriate number. MTH 4-06a Pg 127

Within real-life contexts, I can use scientific notation to express large or small numbers in a more efficient way and can understand and work with numbers written in this form.

MTH 4-06b Pg 127

I can choose the most appropriate form of fractions, decimal fractions and percentages to use when making calculations mentally, in written form or using technology, then use my solutions to make comparisons, decisions and choices.

MNU 4-07a Pgs 128 - 130

I can solve problems involving fractions and mixed numbers in context, using addition, subtraction or multiplication. MTH 4-07b Pg 128

Using proportion, I can calculate the change in one quantity caused by a change in a related quantity and solve real-life problems. MNU 4-08a Pg 131

I can discuss and illustrate the facts I need to consider when determining what I can afford, in order to manage credit and debt and lead a responsible lifestyle. MNU 4-09a Pa 132

I can source information on earnings and deductions and use it when making calculations to determine net income. MNU 4-09b Pg 132

I can research, compare and contrast a range of personal finance products and, after making calculations, explain my preferred choices. MNU 4-09c Pg 132

I can research, compare and contrast aspects of time and time management as they impact on me. MNU 4-10a Pa 133

I can use the link between time, speed and distance to carry out related calculations.

MNU 4-10b Pg 133

I can apply my knowledge and understanding of measure to everyday problems and tasks and appreciate the practical importance of accuracy when making calculations.

MNU 4-11a Pg 134

Through investigating real-life problems involving the surface area of simple 3D shapes, I can explore ways to make the most efficient use of materials and carry out the necessary calculations to solve related problems. MTH 4-11b Pg 134

I have explored with others the practicalities of the use of 3D objects in everyday life and can solve problems involving the volume of a prism, using a formula to make related calculations when required. MTH 4-11c Pg 134

I have discussed the importance of mathematics in the real world, investigated the mathematical skills required for different career paths and delivered, with others, a presentation on how mathematics can be applied in the workplace.

MTH 4-12a Pg 135

Having explored how real-life situations can be modelled by number patterns, I can establish a number sequence to represent a physical or pictorial pattern, determine a general formula to describe the sequence, then use it to make evaluations and solve related problems.

MTH 4-13a Pgs 136 & 137

I have discussed ways to describe the slope of a line, can interpret the definition of gradient and can use it to make relevant calculations, interpreting my answer for the context of the problem. MTH 4-13b Pgs 136 & 137 Having investigated the pattern of the coordinate points lying on a horizontal or vertical line, I can describe the pattern using a simple equation.

MTH 4-13c Pgs 136 & 137

I can use a given formula to generate points lying on a straight line, plot them to create a graphical representation then use this to answer related questions. MTH 4-13d Pgs 136 & 137

Having explored the distributive law in practical contexts, I can simplify, multiply and evaluate simple algebraic terms involving a bracket. MTH 4-14a Pg 138

I can find the factors of algebraic terms, use my understanding to identify common factors and apply this to factorise expressions. MTH 4-14b Pg 138

Having discussed the benefits of using mathematics to model real-life situations, I can construct and solve inequalities and an extended range of equations. MTH 4-15a Pg 138

Points to Consider

When working at Fourth Level teachers are advised to refer to 'Points to Consider' and 'Suggested Written Recording' at Third Level to ensure consistency.

- Pupils should continue to build upon the language used in earlier levels to describe numeracy concepts to **ensure consistency** e.g. use partitioning instead of splitting, negative instead of minus, simplify instead of reduce, evaluate instead of solve, improper fractions instead of vulgar fractions., BIDMAS instead of BODMAS to introduce the term indices.
- Estimation should permeate through all subject areas before calculations are attempted. Opportunities should be provided for all learners to develop estimation skills as often as possible. Encouragement should be given to ensure that this is an automatic process for learners.
- Pupils should **experience challenge** through applying their mathematical knowledge in a variety of contexts across the curriculum.
- Pupils are continuing to develop their understanding of the relationships between fractions, decimal fractions and percentages knowledge. Every opportunity should be maximised to **reinforce these links**.

- To support understanding of the relationship between fractions and decimal fractions it can be useful to display place value charts that show tenths, hundredths e.g. include $\frac{1}{10}$, $\frac{1}{100}$
- Like terms should be gathered together in equations and expressions.
- The use of brackets within operations is critical and time must be taken to ensure pupils have had many opportunities to explore their use.
- Straight line equations should be written in the form y=mx + c.

Suggested Written Recording

Standard written recording that prepares pupils for the expectations of national examinations should be encouraged.		
When working at Forth Level teachers are advised to refer to 'Points to Consider' and 'Suggested Written Recording' at Third Level to ensure consistency.		
• When working with fractions answers should always be simplified e.g. $2/8 \times 2 = 4/8 = \frac{1}{2}$	 Answer should be given to the same number of decimal places as the other numbers in the calculations, unless prompted differently in the question. 	
 Answers should be presented in the same format as the question 		
e.g. if there are mixed numbers in the questions, then the answer should be recorded as mixed numbers.	 Pairs of factors should always be listed with the lowest factor first e.g. (1, 32) (2, 16) (4, 8). 	

ESTIMATION & ROUNDING AIM: Work with tolerance to discuss, explain and solve problems involving the acceptable range of values in which a measurement will be acceptable. MNU 4-01a

Uses tolerance to choose the most appropriate degree of accuracy for real-life calculations, selects and communicates processes and solutions.

Have an understanding of the level of tolerance that would be acceptable in different situations.

National 5 Link ~ Rounding to a given number of significant figures.

When working with decimal numbers, explain your methods and answers using the vocabulary of estimation and rounding including upper and lower bounds.

Express required accuracy using tolerance notation.

Realise that different degrees of accuracy are acceptable in different situations.

Interpret measurements given in tolerance rotation including percentage limits.

Justify whether a measurement meets the required tolerance.

Uses a given tolerance to decide if there is an allowable amount of variation of a specified quantity, for example, dimensions of a machine part.



Apply knowledge of order of operations in new and unfamiliar contexts with and without a calculator.

Use BIDMAS to calculate multi-step calculations including those with brackets.

e.g. (5 x 3) + (24 ÷ 6)

Explore rules for the order of operations in number calculations such as BIDMAS.







e as a
nk ~ Compound interest

percentage and

Use knowledge of percentages to solve problems involving percentages in context.

Calculate percentages in a variety of ways including using technology.

FRCATIONS, DECIMALS, PERCENTAGES AIM: Chooses the most appropriate form of fractions, decimal fractions or percentages when solving problems and can justify choice. MNU 4-07a



PROPORTION AIM: Solve problems involving proportion in real-life situations. MNU 4-08a



between direct and inverse proportion.



Use numerical methods to illustrate direct and inverse proportion.

Earnings & Hours Worked * 4 you work 2 hours you get paid \$14 * 14 you work 3 hours you get paid \$21			
	Number of Loaders	Hours Taken	
	2	4	
	4	2	
	8	1	

MONEY AIM: Use understanding of credit, debit, income, effective budgeting and financial products to make appropriate financial decisions. MNU 4-09a, MNU 4-09b, MNU 4-09c



Applies knowledge of currency conversion to determine best value.

TIME AIM: Use the relationship between distance, speed and time to carry out calculation involving decimal and decimal fraction hours. MNU 4-10a, MNU 4-10b

Carries out calculations involving speed, distance and time involving decimal and decimal fraction hours.

e.g. An iceberg and a ship separated by 440 kilometres are moving directly towards each other. The iceberg is floating at 2 kilometres per hour, whereas the ship is travelling at 27 kilometres per hour. Assuming that neither changes course, how long will it be before the two collide?

Identify and apply the correct formula in a variety of situations.

Calculates time durations across hours, days and months.

Convert hours and minutes to a decimal fraction and vice versa with a calculator if necessary.





MNU 4-11a, MTH 4-11b, MTH 4-11c



Fife Numeracy & Mathematics Progression Pyramid: Fourth Level ~ Page 134

~ Round ber of gures.
estanding of the a and premature ag.
nd justify the level / required to solve real life problems g measurement.
surements could be culate.
ll measurements in
nd understanding that units of measurement will nore accurate answers.







Fife Numeracy & Mathematics Progression Pyramid: Fourth Level ~ Page 136



Fife Numeracy & Mathematics Progression Pyramid: Fourth Level ~ Page 137

Describe a gradient as positive, negative, zero or undefined.

> Recognise that there are other types of sequences *e.g. non-linear sequences.*



Fourth Level Overview

* The numeracy learner statements that are the responsibility of all are shown in italics.

Key Aspect	Experiences and Outcomes	National Benchmarks
Estimation and Rounding Pg 125	Having investigated the practical impact of inaccuracy and error, I can use my knowledge of tolerance when choosing the required degree of accuracy to make real-life calculations. MNU 4-01a	 Rounds answers to a specified significant figure. Demonstrates that the context of the question needs to be considered when rounding. Demonstrates the impact of inaccuracy and error, for example, the impact of rounding an answer before the final step in a multi-step calculation. Uses a given tolerance to decide if there is an allowable amount of variation of a specified quantity, for example, dimensions of a machine part, 235 mm ± 1 mm.
Number and number processes Pg 126	Having recognised similarities between new problems and problems I have solved before, I can carry out the necessary calculations to solve problems set in unfamiliar contexts. MNU 4-03a	 Interprets and solves multi-step problems using the four operations. Applies the correct order of operations in all calculations, including those with brackets.
	I have investigated how introducing brackets to an expression can change the emphasis and can demonstrate my understanding by using the correct order of operations when carrying out calculations. MTH 4-03b	

Key Aspect	Experiences and Outcomes	National Benchmarks
Powers and Roots Pg 127	I have developed my understanding of the relationship between powers and roots and can carry out calculations mentally or using technology to evaluate whole number powers and roots, of any appropriate number. MTH 4-06a	 Shows understanding that square roots of whole numbers can have positive and negative values, for example, √9 = ±3 Uses knowledge of the inverse relationship between powers and roots to evaluate whole number roots of any appropriate number, ³√27 = 3. Uses knowledge of mathematical notation to express numbers in scientific notation.
	Within real-life contexts, I can use scientific notation to express large or small numbers in a more efficient way and can understand and work with numbers written in this form. MTH 4-06b	
Fractions, Decimals and Percentages Pgs 128 -131	I can choose the most appropriate form of fractions, decimal fractions and percentages to use when making calculations mentally, in written form or using technology, then use my solutions to make comparisons, decisions and choices. MNU 4-07a	 Chooses the most efficient form of fractions, decimal fractions or percentages when making calculations. Uses calculations to support comparisons, decisions and choices. Calculates the percentage increase or decrease of a value. Applies addition, subtraction and multiplication skills to solve problems involving fractions and mixed numbers. Uses knowledge of proportion to solve problems in real-life which involve changes in related quantities.

Key Aspect	Experiences and Outcomes	National Benchmarks
	I can solve problems involving fractions and mixed numbers in context, using addition, subtraction or multiplication. MTH 4-07b	
	Using proportion, I can calculate the change in one quantity caused by a change in a related quantity and solve real-life problems. MNU 4-08a	
Money Pg 132	I can discuss and illustrate the facts I need to consider when determining what I can afford, in order to manage credit and debt and lead a responsible lifestyle. MNU 4-09a I can source information on earnings and deductions and use it when making calculations to determine net income. MNU 4-09b	 Applies understanding of credit and debit in relation to earnings and deductions. Uses budgeting skills to manage income effectively and justifies spending and saving choices. Calculates net income by selecting appropriate information. Compares a range of personal finance products. Communicates the impact of financial decisions. Applies knowledge of currency conversion to determine best value.
Key Aspect	Experiences and Outcomes	National Benchmarks
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Time	I can research, compare and contrast a range of personal finance products and, after making calculations, explain my preferred choices. <u>MNU 4-09c</u> I can research, compare and	• Demonstrates effective time management skills, for example, working with
Pg 133	contrast aspects of time and time management as they impact on me. MNU 4-10a	 different time zones or making plans, including across midnight. Carries out calculations involving speed, distance and time involving decimal fraction hours. Calculates time durations across hours, days and months.
	I can use the link between time, speed and distance to carry out related calculations. MNU 4-10b	
Measurement Pg 134	I can apply my knowledge and understanding of measure to everyday problems and tasks and appreciate the practical importance of accuracy when making calculations. MNU 4-11a	 Demonstrates understanding of the impact of truncation and premature rounding. Calculates the area of kites, parallelograms and trapeziums. Uses formulae and calculates the surface area of cylinders, cuboids and triangular prisms. Calculates the volume of triangular prisms and cylinders using formulae.

Key Aspect	Experiences and Outcomes	National Benchmarks
Mathematics - its impact on the world, past, present and future	Through investigating real-life problems involving the surface area of simple 3D shapes, I can explore ways to make the most efficient use of materials and carry out the necessary calculations to solve related problems. <i>MTH</i> 4-11b I have explored with others the practicalities of the use of 3D objects in everyday life and can solve problems involving the volume of a prism, using a formula to make related calculations when required. <i>MTH</i> 4-11c I have discussed the importance of mathematics in the real world, investigated the mathematical skills required for different career paths and delivered, with others, a	 Contributes to discussions and presentations on the role of mathematics in everyday life and in the workplace. Investigates the mathematical skills required for a range of careers, including those
Pg 135	can be applied in the workplace. MTH 4-12a	IN STEM SUBJECTS.

Key Aspect	Experiences and Outcomes	National Benchmarks
Patterns and Relationships Pgs 136 & 137	Having explored how real-life situations can be modelled by number patterns, I can establish a number sequence to represent a physical or pictorial pattern, determine a general formula to describe the sequence, then use it to make evaluations and solve related problems.	 Determines a general formula for the nth term to describe a sequence and uses it to solve related problems, linear examples only. Calculates the gradient of lines in a coordinate diagram. Draws conclusions about the gradient of a line, for example, 'does the ramp meet building regulations?'. Communicates the gradient of vertical and horizontal lines and states the equation of these lines as x = a or y = b or equivalent. Uses a given formula to plot a straight line onto a Cartesian diagram.
	I have discussed ways to describe the slope of a line, can interpret the definition of gradient and can use it to make relevant calculations, interpreting my answer for the context of the problem. MTH 4-13b	• Oses a given formala to plot a sit algin time onto a cal residir diagram.
	the coordinate points lying on a horizontal or vertical line, I can describe the pattern using a simple equation. MTH 4-13c	

Key Aspect	Experiences and Outcomes	National Benchmarks
	I can use a given formula to generate points lying on a straight line, plot them to create a graphical representation then use this to answer related questions. MTH 4-13d	
Expressions and Equations Pg 138	Having explored the distributive law in practical contexts, I can simplify, multiply and evaluate simple algebraic terms involving a bracket. MTH 4-14a I can find the factors of algebraic terms, use my understanding to identify common factors and apply this to factorise expressions. MTH 4-14b	 Expands brackets using the distributive law and simplifies. Solves an extended range of linear equations involving the distributive law, for example, ax ± b = cx ± d, where a, b, c and d are integers. Solves linear inequalities, including on simple closed intervals. Solves problems by expressing the given information appropriately as an equation, in-equation or formula. Evaluates algebraic expressions involving a bracket. Factorises expressions with a numeric common factor.

Key Aspect	Experiences and Outcomes	National Benchmarks
co	Having discussed e benefits of using mathematics to model real-life situations, I can onstruct and solve inequalities and an extended range of equations. MTH 4-15a	

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